INSTITUTES

OF

ARITHMETIC,

ELEMENTARY AND PRACTICAL:

THE

MENSURATION OF SURFACES AND SOLIDS,

AND THE USE OF

LOGARITHMS IN ALL THE PARTS OF ARITHMETIC;

TO WHICH ARE ADDED,

TABLES OF ANNUITIES, LIVES, &c.

THE WHOLE

Designed as a Directory or Text-Book for the Use of Schools.

BY WILLIAM GORDON,

Master of the Mercantile Academy, EDINBURGH.

Longum iter est per precepta, breve et efficax per exempla. SENECA.

EDINBURGH:

Printed for W. CREECH; and Sold by G. and J. ROBINSONS,
LONDON.
M,DCC,LXXXIX.

TO THE HONOURABLE

MR. BARON GORDON,

THE FOLLOWING INSTITUTES,

CALCULATED

TO RENDER THE THEORY AND PRACTICE

OF ARITHMETIC EASY AND FAMILIAR,

IN

TESTIMONY OF HIS ATTACHMENT,

GRATITUDE, AND PARTICULAR REGARD,

ARE

MOST RESPECTFULLY DEDICATED,

BY

HIS MOST OBEDIENT,

MOST OBLIGED,

AND DEVOTED HUMBLE SERVANT,

THE AUTHOR.

Academy, Edinburgh, 20th May, 1789.

TO THE

TEACHERS OF ARITHMETIC.

THE importance of figures to this commercial ſtate is ſo well known, that a work calculated to fix the theory, render the application eaſy, and inſure accuracy and diſpatch, will require no apology. It was originally intended as a directory or text-book, for the uſe of the ſtudents in this academy: and, for that reaſon, the axioms, upon which the whole proceeds, are few and ſimple; but the problems and examples are multiplied and diverſified, to render the application eaſy, and practice familiar. The definitions and rules are ſhort, but expreſſive, and the demonſtrations plain and rational. The leading example to every rule hath the operation, and an illuſtration at large, in at leaſt two different methods, the one to ſerve as a proof of the other; after which, the anſwer immediately follows the queſtion.

As, in every mode of calculation, accuracy and diſpatch are principally to be had in view, various methods of ſolution and expediting practice have been introduced, upon which, ingenuity and invention may ſtill make further improvements, for promoting theſe great and eſſential points.

To give the Maſter an opportunity of explaining to his pupils the nature and uſe of the ſeveral varieties, that occur in counting-houſes, they are diſpoſed and arranged in that natural order, that the one may lead directly to the other.

As nothing is more irkſome to a ſtudent than tedious illuſtrations, prolixity as well as obſcurity hath been cautiouſly avoided, queſtions that have no relation to real buſineſs, as much as poſſible excluded. In a word; it is hoped the whole will be found ſo planned and conducted, as to make a rational and complete ſyſtem of arithmetic, ſufficient, if thoroughly taught, to enable a ſtudent to acquit himſelf with propriety in the counting-houſe, or to proſecute higher ſtudies with pleaſure.

If any difficulty ſhould ariſe in bringing out the anſwer to any of the queſtions, a letter, poſt paid, addreſſed to the Author, will procure an immediate ſolution at large.

Great care hath been taken to reduce the rules in menſuration to a ſhort and eaſy ſcale, and that the price of the book might not be ſwelled too high, the figures are left for the practice of the ſtudent, and the direction of the teacher.

As logarithms are particularly adapted to expedite practice, in computations in Annuities, Compound Interest, and other rules, where a deep Multiplication, Division, Evolution, &c. may be concerned, their use hath been fully explained, illustrated, and exemplified in all the parts of Arithmetic, where they can, with propriety, be applied.

CONTENTS.

A TREATISE OF ARITHMETIC.

INTRODUCTION.

ARITHMETIC is a science, explaining the powers, properties, and affections of numbers, and the art of computing by them with facility, accuracy, and dispatch.

The whole arithmetical *Alphabet* is comprehended in these ten characters, viz.

1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

One, two, three, four, five, six, seven, eight, nine, cipher. From the various combinations or repetitions of which, any number may be expressed, and numerical question resolved.

An *Unit*, or 1, is the first and simplest idea in numbers; but, by the continued addition of another unit, we may carry our ideas of numbers to any proposed length whatever; and, since an unit may be divided into any number of parts, if any part be taken away, the remainder will be a fraction, which may also be decreased at pleasure, by a continued deduction; so that, on the one hand, unity may be increased by addition of units, and on the other, decreased by a deduction of parts, to infinity. Hence an unit is the common barrier between whole numbers and fractions, from whence every increase and decrease proceeds.

Of the *Characters* in the above series, the first nine are significant, and increase in a progression of ones; but the last, or 0, has no value of its own, though it adds to the value of the other nine, by

ſetting them higher in the ſcale of numbers, as will be afterwards explained.

To ſhorten and illuſtrate the method of procedure in arithmetical operations, the following characters have been with propriety adopted:

= *Equality*, ſignifying that the amount, product, remainder, or, in one word, the reſult, however taken, on each ſide of it, are equal.

+ *Addition*, ſignifying that the numbers between which it ſtands are to be added together.

— *Subtraction*, ſignifying that the number after it muſt be taken from that before it.

× *Multiplication*, ſignifying that the numbers between which it ſtands are to be multiplied together.

÷ *Diviſion*, ſignifying that the number which precedes it is to be divided by that which follows it.

: Signifies *Proportion*, and ſtands between the firſt and ſecond term, and between the third and fourth term.

:: Signifies alſo *Proportion*, and ſtands between the ſecond and third term.

The uſe of theſe and ſome other characters will be more conveniently illuſtrated in the ſequel, where they will be reſpectively introduced, as occaſion requires.

CHAP. I.

OF THE ELEMENTS OF ARITHMETIC.

THE whole of this ſcience depends upon five rules, viz. NOTATION, ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION; upon the proper application of which, the moſt complicated proceſſes, curious calculations, and ſurpriſing ſolutions, may be effected.

SECT. I. NOTATION *and* NUMERATION, *with the Axioms deducible therefrom.*

I. NOTATION teacheth the method of writing down, in numerical characters, any number conceived, known, or propoſed.

All numbers increaſe from the right hand to the left; and, in every ſeries, each character has both a ſimple and a local value.

The ſimple value of all the characters was aſcertained in the introduction; and it is the buſineſs of this rule to explain and point out the value aſſumed by any figure from the place in which it ſtands: In order to which, obſerve the following

NOTATION TABLE.

4th Period.						3d Period.						2d Period.						1ſt Period.					
of Trillions.						of Billions.						of Millions.						of Units.					
Hundreds of thouſands	Tens of thouſands	Thouſands	Hundreds	Tens	Units	Hundreds of thouſands	Tens of thouſands	Thouſands	Hundreds	Tens	Units	Hundreds of thouſands	Tens of thouſands	Thouſands	Hundreds	Tens	Units	Hundreds of thouſands	Tens of thouſands	Thouſands	Hundreds	Tens	Units
·9	7	4,	0	1	2·	5	7	3,	8	5	6·	5	2	3,	8	7	8·	5	7	0,	1	2	3
1	0	0,	0	0	0.	1	0	0,	0	0	0.	1	0	0,	0	0	0.	1	0	0,	0	0	0

In the table, every three places, reckoning from the right hand, make just half a period; and the right hand figures of these half periods are termed *Units* and *Thousands* by turns, the middle figure always *Tens*, and the left hand figure always *Hundreds*. Every six places make a full period, distinguished by a *Point*, as every half period is distinguished by a *Comma*.

From the construction of the table it will be obvious,

1. That all numbers increase in a tenfold proportion, from the units place towards the left, *ad infinitum*.

2. That every figure in a series, besides its simple value, has a new value stamped upon it from the rank it bears in the series, called its *local* value.

3. The units place expresseth the number of ones contained in the figure, the tens place, the number of tens, &c.; hence a figure, in any superior place, has ten times the value it would have in the place immediately below it.

4. That a cipher, void as it is of any value of its own, serves to supply vacant places, and raises the value of significant figures on the left of it, in proportion to the distance they are removed from the units place.

Prob. To express, in figures, any number proposed in words.

1. Begin at the left hand and write towards the right.

2. Write every figure in the place and period which the verbal expression points out, and supply vacant places with ciphers.

EXAMPLES.

Seventy millions and seventy.	70.000,070
Seventeen billions, nineteen millions, one hundred and eighty thousands.	17.000,019.180,000
Nineteen thousand and eight millions, nine thousands and fifty-nine.	[illegible]
Twelve thousands and eleven.	[illegible]
Five billions, five millions, five thousands and five.	
Fifteen thousand millions, and fifteen.	
One hundred and eighty thousand billions, ten thousands and nine.	

II. NUMERATION is another branch of this science, which teacheth to read any number expressed by figures.

Prob. To read any series of figures.

1. Begin at the left hand, and read towards the right.

2. To the simple value of each figure join also the name of its place.

3. Conclude every period by expressing its title, excepting the first, and omit the ciphers every where.

EXAMPLES.

37,045,978,604	Thirty-seven thousands forty-five millions, nine hundreds seventy-eight thousands, six hundreds and four.
8475900785970	
59700040005701	
68004003007402	
73201010150111	
57400210901999	

III. ARITHMETICAL AXIOMS.

An *Axiom* is a first principle, or self-evident proposition, which requires no demonstration. The following axioms are necessarily deduced from the nature of numbers, the manner of their notation, or their specific properties.

1. Any number whatever may be increased or diminished at pleasure.

2. All numbers increase in value from the right to the left, in a tenfold proportion, and decrease from the left to the right, in the same proportion.

3. Ten in any inferior place make an unit in the next higher place: hence, if the right hand figure of any number be cut off, the remainder is a just number of tens, and the figure so cut off, the excess.

4. If to any number, one, two, or three ciphers be annexed, it becomes 10, 100, 1000 times greater, &c. than in its simple state.

5. Any number may be resolved into as many constituent parts as it has significant figures, by annexing to each as many ciphers as there are figures on its right.

6. The separate value of a single figure is greater by one, at least, than the value of all the figures on its right.

7. An unit is an even part of every whole number, as every whole number is a multiple of unity.

8. Numbers equally augmented, or diminiſhed, have always the ſame difference.

9. The difference of two unequal numbers added to the leſſer will be equal to the greater; and ſubtracted from the greater will be equal to the leſſer.

10. Any whole is equal to all its parts taken together.

SECT. II. ADDITION. +

ADDITION teacheth to collect ſeveral numbers of the ſame kind into one ſum, or total.

Prob. Two or more numbers given, reſpecting the ſame ſpecies of things, to find their ſum, or total.

1. Place the numbers, one below another, in ſuch order, that all the units, all the tens, all the hundreds, &c. may ſtand in their reſpective columns.

2. Begin at the units place, and proceed with the flux of numbers to the higheſt place.

3. Find the ſum of every column, but ſet down the right hand figure only, carrying the reſt as ſo many units to the next ſuperior place.

4. Under the laſt, or higheſt place, write down the whole ſum.

The reaſon of this rule depends upon axioms 3. and 10.

EXAMPLES.

	(1)		(2)		(3)
Numbers to be added.	374598	Numbers to be added.	86745976	Numbers to be added.	549786745
	647859		57849786		97685737
	786759		3745986		57867597
	879786		897456		89765978
	67845		756978		97856789
	69759		459786		97897845
	74597		97856		97897444
	2901203				

Illuſtration of the firſt example.

Beginning with the column of units $7+9+5+6+9+9+8=53$; the figure 3 then, on the right, is ſet down under units, and that on the left, 5, is added with the column of tens, thus, $5+9+5+4+8+5+5+9=50$: again, ſetting down 0, we proceed with 5 to the next column, thus, $5+5+7+8+7+7+8+5=52$: ſo

putting down 2, we proceed with 5, as before, thus, $5+4+9+7+9+6+7+4=51$; of which 1 is noted: again, $5+7+6+6+7+8+4+7=50$; of which 0 is noted: laſtly, $5+8+7+6+3=29$, which is put down altogether, and completes the ſum 2901203 for the anſwer.

397645	8740598	49786759
874987	6715973	61324352
483275	8567432	78471827
976749	9785976	32639284
678573	4327598	98764172
864798	7600597	12346939
674129	6430798	79218376
978579	7981679	31892735
864596	4730597	65438746
725978	8687471	45672365
674597	9071849	54312677
426745	8743597	56798434
967597	6873295	17819768
68973	6752137	93291343
8496	5987236	87654237
978	748379	23456874
35	67842	87538595
9	3879	23572516
	459	29743853
	87	81367259
	8	75283978
		35827133

Proof of Addition.

Add the given numbers firſt upwards and then downwards, and if both ſummations are right, the totals will agree. Or,

Divide the numbers, after they have been added, into parcels, find the ſum of each of theſe parcels, and collect theſe ſums into one total, which, if right, will agree with that found at once.

EXAMPLES.

37415		
67897		7410567836
34789		978563257
67845		86780375
———	207946	9767835
71897		867375
36789		98789
57897		6785
97657		78572
37859		374597
———	302099	87659
6745		98763
3741		4159
7059		785
6074		———
567		
———	24186	———
534231	———	
	534231	———

More examples to facilitate practice.

1. A certain squire had the following rents paid him, viz. in barley 7489 bolls, in wheat 378 bolls, in rye 1079 bolls, in pease 2847 bolls, in beans 848 bolls, in oats 3417 bolls, and in potatoes 39 bolls; required the amount? *Answer*, 16097.

2. A merchant owes to A, L. 745, to B, L. 85, to C, L. 219, to D, L. 885, to E, L. 415, to F, L. 95, to G, L. 9, to H, L. 1975, to I, L. 847, to K, L. 15, and to L, L. 74; required the amount of his debts? *Answer*, L. 5364.

3. A farmer went to market, and laid out upon oxen L. 216, upon horses L. 318, upon sheep L. 547, upon meal L. 411, upon cows L. 117, upon timber L. 57, upon labouring utensils L. 119, and upon broad cloth, linen, and expences, L. 47; required the total disbursements? *Answer*, L. 1832.

4. Received from A, L. 577, from B, L. 1187, from C, L. 193, from D, L. 845, from E, L. 937, from F, L. 397, from G, L. 3037, from H, L. 919, and from I, L. 10740; how much did I receive in all? *Answer*, L. 18832.

Sect. III. Subtraction. —

Subtraction finds the difference betwixt two similar numbers given; or, by deducing the lesser from the greater, discovers the remainder.

Prob. Two numbers being given, thence to find their difference.

1. Set the greater number uppermost, and the lesser under it, so as each figure in both, of the same local value, may stand in the same column.

2. Begin with the units place, and proceed from the right to the left with the flux of numbers.

3. When the figure in the lesser number can be subtracted from its correspondent in the greater, note down the difference.

4. If there is no difference, write down a cipher.

5. If the figure in the lesser number be of more value than its correspondent in the greater, add 10 to that in the greater mentally, subtract and note the difference; but for every 10 so added to the greater number, add 1 to the next superior figure in the lesser, before subtraction, by *Axiom* 3.

EXAMPLES.

	(1)	(2)	(3)
From	741598765401	901235678894	5637516012
Take	594174897645	374201781970	1749876925
Rem.	147423867756		

Illustration of Example 1.

Because 5 in the lesser number cannot be taken from 1 in the greater, 5 is taken from 1+10, and the remainder is 6. Then, because 10 units=one 10, 1+4 from 0+10=5. Again, 1+6 from 4+10=7; and 1+7 from 5+10=7: Then 1+9 from 10+6=6; and 1+8 from 10+7=8: 1+4 from 8=3: and, as 0 was borrowed, 9−7=2; 5−1=4: but 10+1−4+7; and 4+10−9+1=4. Lastly, 7−5+1=1; and the whole remainder is 147423867756.

Proof of Subtraction.

If the remainder be added to the leſſer number, the ſum will be equal to the greater; or ſubtracted from the greater, the remainder will be equal to the leſſer. *Axiom* 9.

EXAMPLES.

5407597856	497156437	540107659734
597856785	168473987	86748973659
4809741071		
5407597856 added		
597856785 ſubtracted.		

More examples to facilitate practice.

1. A gentleman has of yearly rent L. 1500, and pays of feu-duty L. 89; how much has he free? *Anſwer*, L. 1411.

2. What number added to one thouſand and fourteen will make eighteen hundred and one? *Anſwer*, 787.

3. A merchant, in balancing his books, finds his groſs ſubject amount to L. 11018, and his debts to L. 1319; what is his neat ſtock? *Anſwer*, L. 9699.

4. Suppoſe 5770 years have elapſed ſince the creation, and Troy deſtroyed by the Greeks 2820 years after the creation; how many years have elapſed ſince the deſtruction of Troy? *Anſwer*, 2950.

5. A city was built in the year of the Chriſtian æra 999; how old is it, this preſent year?

6. A man was born in the year 1685; how old is he at this day?

7. Troy was deſtroyed 1184 years before the birth of Chriſt, and Rome was built 748 years before that æra; how many years between the deſtruction of Troy and the building of Rome? *Anſwer*, 436.

Note. If the major, or minor, or both, be made up of partial ſums, theſe partial ſums muſt be added, previous to any ſubtraction.

EXAMPLES.

1. I owe to A, L. 977; to B, L. 415; and to C, L. 819. I have in money L. 899, and in bills almost due, L. 877; what do I require more to pay off my debts?

977	
415	
819	
	2211 *Major.*
899	
877	
	1776 *Minor.*
	435 *Answ.*

2. I went to a market, where I laid out upon goods L. 397, recovered of book debts L. 119, paid off different accounts to the amount of L. 449, sold a bill on London for L. 378, lent a friend L. 299, made ready money sales to the amount of L. 577, and expended one way with another, L. 25; after all I brought home L. 674: how much money did I carry out? *Answer*, L. 770.

3. I owe to A, L. 870, but I compound with him for L. 590; I owe to B, L. 970, but I compound with him for L. 674; I owe to C, L. 877, but I compound with him for L. 599; and I owe to D, L. 1000, but I compound with him for L. 745; what do I save by these compositions? *Answer*, L. 1109.

4. A gentleman receives of yearly rent, viz. from A, L. 137; from B, L. 299; from C, L. 300; from D, L. 877; and from E, L. 457. He allows for family expence, L. 489; servants wages, and other domestic occurrences, L. 199; to pay off tradesmens accounts, L. 218; and for his own pocket, L. 99: how much, supposing these allocations just, will he put out at interest yearly? *Answer*, L. 1065.

5. What number added to $117+577=1779-326$? *Answer*, 759.

Sect. IV. Multiplication. ×

Multiplication teacheth, in a number called the Product, to repeat a number called the Multiplicand as often as there are units in another number called the Multiplier; and hence multiplication is said to supply the place of many additions.

The first and lowest step in multiplication is to multiply one single digit by another, the result of which is called a single product, or rectangle. For this elementary step the learner must commit to memory the following Table:—

Multiplication Table.

2 times	2 is	4
	3	6
	4	8
	5	10
	6	12
	7	14
	8	16
	9	18
	10	20
	11	22
	12	24
3 times	3	9
	4	12
	5	15
	6	18
	7	21
	8	24
	9	27
	10	30
	11	33
	12	36
4 times	4	16
	5	20
	6	24
4 times	7	28
	8	32
	9	36
	10	40
	11	44
	12	48
5 times	5	25
	6	30
	7	35
	8	40
	9	45
	10	50
	11	55
	12	60
6 times	6	36
	7	42
	8	48
	9	54
	10	60
	11	66
	12	72
7 times	7	49
	8	56
	9	63
	10	70
	11	77
	12	84
8 times	8	64
	9	72
	10	80
	11	88
	12	96
9 times	9	81
	10	90
	11	99
	12	108
10 times	10	100
	11	110
	12	120
11 times	11	121
	12	132
12 times	12	144

Prob. To find the product or rectangle of any two numbers.

1. In placing the given numbers, let the major be uppermoſt, and places of the ſame local value under each other.

2. If the multiplier conſiſts but of one place, the product is found by a ſingle multiplication of all the figures in the multiplicand, one after another, into that place, from units to the higheſt, writing down and carrying all along as in addition.

3. If the multiplier conſiſts of many places, the product of the multiplicand into each of theſe muſt be taken, one after another, from the units place in courſe with the flux of numbers, and the loweſt place of each product muſt ſtand below its reſpective multiplier.

4. Add theſe partial products, and the ſum is the product required.

EXAMPLES.

	(1)	(2)	(3)	(4)
Multiplicand	57459	37456	8475937	9075674
Multiplier	9	12	45	609
Product	517131	449472	42379685	81681066
			33903748	54454044
			381417165	5527085466

Illustration of Example (4.)

$9\times4=36$, of which the right hand place is noted, and the other place added with the next product, thus, $9\times7=63+3=66$, of which 6 is noted, and 6 reserved for the succeeding product. Then $9\times6+6=60$; note 0: $9\times5+6=51$; note 1: $9\times7+5=68$; note 8: $9\times0+6=6$; note 6, and reserve nothing. Lastly, $9\times9=81$, which is noted at length, and completes the product of the units place, as nothing can be produced by multiplying into the cipher. All the figures in the multiplicand are multiplied in the same order, and in the same manner, with 6, which, as it stands in the place of hundreds, requires the first figure of its product to stand below the place of hundreds of the preceding. When these products are added together, the sum is just 609 times the given multiplicand, which is all that was required.

Proof of Multiplication.

Make that the multiplier which before was the multiplicand.

EXAMPLE.

	35
745	745
35	
	175
3725	140
2235	245
26075	26075

But the most usual method is by casting the 9s out of the multiplier and multiplicand, and placing the excesses on the right and left sides of a cross; multiply these two figures into one another, and cast the 9s out of their product, placing the excess at the top of the cross; cast the 9s also out of the product of your multiplication, and place the excess at the bottom: then will the figures at top and bottom be the same, if the work be right.—See this accounted for at the end of this chapter.

EXAMPLES.

Multiply	74598345		29601847	
into	805	0	300905	8
		4+0		8+1
	372991725	0	148009235	8
	596786760		266416623	
			88805541	
	60051667725		8907343771535	

More examples to facilitate practice.

59307 × 796	57351063 × 410597
87694 × 358	64789783 × 687547
7800 × 978	94387010 × 504789
78464 × 4207	6878459 × 674057
673000 × 8900	8734787 × 978059
900700 × 58470	9807415 × 876079
	5478109 × 418937

Contractions in Multiplication.

1. When any number of ciphers is annexed to 1 for a multiplier, the product is found by annexing the same number of ciphers to the multiplicand.

Example, 374589745 into 1000=374589745000; for to multiply by 1 produces the multiplicand in its original state, and its value is increased to the required height by annexing the ciphers.

For the same reason, ciphers on the right of either or both factors are left out in the multiplication, but affixed to the product.

EXAMPLES.

7859453000		541072600	
36000	0	5700	3
	0+5		3+7
47156718	0	37875082	3
23578359		27053630	
282940308000000		3084113820000	
3748597300		867503270	
45900		709600	

2. When the multiplier consists of a series of 9s, annex a series of ciphers to the multiplicand equal to the number of 9s, and the product will be found exactly, by subtracting the multiplicand, in its original state, from the multiplicand increased by the ciphers; because any number of 9s wants but one of the same number of ciphers annexed to 1.

74185000	3745910000
74185	374591
Ex. 74185×999=74110815	374591×9999=3745535409

Mul. 8756457 by 999   8432597 by 99

In like manner, if any other digit be in the units place, the product of the given multiplicand into the number of units that digit wants of 10, must be subtracted from the multiplicand increased.

457320000 Multiplicand increased.
2

91464 Product into 2.

Ex. 45732×9998=457228536 Remainder and product.

8974596700000
7

628221769

Ex. 89745967×99993=8973968478231

Mul. 47832970 by 997   843275 by 9991

3. The product of any number of repetends may be found by taking the product of one of them, and making the addition from that product, as if all the products had been regularly taken down. An example will be the best illustration.

	741897
0	6666
6+9	
9	4451382
	4945485402

3978425789	8597632874
7777	888

Illust. As the same product would be four times repeated, the first figure of each moving one step higher in the addition, 2 stands for the first place, 2+8 for the 2d, 2+8+3 for the 3d, 2+8+3+1 for the 4th, 8+3+1+5 for the 5th, 3+1+5+4 for the 6th, 1+5+4+4 for the 7th, 5+4+4 for the 8th, 4+4 for the 9th, and 4 for the 10th, carrying all along as usual.

4. The product of any multiplier between 10 and 20 may be found in one line, by adding to the products in course the nearest inferior place of the multiplicand; which an example will illustrate.

475943	*Illust.* $8\times3=24$. $8\times4+2+3=37$.
18	$8\times9+3+4=79$. $8\times5+7+9=56$.
	$8\times7+5+5=66$. $8\times4+6+7=45$.
8566974	$4+4=8$.

48935757	8756429765	347854987
17	19	15

5. If 1 be in the multiplier's lowest place, its product needs not be put down, but the multiplicand may be added for it in summing the products.

EXAMPLES.

745983	874563	374105
81	451	901
5967864	4372815	3366945
60424623	3498252	337068605
	394427913	
		385723976
		961

6. When the multiplier is any composite number, whereof the parts are easily discovered, multiply by those parts one after another continually, and the last product will be the product sought.

EXAMPLES.

7415738 × 56	897345 × 125	7415012 × 5500
7	5	11
51910166	4486725	81565132
8	5	5
415281328	22433625	40782566000
	5	
	112168125	

847328759 by 960.

7. When the multiplier is a prime number, or too high for an immediate discovery of its component parts, multiply by as many 10s continually as the multiplier consists of places, abating one; multiply the last product by the first digit on the left, the next product in course by the next digit, and so on till all the products, and the multiplicand also, are multiplied by the several digits in the multiplier respectively: the sum of the products of these digits, set in the order of addition, will be the product required.

EXAMPLES.

5×34785 ×425

34785	
10	
347850	×2
10	
3478500	
4	
13914000	
695700	
173925	
14783625	

4×54732×3124

54732		
10		
547320	×3	
10		
5473200	×1	
10		
54732000		
3		
164196000	Product of	3000
5473200		100
1094640		20
218928		4
170982768		3124

38794685732×4567

8. In large operations it will be found convenient to make a table of the products of all the nine digits into the multiplicand, which may be easily effected, thus: Set all the digits under one another, and opposite 1 place the multiplicand, double it for 2, to this sum add the multiplicand for 3, to that sum add the multiplicand for 4. Go on thus till there are products for all the digits. Transfer the particular products out of this table, and their sum will be the product required.

Table.

3785946	1
7571892	2
11357838	3
15143784	4
18929730	5
22715676	6
26501622	7
30287568	8
34073514	9

Multiply	378594600
into	659847000
	26501622
	15143784
	30287568
	34073514
	18929730
	22715676
Product	249814511026200000

Having thus gone through the most material contractions in multiplication, it will not be improper to add such a number of examples as may not only confirm the student's practice, but enable

him, on all occasions, to adopt that method which bids fairest for dispatch, and least liable to error.

Promiscuous examples in all the contractions.

3745689735 × 1000	5478673215 × 7200
8597837459 × 9999	6375987687 × 6300
5674597856 × 9992	5978469784 × 17000
674578635 × 7777	5421067859 × 71700
857458000 × 8970	6473598467 × 88800
457386745 × 19	5745637598 × 16000
573541978 × 547856	2456785978 × 55500
	3415978678 × 11110

A certain island contains 36 counties, each county 37 parishes, each parish 38 families, and each family 39 persons; required the number of parishes, families, and persons in the island? *Answer*, 1332 parishes, 50616 families, 1974024 persons.

How many changes may be rung on 12 bells? *Answ.* 479001600.

A man had a farm consisting of 20 acres, which he offered to sale at 1 guinea for the first acre, 2 for the second, 4 for the third, still doubling till they were all expended; what would the 20 acres amount to at that rate? *Answer*, 1048575 guineas.

Note, There are some other contractions in multiplication, which cannot be performed without division, and for that reason will be subjoined to that rule.

SECT. V. DIVISION. ÷

DIVISION teacheth to find, in a number called the *Quotient*, how oft a number, called the *Divisor*, is contained in another number, called the *Dividend*, and may be said to supply the place of many subtractions.

Prob. To divide one number by another, or to find how often one number is contained in another,

1. Set the divisor on the left of the dividend.
2. From the left of the dividend point off so many figures as will contain the divisor, which is called a *Dividual*.
3. Find how often the divisor is contained in this dividual, and place the answer in the quotient.

4 Multiply the divisor into the quotient figure, set the product under the dividual, subtract the one from the other, and note the remainder below.

5. To the right of the remainder annex the next figure in the dividend, for a new dividual, with which proceed as before.

6. If the divisor be too great for any dividual, place a cipher in the quotient for it, before any new figure be brought down; for the quotient must consist of as many places as there are dividuals.

7. If any thing remain after all, it is called the *Remainder*, and with the divisor forms a fraction, expressing some part of one, which gave rise to vulgar fractions.

EXAMPLES.

(1)

Divisor. Dividend. Quotient.

```
      8)54973014(6871626
 6×8=48......
     --
      69
 8×8=64
     --
      57
 7×8=56
     --
      13
 1×8= 8
     --
       50
 6×8=48
     --
       21
 2×8=16
     --
        54
 6×8=48
     --
        6
        - } Remainder.
        8
```

(2)

```
25)37415978(1496639
   25......
   --
   124
   100
   ---
   241
   225
   ---
   165
   150
   ---
   159
   150
   ---
    97
    75
   ---
   228
   225
   ---
     3
    --
    25
```

36)7489576534(

44)97658459(

Illustration of Example 1.

Because 8, the divisor, is not contained in 5, 54 becomes the first dividual; in which, as 8 is contained 6 times, 6 is annexed

to the quotient, but $6\times8=48$ only; and $54-48=6$. To 6 we annex 9, and $69\div8=8$, but $8\times8=64$, and $69-64=5$; annex 7. Then $57\div8=7$, but $7\times8=56$, and $57-56=1$, to which 3 being annexed, we have $13\div8=1$, and $13-8=5$; annex 0, and we have $50\div8=6$, and $50-6\times8=2$; annex 1. Then $21\div8=2$; but $21-2\times8=5$: annex 4, and $54\div8=6$; but $54-6\times8=6$, the laſt remainder; which, with the diviſor, is $\frac{6}{8}$ of 1.

Here it may not be improper to obſerve,

1. That not only the laſt remainder, but every other remainder, muſt be leſs than the diviſor; for, if it be either greater or equal to the diviſor, it might have been got oftener, and the quotient figure is in conſequence too little.

2. That the quotient figure muſt never expreſs more than the number of times the diviſor is contained in the dividual; for, in that caſe, a greater number muſt be ſubtracted from a leſſer, which is impoſſible.

3. By reviewing the ſteps of the foregoing operation, and reducing the dividuals and quotient figures to their ſeparate values, the reaſon of this manner of operation will be ſufficiently obvious. See *Axiom* 5.

Proof of Diviſion.

Diviſion may be proved by diviſion, by multiplication, and ſtill more expeditiouſly by caſting out the nines.

1. By *Diviſion:* Subtract the remainder from the dividend, and divide the difference by the quotient; the new quotient will be juſt equal to the firſt diviſor.

2. By *Multiplication:* Multiply the quotient by the diviſor, and to the product add the remainder; the ſum will be juſt equal to the dividend.

3. By caſting out the *Nines:* Caſt the nines out of the diviſor and quotient, place the exceſſes on the right and left ſides of a croſs; caſt the nines out of the product of theſe two figures, and add the exceſs to the remainder, if any; out of that ſum caſt alſo the nines, and place the exceſs at the top of the croſs; laſtly, caſt the nines out of the dividend, and place the exceſs at the bottom: then will the top and bottom figures agree, if the work be right.

Example by Division.

```
9)74568(8285          74568
  72...                   3
  ---                 -----
   25           8285)74565(9
   18                74565
   --                -----
    76                   0
    72
    --
     48
     45
     --
      3

75)834279684(        96)7425986(
```

Example by Multiplication.

```
355 ) 1785945 (    5030
      1775...       355
      ----        -----
       1094        1065
       1065       1775
       ----     -------
        295     1785650
                    295
                -------
                1785945
```

The same by casting out the nines.

```
355 ) 1785945 ( 5030
      1775...
      ----
       1094
       1065       3
       ----     8+4
        295       3
```

Here it may not be improper to obſerve, that when, as in the laſt example, the diviſor is large, it will facilitate the operation to conſider how oft the firſt figure of the diviſor is contained in the firſt figure of the dividual; if the number in both are equal, or if, in the firſt two, the dividual conſiſts of more places than the diviſor, then imagine the remainder to be prefixed to the next figure of the dividual; and if the ſecond figure be contained as often in this number, make the ſame trial with the remaining figures, and if any of them fall ſhort, you muſt try the firſt figure one leſs, and go on with the reſt in the ſame way, till all the ſubſequent figures

of the ſame diviſor can be got as often as the firſt was taken. With proper practice in this, the greateſt diviſor will become eaſy.

More examples to facilitate practice.

347)5986741597(	37451)2015978368(
845) 973407459(	45678) 321347598(
975)8407415637(	573489) 123074156(
4159)3745878597(	348769) 374567859(
6754)5978674158(	810745) 801234567(
3741)8567483759(	976415) 374159764(
86754)9789765978(	1156741) 612345979(
47151)8735015178(	3140789) 307459787(
6745)7418567345(	4070456) 112347897(
	9074159) 807234156(

Contractions in Diviſion.

1. If there are ciphers on the right of the diviſor, point them off; point off alſo an equal number of places from the right of the dividend, and find the quotient ariſing from the places on the left of the points, which will have the ſame effect as if the diviſion had been made for all the places in both factors. If the places pointed off from the dividend were ſignificant, annex them to the laſt remainder, and bring down the whole diviſor for the other member of the fraction.

EXAMPLES.

64800)8967823(138$\frac{25423}{64800}$	78000)475976000(6102$\frac{20}{78}$
648	468
2487	79
1944	78
5438	176
5184	156
25423	20
500)3798657435(	8900)765483278(

Corol. Hence, to divide by any number of ciphers, with 1 prefixed to them, it will be only necessary to point off places for the ciphers from the right of the dividend; for the places on the left of the point will be the quotient, and those on the right will be the remainder.

EXAMPLES.

100)745835(7458 $\frac{35}{100}$ 1000)415978000(415978

2. When the divisor is small, or may be made so, by cutting off ciphers, the operation may be performed mentally, and the quotient set under the dividend, so as its left hand figure may stand under the right hand figure of the first dividual, and the remainder subjoined by way of fraction to the quotient.

EXAMPLES.

2)4978356	4)5783497	6)7415987	8)745978
2489178			
3)7415978	5)67859735	7)58974597	9)741597
2471992 $\frac{2}{3}$			

1100)4759789746
4327081 $\frac{645}{1100}$

9000)578468573897

12000)7489756745

12000)74159786739 78

3. If the divisor is a composite number, the quotient may be found by dividing by the component parts, one after another continually, and the last quotient will be the answer.

For the true remainder, multiply the last remainder into all the preceding divisors continually, adding the several remainders to the products of the divisor to which they belong: The last product is the total remainder.

EXAMPLES.

$$8)74859783 \div 56 = 8 \times 7$$

Rem.

$$7)9357472 - 7$$

Qu. $1336781 - 5$

$5 \times 8 + 7 = 47$ total rem.

$$5)41589769 \div 125$$

$$5)8317953 - 4$$

$$5)1663590 - 3$$

$$332718\tfrac{19}{125}$$

$$4)234786 \div 168$$

$$7)58696 - 2$$

$$6)8385 - 1$$

$1397 - 3$ or $\frac{90}{168}$

$$4)478976{,}845 \div 504000$$

$$6)119744$$

$$7)19957 - 2$$

$$3)2851$$

$$950\tfrac{176845}{504000}$$

For Rem.

$$1 \times 7 \times 6 + 2 = 44 \times 4 = 176$$

497685600 by 1120

$97845678596 \div 14400$

Note, When the laſt quotient is multiplied back continually into the ſeveral diviſors or component parts, it will certainly produce the dividend, abating the remainder: For the ſame reaſon, then, when the laſt remainder is multiplied back into all the diviſors before it, and the partial remainders added in courſe, the reſult muſt certainly be equal to the remainder found at one operation.

The operation may be frequently made very ſimple, by dividing both factors by any number or numbers that will meaſure them without a remainder; for it is the ſame thing to divide 4 by 2, as to divide 16 by 8.

EXAMPLES.

4)468	1692	12	84,0	7896,0	12	1728	4147200
9)117	423		7	658	12	144	345600
13	47		1	94		13	2400
	$3\frac{8}{11}$						

4. As in multiplication, it may alſo, in diviſion, be ſometimes proper to conſtruct a table for the products of the diviſor into all the nine digits, whence the products and quotient figures are taken at once, as in the

EXAMPLE.

457325	1
914650	2
1371975	3
1829300	4
2286625	5
2743950	6
3201275	7
3658600	8
4115925	9

```
40097675021343 ( 87678729
3658600
-------
3511675
3201275
-------
 3104000
 2743950
 -------
  3600502
  3201275
  -------
   3992271
   3658600
   -------
    3336713
    3201275
    -------
     1354384
      914650
     -------
      4397343
      4115925
      -------
       281418
```

6
o+8
6

54769)83278965(

Prob. 2. To divide by a whole number and a fraction.

Multiply the whole number by the lower member or denominator of the fraction, and to that product add the upper member or numerator for a new divisor. Multiply also the dividend into the same denominator, and the product will be a new dividend.

EXAMPLES.

$$
\begin{array}{r|r}
47\frac{1}{2} & 59473685 \\
\underline{2} & 2 \\
\hline
95 & 118947370 \\
\hline
 & 1252077\frac{11}{19}
\end{array}
\qquad\qquad
\begin{array}{r|r}
384\frac{2}{3} & 59473215978 \\
\underline{3} & 3 \\
\hline
1154 & 178419647934 \\
\hline
 & 154609746\frac{1050}{1154}
\end{array}
$$

More examples to facilitate practice.

$$
\text{Divide} \left\{ \begin{array}{r} 3745675989 \\ 876978597 \\ 49763451 \\ 367145978 \\ 859745683 \\ 597856783 \end{array} \right\} \text{by} \left\{ \begin{array}{l} 55\frac{5}{6} \\ 69\frac{7}{8} \\ 158\frac{5}{9} \\ 27\frac{8}{9} \\ 64\frac{3}{5} \\ 93\frac{3}{7} \end{array} \right.
$$

Prob. 3. To multiply by a whole number and a fraction.

Find the product of the integral part of the multiplier, as before, and take parts of the multiplicand by division for the fraction, and add to the integral part of the product, as in the

EXAMPLES.

$$
\begin{array}{rr}
5\,) & 59784597 \\
 & 38\frac{1}{5} \\
\hline
 & 478276776 \\
 & 179353791 \\
 & 11956919\frac{2}{5} \\
\hline
 & 2283771605\frac{2}{5}
\end{array}
\qquad\qquad
\begin{array}{rrl}
2\,) & 64973589 & \\
 & 9\frac{5}{8} & \\
\hline
 & 584762301 & \\
4\,) & 32486794\frac{1}{2} & \text{for } \frac{4}{8} = \frac{1}{2} \\
 & 8121698\frac{5}{8} & \\
\hline
 & 625370794\frac{1}{8} &
\end{array}
$$

$$
\begin{array}{r}
374893274 \\
56\frac{3}{4} \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r}
879328597 \\
789\frac{5}{6} \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r}
37198657 \\
456\frac{3}{7} \\
\hline
\end{array}
$$

Note, Multiplication may be sometimes performed very expeditiously by division, when the multiplier is any even part of 100, 1000, 10000, &c. thus: Annex as many ciphers to the multiplicand as there are places in the multiplier, and divide by that number which would stand in the quotient, were these ciphers and 1 prefixed to them divided by the multiplier.

EXAMPLES.

Multiply 74158075 by 25
4)7415807500 for $\frac{100}{25}=4$

1853951875 Product.

Multiply 47568 by 75
2)4756800

2)2378400
1189200

3567600

More examples to facilitate practice in Division.

$$\text{Divide}\left\{\begin{array}{r} 745978456 \\ 897459785 \\ 597645978 \\ 69764785 \\ 59786478 \\ 58467859 \\ 69748572 \end{array}\right\}\text{ by }\left\{\begin{array}{r} 420 \\ 2100 \\ 515 \\ 3605 \\ 33\frac{1}{8} \\ 195\frac{2}{5} \\ 63\frac{7}{8} \end{array}\right.$$

QUERIES.

1. What is the reason, that, in Addition, the number of tens, in the sum of one column, is always carried to be added with the next column? *Answer*, Because, in the nature of numbers, 10 in any column on the right make exactly 1 in the next to the left.

2. In addition, when only the surplus of tens is noted in all the other places of the sum throughout, why is the whole sum of the last or left hand column noted complete? *Answer*, The surplus of tens is noted in its proper place, and the tens carried to be added as before; but, as there is no new column, they must be noted without increase.

3. In Subtraction, when any figure in the minor is greater than its correspondent in the major, why is the major increased by 10? *Answer*, In the scale of numeration, all numbers increase in a de

cuple proportion; therefore, if before ſubtraction, the major be increaſed by 10, and after ſubtraction the minor be increaſed by 1, the ſtate of neither will be altered; for 1 in the one place is = 10 in the other, and the ſame difference will ſubſiſt between 17 and 26 as between 7 and 16.

4. How comes it, that, in Multiplication, the firſt figure to the left of the product of every figure in the multiplier, muſt ſtand immediately below its multiplying figure? *Anſwer*, The units place in the multiplier is a digit, the ſecond place is ſo many tens, the third ſo many hundreds, &c. and therefore their reſpective values muſt be retained in the product, otherwiſe they would only be conſidered as ſo many ones, not as ſo many tens, hundreds, &c. their true local value.

5. How comes it, that a proceſs in Diviſion gives the quotient complete, when it is taken by dividuals partially, and not all at once? *Anſwer*, Diviſion is the converſe of Multiplication, and therefore the proceſs muſt alſo be the direct contrary, as the proof by multiplication evinceth; but, more particularly, ſince the dividend is taken into as many dividuals as poſſible, and the quotient taken out of the firſt dividual as near as poſſible, the defect, if any, makes the foundation of the next dividual; and this proceſs being repeated as oft as there are places in the dividend to bring down, or figures to mark in the quotient, it will be evident, that all the parts of the dividend have been added, and the number of times the diviſor hath been found in thoſe parts ſeparately found; and ſince all the parts, taken together, are equal to the whole, it muſt follow, that however oft the diviſor is contained in theſe partial dividuals, ſo oft muſt the diviſor be contained in the dividend, of which they are compoſed.

6. How comes it, that to multiply or divide by the component parts of any number, has the ſame effect as the common methods already accounted for? *Anſwer*, Becauſe the product in the one caſe, and the quotient of the parts in the other, is juſt equal to the multiplier or diviſor; for, if 4 times any multiplicand were taken, and then 6 times that product, this laſt product would be 24 times the multiplicand, which is juſt what was required. In like manner, if $\frac{1}{4}$ of any dividend were taken, and that quotient divided by 6, the laſt quotient would be juſt $\frac{1}{24}$ of the dividend, which was the thing required.

7. In multiplying or dividing by a number to which ciphers are annexed, why are the ciphers neglected in the operation? *Anſwer*, To ſave the trouble of writing them; for, to multiply by 10, 100, 1000, &c. requires only to annex the ciphers in the multiplier to the multiplicand; becauſe, to multiply by 1 does not alter a ſingle figure in the multiplicand, and to divide by 1 and any number of ciphers, as diviſion is the converſe of multiplication, requires only

to cut off from the right of the dividend a number of places equal to the number of ciphers in the divisor, for the remainder; then will the remaining figures of the dividend to the left constitute the quotient: Wherefore, in all cases, multiplication, or division, performed without the ciphers affixed to the multiplier or divisor, if they are annexed to the product in the one case, and places for them struck off from the right of the dividend in the other, the result must be the same as if the ciphers had been carried through the whole process.

8. What influence can it have in the proof of multiplication or division, to cast out the 9s? *Answer*, Divide any significant figure with or without ciphers, the remainder will be equal to the significant figure, or figures, taken in their simple value. For instance, divide 8 by 9, the quotient is 0, and the remainder 8; divide 80 by 9, the quotient is 8, and the remainder 8; divide 800 by 9, and the quotient will be 88 and remainder 8, *ad infinitum*: consequently, if any number be divided by 9, the remainder will be equal to the sum of the figures of the said number, taken in their simple value, or to the excess above the 9s contained in said sum; for, if the number be resolved into its constituent parts, the significant figures of each will be the remainder of that part, when divided by 9, and consequently the remainders of the several parts will be the figures of the given number, out of which, if the 9s are taken, the excess will be the remainder: and, since the remainder arising from a number divided by 9 is found by adding the figures of said number, the proof of any process will always come out if nothing is wrong in the operation; but, it may also come out, though something in the process be wrong, if the sum of the figures be the same.

9. In multiplying by a whole number and a fraction, why are parts of the multiplicand taken for the fraction? *Answer*, The product of any multiplicand by 1 is = the multiplicand; consequently, the product of the multiplicand into any part of 1 must be that part of the multiplicand the fraction represents.

10. In dividing by a whole number and a fraction, why are divisor and dividend reduced to the denomination of the fraction? *Answer*, Because then both factors become homogeneal, and, being equally increased, the quotient will still be the same, as by any other true division whatever.

CHAP. II.

Application of the Elements to relative Integers, and their Subdivisions.

INTRODUCTION.

Concerning the Diviſion of Integers into their common inferior Denominations.

In order to accommodate relative units in the various articles of buſineſs, to the common exigencies of life, ſome certain denomination hath been fixed on, by the common conſent of a nation, as the higheſt; that ſtandard denomination hath, by the ſame conſent, been divided into a certain number of parts equal to the whole; each of theſe again into another number of equal parts, and each of theſe again into another number, &c. till it was thought proper to go no farther. Hence the invention of the following tables, which ought all to be committed to memory.

Sect. I. Applicate Addition.

1. *Of Money.*

The current coins in Britain are copper farthings and halfpence; ſilver ſixpences, ſhillings, and crowns and half crowns, each crown valued at five ſhillings, and the half crown at two ſhillings and ſixpence; the guinea, valued at twenty-one ſhillings, the half guinea, ten ſhillings and ſixpence, quarter guinea, five ſhillings and threepence, and five guinea piece = L. 5 : 5 : 0, all gold coins. Beſides theſe, we have ſometimes Spaniſh and Portugueſe pieces of gold and ſilver, which paſs at the reſpective prices current at the time.

In England they always reckon in Sterling money, as they do alſo in Scotland, excepting in ſome country places, where they reckon in Scots money, which is juſt $\frac{1}{12}$ of the Sterling; but both are divided the ſame way, as in the following

MONEY TABLE.

4 farthings	=	1 penny.				
48	=	12	=	1 ſhilling.		
960	=	240	=	20	=	1 L. pound.

EXAMPLES.

Borrowed from sundries.				Paid to sundries.				Lent to sundries.			
L.	784	19	9½	L.	597	11	11½	L.	47569	18	8¾
	583	17	8½		539	19	11¾		59783	17	11½
	647	11	11¼		599	18	10½		9874	15	9½
	568	17	9½		498	17	11½		8385	17	11½
	678	18	8¾		555	11	10½		6749	11	9½
	567	14	5½		567	14	5½		8473	19	10½
	977	13	9½		949	16	7½		648	11	9¾
	644	14	7½		597	11	9½		575	19	11½
	778	11	11½		765	19	10½		697	17	10½
	387	16	9½		369	18	11½		98	18	11½
									76	17	10¾
Tot.	6620	17	7¼								

Made the following remittances to my correspondent to answer my several draughts; what is the sum of each?

Draughts.				Remittances.			
L.	978	11	11½	L.	919	19	10¾
	1849	17	10¾		1911	17	11½
	645	18	9½		878	11	10¾
	1176	19	4¼		967	10	11½
	945	17	7½		876	11	10¼
	785	18	9¾		987	10	11½
	967	17	11½		741	11	9¾
	1796	18	8¾		857	18	10½
	943	17	11½		1817	11	11½
	897	18	9½		976	19	7½
	84	15	5¼		126	11	5¼
	385	17	7½		317	19	9½
	95	18	11½		87	17	8¾

Illustration of Example 1.

In adding up the column of farthings, I find they amount to 5¼d. wherefore I note down the ¼, and carry 5 to be added with the pence, the column of which amounts to 91d. or 7s. and 7d. wherefore I note the pence, and carry 7s. to their respective column, the units part of which I find amounts to 57, wherefore I note 7, and carry 5 to be added with the column of tens, amounting to 15, which being seven times 20, and 1 over, I note the 1, and carry L. 7 to be added with the units place of the pounds, or integral part of the example.

2. *Troy Weight.*

This weight is used for jewels, gold, silver, liquors, and bread, and divided as in the following

TABLE.

24 grains	= 1 dwt. pennyweight.			
480	= 20	= 1 oz. ounce.		
5760	= 240	= 12	= 1 lb pound.	

EXAMPLES.

Bought of silver.

lb.	oz.	dwt.	gr.
798	11	19	21
864	10	18	22
375	11	17	23
847	10	14	18
975	11	19	21
845	10	14	17
67	9	13	12
68	7	15	15
39	8	18	17
6	7	15	19
9	8	8	9
4902	2	18	2

lb.	oz.	dwt.	gr.
397	11	18	18
648	10	19	20
475	9	17	21
67	8	15	19
8	7	8	17
35	11	19	22
67	10	17	23
96	11	18	17
876	11	11	14
98	10	19	15
86	11	14	23
87	10	17	22
99	9	19	19
77	8	18	22
84	11	17	23
194	10	15	19
376	9	14	17
598	10	16	18
674	6	19	19

lb.	oz.	dwt.	gr.
976	10	19	21
847	11	17	22
385	10	14	23
875	11	17	21
97	10	18	15
8	11	9	22
67	11	19	21
85	10	11	11
97	11	10	11
48	10	17	17
77	11	14	22
85	11	11	11
776	10	19	21
845	9	18	22
786	8	17	23
45	11	15	19
6	10	14	11
7	11	18	22
9	4	13	19

In the column of grains I first add the units, and find they amount to 54, wherefore I note 4, and carry 5 to the column of tens, whose sum is 19, so that the whole is 194; in which 24 will be found 8 times, and 2 over; wherefore I note the 2, and carry 8 to the column of dwts, which is added as shillings, &c.

3. *Apothecaries Weight.*

This is a deduction from Troy weight, by which medicines are weighed out for composition, though they are bought and sold by avoirdupoise. The ℔ is divided as in the following

TABLE.

20 grains	=	1 ℈ scruple.						
60	=	3	=	1 ʒ dram.				
480	=	24	=	8	=	1 ℥ ounce.		
5760	=	288	=	96	=	12	=	1 ℔. pound.

EXAMPLES.

℔	℥	ʒ	℈	gr.	℔	℥	ʒ	℈	gr.	℔	℥	ʒ	℈	gr.
197	11	7	2	19	987	3	4	1	18	84	11	7	2	18
684	10	6	1	18	568	4	5	2	19	96	10	6	1	19
375	11	3	2	17	674	5	1	2	18	56	9	5	2	17
586	9	2	1	19	695	3	2	2	19	97	8	4	1	14
385	11	5	2	16	875	8	1	1	17	56	7	3	2	15
548	9	4	1	14	945	11	7	2	19	97	8	3	1	16
687	8	3	2	11	467	10	6	1	18	84	10	2	1	17
497	10	6	1	15	57	11	5	2	17	96	11	1	1	18
849	9	7	2	13	68	10	4	1	11	45	11	1	1	17
572	8	3	1	11	75	11	3	2	9	64	9	3	1	19
67	9	1	1	19	84	10	2	1	8	75	8	1	1	18
8	4	0	0	15	67	11	1	2	7	67	11	6	2	19
9	9	6	1	0	42	10	5	1	18	84	10	7	1	13
87	11	7	2	19	47	9	6	2	14	94	11	7	2	18
34	10	6	1	18	35	8	7	2	19	84	9	6	1	17
89	8	5	0	17	64	7	3	1	17	78	8	7	2	15
54	9	4	1	15	96	11	3	2	18	64	7	7	1	12
65	11	3	1	16	88	10	1	1	11	97	3	4	2	19
94	10	2	2	18	45	11	1	2	12	35	1	3	1	14

4. *Avoirdupoise Weight*

Serves for coarse commodities, as iron, lead, salt, hemp, flax, rosin, butter, cheese, and all grocery goods whatever. The table is divided into two parts, called the greater and lesser, as follow:

TABLE *of the greater.*

28 lb	=	1 quarter.				
112	=	4	=	1 C weight.		
2240	=	80	=	20	=	1 tun.

TABLE *of the lesser.*

16 drams	=	1 oz.				
256	=	16	=	1 lb.		
4096	=	256	=	16	=	1 stone.

Note, It hath been found, that 1 lb avoirdupoise, weighs 14 oz. 11 dwt. 15½ grains Troy; but 1 ounce Troy is heavier than the ounce avoirdupoise by 32 grains.

EXAMPLES.

Tun.	C.	qr.	lb	St.	lb	oz.	dr.	Tuns.	C.	qr.	lb	oz.
748	19	3	25	79	15	15	15	597	19	3	27	15
687	18	2	27	87	14	14	14	468	17	2	24	14
75	14	1	26	95	13	12	11	787	11	3	25	11
98	17	2	24	147	12	11	13	898	10	2	26	9
87	15	3	23	987	15	14	13	427	11	1	11	11
199	18	2	21	48	13	14	15	858	18	2	19	14
57	14	1	20	97	8	7	5	978	17	1	14	15
89	16	2	24	49	10	12	11	887	15	2	19	13
57	17	1	25	45	11	15	14	598	13	1	24	11
68	14	3	22	98	10	11	12	742	19	2	22	14
9	17	1	21	38	11	10	9	87	15	1	0	19
6	18	2	23	74	10	11	8	9	0	3	27	0
7	17	1	21	65	11	10	9	6	8	2	0	12
4	15	3	19	75	10	11	7	9	19	3	9	9
37	13	3	24	85	14	2	19	874	19	3	27	0
85	14	2	25	37	15	1	20	849	17	2	26	0
74	15	1	27	396	17	2	21	718	19	1	24	0
89	17	2	21	84	18	1	22	876	17	2	12	0
67	18	1	23	97	17	2	21	974	15	1	22	0
7	11	3	26	88	18	1	22	819	18	2	21	0

5. *Wool Weight*

Was originally calculated for its own purpose, and divided as in the following

TABLE.

7 lb	=	1 clove.										
14	=	2	=	1 stone.								
28	=	4	=	2	=	1 todd.						
182	=	26	=	13	=	$6\frac{1}{2}$	=	1 wey.				
364	=	52	=	26	=	13	=	2	=	1 sack.		
4368	=	624	=	312	=	156	=	24	=	12	=	1 last.

EXAMPLES.

L.	S.	W.	T.	St.	C.	lb.
38	11	1	6	1	1	4
79	10	1	$6\frac{1}{4}$	1	0	3
97	9	1	$5\frac{1}{2}$	1	1	4
83	10	1	6	0	0	6
79	9	0	$5\frac{1}{4}$	1	1	5
87	8	1	$4\frac{1}{2}$	1	1	3
478	9	1	$6\frac{1}{4}$	1	1	3
897	11	1	$5\frac{1}{2}$	1	1	4
86	10	1	$4\frac{1}{4}$	1	1	5
94	11	1	$5\frac{1}{2}$	1	1	6
78	8	1	$3\frac{1}{4}$	1	1	5
84	7	1	$2\frac{1}{2}$	1	1	4
9	8	1	$4\frac{1}{2}$	1	1	3
7	11	1	$6\frac{1}{4}$	1	1	6
9	7	1	4	1	1	5

L.	S.	W.	T.	St.	C.	lb.
396	11	1	$3\frac{1}{2}$	1	1	5
674	10	1	$2\frac{1}{2}$	1	1	6
879	11	1	5	1	1	4
387	10	1	$4\frac{1}{2}$	1	1	3
98	9	1	$5\frac{1}{2}$	1	1	2
87	8	1	6	1	1	3
84	10	1	$6\frac{1}{4}$	1	1	5
97	11	1	$4\frac{1}{2}$	1	1	4
39	10	1	$3\frac{1}{2}$	1	1	6
47	11	1	$4\frac{1}{2}$	1	1	5
93	10	1	5	1	1	4
68	9	1	$3\frac{1}{2}$	1	1	5
76	4	1	$5\frac{1}{2}$	1	1	3
98	8	1	$3\frac{1}{4}$	1	1	6
74	3	1	6	1	1	5

Note, In adding the todds in the last example, sum the fractions as farthings, and carry the units to the integral part; double the sum of the integral part, and divide by 13, the double of $6\frac{1}{2}$, the quotient will give the number to be carried; but the remainder will be double the number to be noted.

6. *Wine Measure*

Is used for all spirits, mead, perry, cyder, vinegar, oil, honey, as well as wine.

TABLE.

Solid inches						
$28\frac{7}{8}$ =	1 pint.					
$57\frac{3}{4}$ =	2 =	1 quart.				
231 =	8 =	4 =	1 gallon.			
14553 =	504 =	252 =	63 =	1 hhd.		
29106 =	1008 =	504 =	126 =	2 =	1 pipe,	
58212 =	2016 =	1008 =	232 =	4 =	2 =	1 tun.

Note, 1 gallon wine = 8lb Troy,
18 gallons = 1 runlet,
$31\frac{1}{2}$ gallons = 1 barrel,
42 gallons = 1 tierce,
10 gallons = 1 anchor, anker, or keg.

EXAMPLES.

Tuns.	Pipes.	Hhds.	Gal.	Pints.
547	1	1	62	7
539	1	1	61	6
574	1	1	59	4
387	1	1	58	3
848	1	1	54	2
75	1	1	48	6
35	1	0	36	5
97	0	1	27	3
85	1	0	0	6
9	0	1	24	0
87	1	1	61	7
45	1	1	59	6
97	1	1	54	7
989	1	1	62	5
784	1	1	55	3
849	1	1	54	7

Tuns.	Pipes.	Hhds.	Gal.	Pints.
8475	1	1	62	5
9769	1	1	45	6
4379	1	1	9	7
687	1	1	19	4
976	1	1	57	3
674	1	1	46	4
789	1	1	59	3
657	1	1	48	5
978	1	1	49	7
846	0	0	54	5
97	1	1	39	6
48	1	1	45	5
87	1	1	36	4
8	1	1	35	3
9	1	1	38	1
8	0	1	48	2

7. *English Ale Measure.*

TABLE.

Solid inches							
$31\frac{1}{2}$ =	1 pint,						
63 =	2 =	1 quart,					
252 =	8 =	4 =	1 gallon,				
2016 =	64 =	32 =	8 =	1 firkin,			
4032 =	128 =	64 =	16 =	2 =	1 barrel,		
8064 =	256 =	128 =	32 =	4 =	2 =	1 kilderkin,	
12096 =	384 =	192 =	48 =	6 =	3 =	$1\frac{1}{2}$ =	1 butt.

EXAMPLES.

Butts.	(48) Gal.	(4) Qts.	(2) Pints.
3	47	3	1
69	44	2	1
78	42	1	1
8	41	1	1
9	45	2	1
8	13	3	0
9	18	1	1
5	44	3	1
6	41	2	1
9	42	1	1
8	45	2	1
9	17	1	1
56	44	3	1
78	38	2	1
97	39	3	1
8	34	1	1
6	17	1	1
7	18	1	1
4	37	3	1
8	47	3	1

Butts.	Bar.	Gal.	Pints.
7859	2	15	3
8764	1	14	2
9769	2	15	3
7857	1	13	6
6437	2	11	4
7893	1	12	2
6875	1	13	2
7843	1	14	3
8765	2	13	2
9737	1	14	5
8328	2	13	3
7113	1	10	2
8475	1	11	3
6759	2	9	7
376	2	8	3
77	2	9	1
88	1	4	2
67	2	11	3
93	1	12	1
8	2	15	3

8. *Ale Measure Scotch.*

TABLE.

Solid inches

$6\frac{23}{48}$ = 1 gill,

$25\frac{11}{12}$ = 4 = 1 mutchkin,

$103\frac{2}{5}$ = 16 = 4 = 1 pint,

$829\frac{1}{5}$ = 128 = 32 = 8 = 1 gallon,

$13429\frac{1}{5}$ = 2048 = 512 = 128 = 16 = 1 butt.

EXAMPLES.

B.	Gal.	P.	M.	Gills.
1	15	7	3	3
0	8	4	1	0
3	12	4	2	2
2	4	6	3	1
1	0	5	2	2
4	10	3	1	3
98	13	7	3	3
79	14	6	2	3
884	15	4	3	2
976	13	5	1	3
847	14	3	2	1
98	13	1	1	3
74	15	2	2	3
85	12	3	1	2
97	13	2	1	1
88	14	3	2	2
99	13	2	1	3
84	11	1	3	2
9	14	2	2	2
8	15	3	3	3

B.	Gal.	P.	Gills.
9784	15	7	15
8419	14	6	14
3976	13	5	13
9764	12	4	12
3796	10	5	13
4987	11	7	14
876	10	6	12
977	13	5	10
426	14	3	7
978	13	2	9
647	12	3	8
198	11	7	7
64	14	4	9
85	12	6	14
96	13	3	4
76	11	6	7
98	12	5	13
49	8	4	14
57	9	3	9
43	13	6	13

8. *English Corn Measure.*

Pints
2 = 1 quart,
4 = 2 = 1 pottle,
8 = 4 = 2 = 1 gallon,
16 = 8 = 4 = 2 = 1 peck,
64 = 32 = 16 = 8 = 4 = 1 bushel,
256 = 128 = 64 = 32 = 16 = 4 = 1 coomb,
512 = 256 = 128 = 64 = 32 = 8 = 2 = 1 quarter,
2048 = 1024 = 512 = 256 = 128 = 32 = 8 = 4 = 1 chal.
2560 = 1280 = 640 = 320 = 160 = 40 = 10 = 5 = $1\frac{1}{4}$ = 1 tun or wey,
5120 = 2560 = 1280 = 640 = 320 = 80 = 20 = 10 = $2\frac{1}{2}$ = 2 = 1 last.

Note, The gallon contains $268\frac{4}{7}$ solid inches, and the bushel $2150\frac{2}{5}\frac{1}{0}$.
5 pecks = 1 bushel water measure.

9. *Scots Corn Measure.*

TABLE.

Lippies or 4ths
4 = 1 peck,
16 = 4 = 1 firlot,
64 = 16 = 4 = 1 boll,
1024 = 256 = 64 = 16 = 1 chalder.

Note, The Scots firlot contains $2688\frac{1}{4}$ solid inches.

EXAMPLES.

1. English.

Ch.	Q.	B.	P.	G.	Pints.
136	3	7	3	1	7
589	1	4	2	0	6
78	2	6	2	1	0
0	2	5	1	0	3
55	3	1	2	1	6
976	2	5	1	1	5
89	3	4	1	1	7
397	3	7	3	1	7
985	2	6	2	1	6
847	1	5	1	1	5
785	1	3	3	0	4
876	2	1	2	1	3
987	1	2	1	0	4
876	2	1	3	1	3
987	2	2	3	0	5
793	3	2	1	0	3
543	1	1	2	1	4
785	3	5	3	0	6
479	1	5	2	1	7
674	3	2	1	0	5

2. Scots.

Ch.	B.	F.	P.	L.
778	11	3	2	1
557	12	2	1	2
978	10	1	2	1
678	11	3	1	2
97	10	2	1	1
8	11	1	1	2
449	15	3	3	3
778	13	2	1	3
849	12	3	3	3
976	13	2	2	1
878	12	1	3	3
987	11	2	2	3
876	13	3	3	1
786	11	1	1	3
497	12	3	3	1
588	13	1	1	2
678	12	2	1	2
979	13	3	2	3
789	15	2	3	1
379	14	1	2	3

10. *English Long Measure.*

TABLE.

Inches
12 = 1 foot,
36 = 3 = 1 yard,
198 = $16\frac{1}{2}$ = $5\frac{1}{2}$ = 1 pole,
7920 = 660 = 220 = 40 = 1 furlong,
63360 = 5280 = 1760 = 320 = 8 = 1 mile.

EXAMPLES.

Miles.	Furl.	Yds.	Feet.	Inches.
119	7	119	2	11
857	6	210	1	9
945	5	215	2	8
675	4	119	1	10
49	3	212	2	9
8	2	218	2	8
74	4	217	2	7
85	3	118	1	4
97	1	125	2	3
49	2	149	2	5
68	1	110	1	9
64	2	212	2	6
75	2	118	1	7
56	3	36	2	8

Miles.	Furl.	Poles.	Yds.	Feet.	Inches.
798	7	35	$4\frac{1}{2}$	2	11
879	6	36	5	2	10
478	5	37	$5\frac{1}{4}$	2	11
979	4	38	$4\frac{3}{4}$	1	9
848	3	29	3	2	8
756	1	32	$2\frac{1}{2}$	1	7
549	5	34	3	2	7
679	4	33	$2\frac{3}{4}$	2	11
897	3	28	$5\frac{1}{4}$	1	10
978	2	18	$4\frac{1}{2}$	1	11
119	1	17	$5\frac{1}{4}$	1	10
67	2	28	$4\frac{1}{2}$	2	7
88	1	39	4	1	8
97	1	34	3	2	7

11. *Scots Long Measure.*

TABLE.

Inches

12 = 1 foot,

37 = $3\frac{1}{12}$ = 1 ell,

222 = $18\frac{1}{2}$ = 6 = 1 fall,

8880 = 740 = 240 = 40 = 1 furlong,

71040 = 5920 = 1920 = 320 = 8 = 1 mile.

Note, 3 English miles = 2 Scots or computed miles.

EXAMPLE.

Miles.	Furl.	Falls.	Ells.	Feet.	Inches.
7984	7	39	5	2	11
8796	6	37	4	1	10
976	5	25	3	2	9
846	4	27	3	1	6
954	3	29	1	0	7
695	2	35	2	2	5
36	1	36	1	1	4
9	6	38	2	2	3
87	2	33	1	1	3
8	1	30	2	2	10

12. *Cloth Measure.*

TABLE.

4 nails	= 1 quarter,		
16	= 4	=	1 yard.

Note,	$4\frac{1}{2}$ yards	=	1 ell English,
	$\frac{3}{4}$ yard	=	1 ell Flemish,
	185 Engl. feet	=	180 Scots.

EXAMPLE.

Yds.	Qrs.	N.
26	2	3
17	3	0
9	1	2
10	2	1
20	1	3
11	0	1
6	1	2
18	2	2

13. *English Land Measure.*

TABLE.

Square inches						
144=	1 square foot,					
1296=	9=	1 square yard,				
39204=	$272\frac{1}{4}$=	$30\frac{1}{4}$=	1 square pole,			
1568160=	10890=	1210=	40=	1 square rood,		
6272640=	43560=	4840=	160=	4=	1 acre,	
4014489600=	27878400=	3097600=	102400=	2560=	640=	1 mile

EXAMPLE.

Acres.	Roods.	Poles.	Yards.
748	3	39	$29\frac{1}{4}$
673	2	27	25
38	3	24	$27\frac{1}{2}$
68	2	28	$28\frac{1}{2}$
79	3	29	25
83	3	25	28
76	1	19	29
8	2	17	17

14. *Scots Land Measure.*

TABLE.

Links					
$17\frac{9}{25}$=	1 ell,				
625 =	36=	1 fall,			
2500 =	1440=	40=	1 rood,		
10000 =	5760=	160=	4=	1 acre,	
6400000 =	3686400=	102400=	2560=	640=	1 mile.

Note, $78\frac{534}{625}$ ſquare inches = 1 ſquare link,
787 Scots acres = 1000 Engliſh nearly.

EXAMPLE.

Acres.	Roods.	Falls.	Ells.	Feet.
779	3	37	35	8
476	2	27	39	7
685	3	28	37	6
47	1	29	18	5
697	3	24	24	4
86	1	21	22	5
68	3	22	27	6
9	3	24	19	4
8	2	19	29	3
7	3	11	12	2
8	3	27	13	3
78	1	12	12	7
97	2	32	18	4
85	3	18	19	5
76	2	17	14	3
87	3	14	15	3
96	2	18	22	4
78	1	17	11	5

15. *Time.*

TABLE.

″ ſeconds
60 = 1′ minute,
3600 = 60 = 1 hour,
86400 = 1440 = 24 = 1 day,
31556937 = 525949 = 8765 = 365 5h. 49′ 57″ = 1 year.

For eaſe in calculation, the year is made to conſiſt of 365 days 6 hours, and every fourth or leap year of 366 days; hence the three intermediate or common years conſiſt of 365 days, divided into months, as under:

January	31 days,	July	31 days,
February	28, and in leap year 29,	Auguſt	31,
March	31,	September	30,
April	30,	October	31,
May	31,	November	30,
June	30,	December	31.

F 2

EXAMPLES OF

Long Measure.

Leag.	Mil.	Fur.	Yds.	Feet.	Inch.
743	1	7	118	2	11
876	2	6	119	2	10
764	1	5	219	2	11
859	2	4	210	2	9
785	1	3	211	2	8
678	2	5	185	2	6
597	1	4	217	1	7
678	1	3	118	2	6
987	2	5	117	2	8
85	1	3	218	1	9
97	2	5	212	3	8
56	2	4	115	1	1
68	1	3	215	2	0
75	2	4	119	2	11

Land Measure.

Acres.	Rd.	Sq.P.	Sq.Yd.	Sq.F.	Sq.In.
574	2	39	29	8	119
684	3	19	19	7	120
564	2	29	29	8	114
974	3	25	25	3	125
867	2	15	15	4	135
976	3	38	15	5	117
657	2	37	18	6	135
849	3	32	19	7	121
578	3	33	17	8	111
849	2	36	14	6	119
765	1	34	26	8	141
387	3	32	27	7	115
849	2	34	28	8	134
976	1	25	28	8	119

These examples, it is presumed, may make one sufficiently versant in addition; but if, in any instance, they are found insufficient, that defect may be easily supplied by the master, who may set more examples, or rather put the student on setting them himself.

Sect. II. Applicate Subtraction.

	(1)				(2)				(3)		
From L.	741	11	$3\frac{1}{4}$	From L.	4179	9	$3\frac{1}{2}$	From L.	907	6	$8\frac{1}{4}$
Take	497	15	$4\frac{1}{2}$	Take	3789	19	$9\frac{3}{4}$	Take	698	16	$10\frac{1}{2}$
Rem. L.	243	15	$10\frac{3}{4}$	Rem. L.				L.			

Illustration of Example 1.

Beginning with the lowest place, because $\frac{1}{2}$d. is more than $\frac{1}{4}$d. take $\frac{1}{2}$d. from 1d. and to the remainder $\frac{1}{2}$ add $\frac{1}{4}$ for the remainder to be noted, viz. $\frac{3}{4}$: For the 1d. borrowed, add 1 to $4=5$, which being also greater than 3 in the major, subtract 5d. from 12d. *i. e.* 1s. and there are left 7d. which added to 3d. make the remainder

10d. Carry the shilling borrowed to 15, and it will make 16s. in the minor; but there being only 11s. in the major, 16 must be subtracted from 20, *i. e.* L. 1, and then there will remain 4s. which, added to 11s. in the major, make the remainder 15s. Carry L. 1 borrowed to 7 in the minor of pounds, which denomination being all integers, requires no farther illustration.

QUESTIONS.

1. A factor received on his constituent's account, viz. from A, L. 318 : 19 : $6\frac{1}{2}$; from B, L. 119 : 11 : $11\frac{1}{2}$; from C, L. 417 : 19 : $9\frac{3}{4}$; and from D, L. 97 : 11 : $8\frac{1}{2}$; out of which he paid to E, L. 217 : 3 : $4\frac{3}{4}$; and to F, L. 319 : 9 : $3\frac{1}{4}$. How much ought he to remit to his constituent? *Answer*, L. 417 : 10 : $3\frac{1}{4}$.

2. A gentleman went to a horse-race, where he gained a prize of L. 52 : 10 : 0; lost by betting against A, L. 73 : 10 : 0; won off B, L. 85 : 11 : $6\frac{1}{2}$; lost to C, L. 210 : 0 : 0; recovered an old debt of L. 417 : 18 : 0; and paid debts owing by him to the amount of L. 517 : 10 : 10; his expences came to L. 15 : 9 : 11; and he carried home with him L. 617 : 18 : 9. How much money had he in his pocket when he set out? *Answer*, L. 878 : 9 : $11\frac{1}{2}$.

3. I have L. 479 : 11 : 11 in my pocket, a London bill, which, with the premium, will bring L. 218 : 17 : $8\frac{1}{2}$, and a discountable bill worth L. 298 : 18 : 8; how much must I borrow to pay an acceptance of L. 1000? *Answer*, L. 2 : 11 : $8\frac{1}{4}$.

4. A man had two sons, between whom he divided his fortune L. 7856 : 13 : 4, giving to the elder, A, L. 1179 : 18 : 0 more than he gave to the younger, B; how much had each?

Answer, To A, L. 4518 : 5 : 8
To B, 3338 : 7 : 8

5. A and B together owe me L. 376 : 17 : 6; A owes me L. 87 : 9 : 6 more than B; required the particulars?

Avoirdupoise Weight.

	Tuns.	Cwt.	Quart.	lb.	oz.
From	597	12	1	15	9
Take	398	18	2	22	14
Rem.					

Troy Weight.

	℔.	oz.	dwt.	gr.
From	75	8	10	15
Take	49	10	15	20
Rem.				

Apothecaries Weight.

	℔.	℥	ʒ	℈	gr.
From	19	3	4	0	15
Take	9	5	6	1	17
Rem.					

Corn Measure Engl.

	L.	Q.	B.	P.	G.
From	177	1	3	2	0
Take	89	3	6	3	1
Rem.					

Corn Meas. Scots.

	Ch.	Bol.	Fir.	P.	L.
From	174	10	2	1	1
Take	84	12	3	2	2
Rem.					

Coal Measure.

	Ch.	Qrs.	B.	P.
From	141	0	7	1
Take	89	2	7	3
Rem.				

Wine Measure.

	Tuns.	Pip.	Hgs.	Gal.
From	456	1	1	37
Take	97	1	1	46
Rem.				

Ale Measure Eng.

	Hgs.	Kil.	Fir.	Gal.
From	514	1	0	7
Take	96	2	1	8
Rem.				

Ale Meas. Scots.

	Hgs.	Gal.	Pts.
From	111	10	4
Take	54	14	6
Rem.			

Cloth Measure.

	Yds.	Q.	N.
From	547	1	2
Take	498	2	3
Rem.			

Long Meas. Eng.

	Mil.	Fur.	Yds.	F.	In.
From	217	3	118	1	9
Take	129	5	213	2	10
Rem.					

Long Meas. Scots.

	Mil.	Fur.	Ch.	F.	Ell
From	114	4	8	1	3
Take	19	7	9	2	4
Rem.					

Land Measure Eng.					Land Meas. Scots.					Superf. Meas.		
	Acres.	Rds.	Sq. P.	Sq Y.	Acr.	Rds.	Sq. Fal.	Sq. E.		Yds.	F.	In.
From	17	1	27	14	74	1	33	31		143	3	119
Take	9	3	33	25	67	1	35	33		85	5	125
Rem.												

QUESTIONS

To improve the invention, and facilitate practice.

1. A goldsmith purchased silver, viz. from A, 15 lb. 11 oz. 19 dwt. 10 gr. from B, 17 lb. 10 oz. 18 dwt. 22 gr. and from C, 18 lb. 9 oz. 14 dwt. 21 gr. Of which he sold to one 12 lb. 8 oz. 18 dwt. 19 gr. to another 13 lb. 5 oz. 14 dwt. 22 gr. to a third 3 lb. 5 oz. 7 dwt. 23 gr. and to a fourth 8 lb. 11 oz. 18 dwt. 12 gr. How much is on hand? *Answer*, 14 lb. 13 dwt. 1 gr.

2. Bought one hogshead of sugar, containing 14 Cwt. 2 qr. 15 lb. of which I retailed out to A, 3 Cwt. 1 qr. 14 lb. to B, 1 Cwt. 2 qr. 15 lb. to C, 2 Cwt. 1 qr. 17 lb. and to D, 3 Cwt. 2 qr. 14 lb. What remains on hand? *Answer*, 3 Cwt. 2 qr. 11 lb.

	C.	Q.	B.	P.	
Bought corn, viz. from A,	54	3	5	3	
from B,	18	2	7	2	
from C,	42	1	6	$1\frac{1}{2}$	
from D,	59	3	5	$3\frac{1}{2}$	
					in all.
Of which sold to E,	48	2	7	$3\frac{1}{2}$	
to F,	32	3	5	$2\frac{1}{4}$	
to G,	36	2	5	2	
					in all.

What remains on hand?

3. A gentleman had an estate of 575 acres 1 rood 18 poles 19 square yards; he farmed off 499 acres 3 roods 30 poles 25 yards: How much remains in his own hand? *Answer*, 75 acres 1 rood 27 poles $24\frac{1}{4}$ yards.

4. A wine merchant bought in 48 tuns 2 hogsheads 15 gallons of wine from one person, sold to another 64 tuns 3 hogsheads 19 gallons; bought from another 52 tuns 3 hogsheads 25 gallons, from

another 43 tuns 2 hogsheads 35 gallons; he sold to another 32 tuns 19 gallons, and to another 45 tuns 18 gallons. On examining his cellars, he found there were still remaining 93 tuns 3 hogsheads 45 gallons: How much had he on hand before the commencement of these transactions? *Answer*, 90 tuns 3 hogsheads 26 gallons.

5. There are four hogsheads of sugar, weighing each gross 15 Cwt. 3 qrs. 21 lb. the weight of each empty hogshead is $47\frac{1}{2}$ lb. required the weight of the sugar? *Answer*, 62 Cwt. 6 lb.

6. The planet Venus revolves round the sun in 224 days 16 hours 49 minutes 24 seconds; and Mercury in 87 days 23 hours 15 minutes 53 seconds: required the distance of their periodical times? *Answer*, 136 days 17 hours 33 minutes 31 seconds.

7. A merchant, in balancing his books, finds due him in open accounts L. 1578 : 10 : 6, in bills L. 978 : 14 : $7\frac{1}{2}$, and in goods at prime cost L. 5711 : 8 : $4\frac{1}{4}$; he owes to A, L. 454 : 17 : 6, to B, L. 197 : 11 : $7\frac{1}{4}$, and to C, L. 91 : 17 : 10: What is his neat estate? *Answer*, L. 7524 : 6 : $6\frac{1}{4}$.

8. A brewer had in his cellar 68 butts 2 barrels 15 gallons of beer. In making the circuit of his customers, he promised to A 25 butts 1 barrel 25 gallons, to B 13 butts 1 barrel 16 gallons, and to C 20 butts: How much will remain after serving these customers? *Answer*, 9 butts 2 barrels 10 gallons.

SECT. III. APPLICATE MULTIPLICATION.

(1)

L.	s.	d.
74	15	$3\frac{1}{2}$
		9
672	17	$7\frac{1}{2}$

(2)

lb.	oz.	dwt.	gr.
57	11	19	22
			6
347	11	19	12

(3)

Tuns.	C.	Q.	lb.	oz.
15	13	3	15	11
				7
109	17	0	25	13

Illustration of Example 3.

Since $7 \times 11 = 77$ oz. we have 13 oz. after 4 lb. are taken out; so we note 13, and carry 4 to the next product. Since $7 \times 15 + 4 = 109$, deduce 3×28, or 3 qrs. and 25 lb. remain to be noted. Again, since $7 \times 3 + 3 = 24$, we have just 6 Cwt. without a remainder. Further, since $7 \times 13 + 6 = 97$, we have 4 tuns 17 Cwt.; note the Cwt. Then, lastly, $7 \times 15 + 4 = 109$ tuns, which completes the process.

In this manner all operations are performed in multiplication, when the multiplier consists only of a single digit; but when it consists of more places than one, it must be disposed into component parts, as in the following

EXAMPLES.

1. There are 56 packs of wool, each consisting of 3 lasts 7 sacks 1 wey $5\frac{1}{2}$ todds 1 stone: how much is the whole?

L.	S.	W.	T.	St.
3	7	1	$5\frac{1}{2}$	1
				8
29	3	1	$2\frac{1}{2}$	0
				7
205	1	1	$4\frac{1}{2}$	0

2. Bought 36 pieces of linen, at L. 4 : 15 : $11\frac{1}{2}$ *per* piece; what is the amount of my bill? *Answer*, L. 172 : 14 : 6.

3. Bought 52 pieces, at L. 3 : 11 : $9\frac{3}{4}$ *per* piece; what is the amount of my bill? *Answer*, L. 186 : 14 : 3.

4. There are 65 hogsheads, containing each 9 Cwt. 3 qrs. 14 lb. of sugar; what is the weight of the whole? *Answer*, 641 Cwt. 3 qr. 14 lb.

5. Each of 34 casks contains 68 lb. 11 oz. 18 dwt. 20 gr. of biscuit; what is the whole weight? *Answer*, 2345 lb. 10 oz. 0 dwt. 8 gr.

6. In a year of scarcity 17 ships arrived at different ports, each with 87 chalders 14 bolls 3 firlots 2 pecks of corn; what was the total amount of the importation? *Answer*, 1494 chalders 12 bolls 3 firlots 2 pecks.

7. Bought 97 dozen pairs mens shoes, at 47s. $11\frac{1}{2}$d. *per* dozen; what was the amount of my bill? *Answer*, L. 232 : 11 : $11\frac{1}{2}$.

8. Bought 87,000 nails, at 5s. $9\frac{3}{4}$d. *per* 1000; what is the charge? *Answer*, L. 25 : 5 : $8\frac{1}{4}$.

9. There are 126 stacks of corn, each computed to contain 3 loads 3 quarters 5 bushels 2 pecks; how much may be contained in the whole? *Answer*, 470 loads 4 quarters 5 bushels.

Questions in Multiplication.

10. A farmer had a 99 years lease, and paid yearly L. 149 : 17s. $6\frac{1}{4}$d. In settling with his landlord, at the expiration of the lease, he

produced receipts for L. 14817 : 19 : 9. How much more should he have paid to procure a discharge of the 99 years rents? *Answer*, L. 19 : 18 : 11½.

11. Bought 379 gallons of rum at 6s. 9d. per gallon, and 278 Cwt. of sugar at L 2 : 11 : 6 per Cwt. for how much must I draw on my banker to pay the purchase? *Answer*, L. 843 : 15 : 3.

12. To compensate an old debt, my friend at Nevis procured me a remittance of sugar to the amount of 375 Cwt. 2 qrs. neat, when weighed here; making allowance for shrinkage and short weight, I computed it cost me per Cwt. in Sterling as under:

Prime cost, per Cwt. L.	1	3	6
Duty and charges,	0	6	6
Freight,	0	3	6
In all, L.	1	13	6

I sold it again at L. 2 : 5 : 6 per Cwt. What was the whole cost, what did the sales amount to, and what was the gain? *Answer*, The sugar cost L. 628 : 19 : 3, it brought L. 854 : 5 : 3; consequently there was cleared on it L. 225 : 6 : 0.

13. There is a floor 18 feet 9 inches in length, and 16 feet 4 inches in breadth; how may square feet doth it contain?

	Feet.	Inch.	
	3)		
Length,	18	9	
		4	
	75	0	4×4=16, and 4 inches = ⅓ of a foot; and to divide by 3 is the same thing as to multiply by ⅓.
		4	
	300	0	
	6	3	Product of 4 inches = ⅓
	306	3	*Answer*.

14 In an area, 24 feet 7 inches long, and 18 feet 5 inches broad, how many square feet? *Answer*, 452 feet $8\frac{11}{12}$ inches, or 452 square feet and 107 square inches.

15. There is a box 8 feet 9 inches 5 lines long, 7 feet 3 inches broad, and 3 feet 5 inches thick; what is its tunnage?

	Feet.	Inch.	Lin.
4)			
	8	9	5
			7
	61	5	11
	2	2	$4\frac{1}{4}$
3)	63	8	$3\frac{1}{4}$
			3
	191	0	$9\frac{3}{4}$
4)	21	2	9
	5	3	$8\frac{1}{4}$
40)	217	7	3
	$5\frac{17}{40}$ tuns.		

16. In a piece of polished stone, 18 feet 6 inches long, 2 feet 4 inches broad, and 2 feet 3 inches deep; how many solid feet? *Answer*, 97 feet $1\frac{1}{2}$ inch, or 97 solid feet 216 solid inches; for, as $1\frac{1}{2}$ inches is the $\frac{1}{8}$ of 12, so 216 is $\frac{1}{8}$ of 1728, the solid inches in a foot.

17. There is a court, 124 feet 9 inches by 110 feet 6 inches, paved with stones 12 inches square; how many of these stones did the pavier employ? *Answer*, 13784, and 1 of $10\frac{1}{2}$ inches square.

18. There is a factory 311 feet 4 inches 7 lines, by 36 feet 7 inches 5 lines; how many square feet doth it contain? *Answer*, 11402 feet 2 inches 4 lines 11 seconds 11 thirds.

19. The floor of a weaver's factory is 144 feet 7 inches, and 9 lines, by 14 feet 6 inches and 2 lines; what is the superficial content? *Answer*, 2099 feet 4 inches 5 lines 9 seconds 6 thirds.

20. The floor of another is 259 feet 10 inches, 8 lines, by 18 feet 5 inches and 4 lines; required the superficial content? *Answer*, 4793 feet 6 inches 0 lines 10 seconds 8 thirds.

Reduction by Multiplication.

In all questions of this nature, multiply continually by those numbers or denominations, beginning with the highest, which the tables, in the introduction to this chapter, point out, adding in course the same denominations when given respectively.

EXAMPLES.

1. In L. 478 : 19 : $8\frac{1}{4}$, how many farthings?

L. 478	19	$8\frac{1}{4}$
20		
9579	shillings.	
12		
114956	pence.	
4		
459825	farthings.	

Because, by the table, 20s. = L. 1, we multiply the given pounds by 20, and add 19s. to the product in course. Because 12d. = 1s. we multiply the shillings by 12, and add the given 8d. to the product. In like manner we multiply by the number of farthings in a penny, viz. 4, and to the product add 1 farthing given; and this last product is the answer.

2. In 47 Cwt. 1 qr. 20lb. how many ounces? *Answer*, 84992.

3. In 482 lb. 7 oz. 13 dwt. 21 gr. troy, how many grains? *Answer*, 2780013.

4. In 48 loads 12 bushels of corn, how many gallons? *Answer*, 15456.

5. In 72 hogsheads of wine how many pints? *Answer*, 36288.

6. The circumference of the earth is 360 degrees, and each degree contains $69\frac{1}{2}$ statute miles; how many inches does it measure? *Answer*, 1585267200.

7. How many hours and minutes from the creation to the end of the year 1787, it being generally counted 5791 years? *Answer*, 3045833280 minutes.

8. In 96 Scots acres, how many square ells? *Answer*, 5529600.

9. In 172 butts 15 gallons ale, Scottish measure; how many pints? *Answer*, 22136.

10. In 786 tuns 1 pipe 120 gallons of wine; how many pints? *Answer*, 1586544.

11. In 755 lasts 11 sacks 1 wey $4\frac{1}{2}$ todds; how many lbs. of wool? *Answer*, 3302152 lbs.

12. In 378 acres 2 roods 22 poles $13\frac{1}{4}$ square yards of land; how many square feet English measure? *Answer*, $164935683\frac{1}{4}$.

SECT. IV. APPLICATE DIVISION.

Here obſerve, 1. That, when a remainder ariſes, it muſt be reduced to the next inferior denomination, and the diviſion continued.

2. If the diviſor is a compoſite number, it will be beſt to uſe the component parts; and, in general, to uſe the contractions pointed out in diviſion of integers when practicable.

EXAMPLES.

Divide L. 672 : 17 : $7\frac{1}{2}$ among nine men equally.

```
9 ) 672   17   7½  (  74   15   3½ to each.
    63
   ----
     42
     36
    ----
      6
     20
    ----
    137
      9
    ----
     47
     45
    ----
      2
     12
    ----
     31
     27
    ----
      4
      2
    ----
      9
      9
    ----
      0
```

By a little practice, the remainder may be eaſily reduced mentally, and then the work will ſtand thus:

```
9 ) 672   17   7½
   -----------------
     74   15   3½
```

This requires no illuſtration, but the ſame example done at length.

2. Divide L. 374 : 18 : 6 among 36 men equally. *Anſwer*, L. 10 : 8 : $3\frac{1}{6}$.

3. 36 pieces of linen cost L. 172 : 14 : 6; what was that per piece? *Answer*, L. 4 : 15 : 11$\frac{1}{4}$.

4. 52 pieces of linen cost L. 186 : 14 : 3; what is that per piece? *Answer*, L. 3 : 11 : 9$\frac{1}{4}$.

5. 65 hogsheads of sugar weigh 641 Cwt. 3 qr. 14 ℔.; how much is contained in each? *Answer*, 9 Cwt. 3 qr. 14 ℔.

6. In 34 casks are contained 2345 ℔. 10 oz. 0 dwt. 8 gr. troy of biscuit; how much is packed in each? *Answer*, 68 ℔. 11 oz. 18 dwt. 20 gr.

7. Bought 97 dozen pairs of shoes and paid in whole L. 232, 11s. 11$\frac{1}{2}$d.; what was that per dozen? *Answer*, L. 4 : 7 : 11$\frac{1}{2}$.

8. Bought 87 M. nails for L. 25 : 5 : 8$\frac{1}{4}$; what did they cost per M.? *Answer*, 5s. 9$\frac{3}{4}$d.

9. Bought 12 reams of paper for L. 5 : 2 : 6; what did it cost per ream? *Answer*, 8s. 6$\frac{1}{2}$d.

10. Bought 96 barrels of herrings for L. 166 : 2s.; what is that per barrel? *Answer*, L. 1 : 14 : 7$\frac{1}{4}$.

11. Bought 123 gallons of brandy for L. 47 : 10 : 8$\frac{1}{4}$; what is that per gallon? *Answer*, 7s. 8$\frac{3}{4}$d.

12. A gentleman has of neat yearly income L. 174 : 2 : 8$\frac{1}{2}$; how much may he spend a-day, reckoning 365 days to a year? *Answer*, 9s. 6$\frac{1}{2}$d.

13. At the expiration of a lease of 99 years, the landlord found, by his books, he had received in full of rent L. 14837 : 18 : 8$\frac{1}{4}$; what did the farmer pay a-year? *Answer*, L. 149 : 17 : 6$\frac{3}{4}$.

14. 126 stacks of corn, all of equal content, are computed to contain 722 loads 4 quarters 5 bushels; how much is in each stack? *Answer*, 5 loads 3 quarters 5 bushels 2 pecks.

15. The planet Mercury revolves round the sun in 88 of our days; how many revolutions will he perform in 17 years and 307 days? *Answer*, 74.

16. Bought 178 chalders 14 bolls of meal for L. 887 : 19 : 6; what did it cost per boll? *Answer*, 6s. 2$\frac{1}{2}$d.

17. Bought 577 spindles of yarn for L. 221 : 10s. I propose to gain L. 50 on the sales; how may I retail it per spindle, to compass my design? *Answer*, 9s. 4$\frac{3}{4}$d.

Reduction by Division.

Here observe, that all lower denominations are reduced to higher, by dividing by as many of the lower as make one of the greater.

QUESTIONS.

1. In 459825 farthings, how many pounds, shillings, and pence?

$$\begin{array}{r l}
4\ ) & 459825 \\
\hline
12\ ) & 114956\tfrac{1}{4} \\
2{,}0\ ) & 957{,}9 \quad 8 \\
\hline
& 478 \quad 19 \quad 8\tfrac{1}{4} \quad \textit{Answer.}
\end{array}$$

2. In 84992 ounces avoirdupoise, how many Cwt. quarters, pounds, &c.? *Anſwer*, 47 Cwt. 1 qr. 20 ℔.

2. In 2780013 grains troy, how many lb. &c. *Anſwer*, 482 lb. 7 oz. 13 dwt. 21 gr.

4. In 15456 gallons, how many loads, &c.? *Anſwer*, 48 loads 12 buſhels.

5. In 36288 pints, how many hogſheads of wine? *Anſwer*, 72.

6. Betwixt London and York the diſtance is computed to meaſure 9567360 inches; required the diſtance in miles? *Anſwer*, 151.

7. In 22136 pints, how many butts Scotch meaſure? *Anſwer*, 172 butts 15 gallons.

8. In 1586544 pints of wine, how many tuns? *Anſwer*, 786 tuns 1 pipe 120 gallons.

9. In 3302152 ℔. of wool, how many laſts? *Anſwer*, 755 laſts 11 ſacks 1 wey $4\frac{1}{2}$ todds.

10. In $164935 68\frac{1}{4}$ ſquare feet Engliſh, how many acres? *Anſwer*, 378 acres 2 roods 22 poles $13\frac{1}{2}$ yards.

Application of Multiplication and Diviſion in Mixt Reduction.

This ſpecies of reduction is a combination of the other two, and muſt always be adopted when the one ſpecies is not contained in the other a preciſe number of times. In all queſtions of this kind reduce one of the given numbers, and alſo one of that required, to the loweſt denomination in the aſſigned value of either; multiply by the firſt, and divide the product by the ſecond; then will the quotient be the anſwer.

QUESTIONS.

1. I want to exchange 317 moidores, at 27s. each, for guineas of 21s. each; how many of the latter ought I to receive?

```
    317
     27
  -----
   2219
   634
  -----
3 ) 8559
  -----
7 ) 2853
  -----
   407 4/7 Answer.
```

Here ſhillings is the loweſt name; conſequently, to multiply by 27, reduces the moidores to ſhillings; and to divide by 3×7, or 21, finds the guineas contained in thoſe ſhillings.

27. In 748 piſtoles, at 16s. 6d. how many half guinea pieces, at 10s. 6d.?

```
      (1)                        (2)
      748                        748
       33 ſixpences               11
    -----                      ------
     2244                  7 ) 8228
    2244                       ------
    -----                      1175 3/7
3 ) 24684
    -----
7 )  8228
    -----
     1175 3/7
```

Since there are 33 ſixpences in a piſtole, and 21 of the ſame name in 10s. 6d. the firſt is uſed as a multiplier, and the ſecond as a diviſor.

It would have been ſtill ſhorter, had both factors been farther abridged, as they are each commenſurable by 3, as in the ſecond operation: And this method of abridging the terms ought never to be omitted when practicable, as it is a maxim that will always hold, in all computations whatever, that ſhort proceſſes are always ſureſt.

3. In 745 French arpens, each containing $1\frac{3}{16}$ acre Engliſh, how many Engliſh acres?

$$\left.\frac{745 \times 19}{4 \times 4}\right\} = 884\tfrac{11}{16} \textit{ Anſwer.}$$

4. In 745 French leagues, 25 of which make a degree, how many ſtatute miles, $69\frac{1}{2}$ to a degree?

$$\left.\frac{745 \times 69\frac{1}{2}}{5 \times 5}\right\} = 2071\tfrac{1}{10} \textit{ Anſwer.}$$

5. In 845 geographical miles, how many ſtatute miles?

$$\left.\frac{845 \times 69\frac{1}{2}}{60}\right\} = 978\tfrac{19}{24} \textit{ Anſwer.}$$

6. In 5745 merks, at 1s. $1\frac{1}{3}$d. how much Sterling? *Anſwer*, L. 319 : 3 : 4.

7. In L. 4573 Scots, at 1s. 8d. how much Sterling? *Anſwer*, L. 381 : 1 : 8.

8. In 97415 yards Engliſh, how many Scots ells? *Anſwer*, $92079\frac{17}{37}$.

9. In 4597 Scots pints, how many Engliſh gallons? *Anſwer*, $2063\frac{8}{693}$.

10. In 6734 French crowns, at $31\frac{1}{4}$d. how many rix-dollars of Hamburg, at 4s. 6d.? *Anſwer* 3959.

11. A gentleman was robbed of the following ſums, viz.

57 pieces of L. 3 : 12 : 0 each,
87 pieces of 36s. each,
174 pieces of 16s. 6d. each,
268 pieces of 27s. each,
137 pieces of 4s. 9d. each,
278 guineas, at 21s. each,
125 quarter ditto, at 5s. 3d. each,
125 notes of 30s. each, and
26 notes of L. 5 each;

What was the total amount of his loſs?

Anſwer, L. 1541 : 18 : 0.

12. The diſtance of Conſtantinople from Vienna is 700 miles; how oft will a coach wheel, 6 yards in circumference, turn round in travelling from the one city to the other? *Anſwer*, $205333\frac{1}{3}$.

13. A traveller wants to be accommodated in moidores at 27s.; piſtoles at 16s. 6d.; 4s. 9d. pieces, and quarter guinea pieces, of each an equal number; how many will he get of each ſort for L. 676 : 1 : 3? *Anſwer*, $253\frac{15}{214}$ of each?

14. It is computed, that 1000 Engliſh acres are equal to 787 acres by the Scotch chain; how many Engliſh acres are contained in 451 Scotch? *Anſwer*, 573 : 0 : 9 : $34\frac{494}{787}$.

15. In 785 wine, how many ale gallons? *Answer*, $719\frac{147}{252}$.

16. Of Jamaica currency, L. 7 is equal to L. 5 Sterling; how much Sterling will equal L. 976 Jamaica currency? *Answer*, $697\frac{1}{7}$.

17. The *par* of an English shilling in Ireland is 13d.; what will L. 756 : 19 : 0 English Sterling be worth in Ireland at that rate? *Answer*, L. 820 : 0 : 7 Irish.

18. In Glasgow 22 ounces make a pound Tron weight, and in Edinburgh $17\frac{1}{2}$; how much weight at Edinburgh will equiponderate 87 stones 11 lb. at Glasgow? *Answer*, 110 stones 3 lb. $13\frac{1}{2}$ oz.

19. There are 56 Scots miles in a degree, and $69\frac{1}{2}$ statute; how many English miles between Edinburgh and Inverness, supposing the reckoning in computed miles to be 130? *Answer*, $161\frac{12}{56}$.

QUERIES.

1. In adding broken denominations, why is the general rule departed from, of carrying the surplus of ten, or tens? *Answer*, Because, though the flux of integers be in a decuple proportion, the flux of the subdivisions of a relative unit are not; and yet the reason for carrying at 10, in addition of integers, furnishes a reason also for the manner of adding subdivisions: For, since 12 pence = 1 shilling, it is as reasonable that I carry the twelves, in the sum of pence, to be added with the column of shillings, as it is to carry the tens found in the column of units to be added with the tens in the next column.

2. Why, in the same case, is the rule departed from in subtraction? *Answer*, Properly speaking, it is not departed from; for, whatever is borrowed of one subdivision is paid back in specie to the next in course; and, when the opposite sides of an account are equally increased or diminished, the same difference will invariably exist. The same reason holds good in multiplication and division.

CHAP. III.

OF VULGAR FRACTIONS.

INTRODUCTION.

Before we proceed to Fractions, it may, perhaps, not be improper here to deſcribe the genus or ſpecies of numbers that ſeem moſt uſeful and neceſſary, whether integral or fractional.

1. Number, in general, may be defined, a collection or aſſemblage of ſeveral units, expreſſing the ratio of a quantity of any kind to another quantity of the ſame kind.

2. An *Integer*, or *Whole number*, is an unit, or a collection of units; as 1, 7, 59, &c.

3. A *Fraction*, or *Broken number*, is any part or parts of an unit, ariſing from diviſion, expreſſed by two numbers and a line drawn between them. Of theſe numbers, that which expreſſes the number of parts is the numerator, as and that which expreſſes the quality of thoſe parts is the denominator, as $\dfrac{5}{6}$

Theſe numbers are together read five-ſixth parts of 1, or ſimply five-ſixths; importing, that 1 is divided into 6 parts, and that the fraction expreſſeth 5 of them.

4. A *Mixt number* is compoſed of an integer and a fraction annexed to it, as $4\frac{3}{4}$, $5\frac{1}{3}$, &c.

5. A *Proper fraction* is always leſs than 1, and conſequently has its numerator leſs than its denominator; as $\frac{1}{4}$, $\frac{7}{8}$, $\frac{4}{5}$, &c.

6. An *Improper fraction* is either equal to or greater than 1; and in the one caſe its numerator is equal to and in the other greater than its denominator; as, $\frac{4}{4}$, $\frac{3}{3}$, $\frac{8}{8}=1$, and $\frac{5}{4}=1\frac{1}{4}$, $\frac{6}{3}=2$, $\frac{9}{2}=4\frac{1}{2}$, &c.

7. A *Compound fraction* is compoſed of ſeveral fractions connected with the particle *of*, every preceding one of which is ſome part of the ſucceeding; as $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{1}{3}$, which implies that the unit is divided into three parts, one of which is expreſſed by $\frac{1}{3}$; this $\frac{1}{3}$ is again ſubdivided into two parts, one of which is expreſſed by $\frac{1}{2}$; and this $\frac{1}{2}$ again is ſubdivided into four parts, one of which is expreſſed by $\frac{1}{4}$. In like manner, ſince 4d. is $\frac{1}{3}$ of 1s. and 1s. is $\frac{1}{20}$ of a pound, 4d. may be expreſſed fractionally $\frac{1}{3}$ of $\frac{1}{20}$ of L. 1. In contradiſtinction, a fraction having only one numerator and one denominator is called a *Simple fraction*.

8. An *Even number* is ſuch as 2, or any even number, can meaſure without a remainder.

9. An *Odd number* is ſuch, as 1, or ſome uneven number, can meaſure without a remainder.

10. A *Compoſite number* is that which can be diſpoſed into component parts.

11. A *prime number* is that which can be meaſured by unity only.

12. A number which meaſureth another number is called an *aliquot* or *Even part* of that number, and the number ſo meaſured is called a *Multiple* of that even part.

13. *A Square number* is the product of any number multiplied by itſelf, of which the number ſo multiplied is called the *Root.*

14. *A Cube number* is produced by multiplying the ſquare into the root.

15. *A Biquadrate number* is produced by multiplying the cube into the root.

16. *A Surſolid number* is produced by multiplying the biquadrate into its root.

17. *Similar numbers* are ſuch as can be ranged into rectangles whoſe ſides are proportional.

18. *Proportional numbers* are ſuch as bear the ſame ratio or relation to each other.

SECT. I. REDUCTION OF VULGAR FRACTIONS.

Caſe 1. *To reduce a whole number to a fraction of an aſſigned denominator.*

1. Multiply the whole number into the aſſigned denominator, for a numerator to the fraction.

2. Place the aſſigned denominator below the numerator found as above, to complete the fraction.

EXAMPLES.

1. Reduce 8 to a fraction whoſe denominator ſhall be 9.

P. 1. $8 \times 9 = 72$

P. 2. $\frac{72}{9}$

2. Reduce 15 to a fraction whoſe denominator ſhall be 5.

Anſwer, $\frac{75}{5}$.

3. Reduce 21 to a fraction whose denominator shall be 7.
Answer, $\frac{147}{7}$.
4. Reduce 17 to a fraction whose denominator shall be 16.
Answer, $\frac{272}{16}$.
5. Reduce 50 to a fraction whose denominator shall be 15.
Answer, $\frac{750}{15}$.
6. Reduce 86 to a fraction whose denominator shall be 20.
Answer, $\frac{1720}{20}$.

Corol. Hence it will be easy to conceive, that, if 1 be the assigned denominator, which is always the case when a whole number is to be expressed fractionally, and no other denominator assigned, to subscribe 1 for the denominator is all that is necessary.

Case II. *To reduce improper fractions to equivalent whole or mixt numbers.*

1. Divide the numerator by the denominator, and the quotient will be the integral part; and, when there is no remainder, will be the answer.

2. Place the remainder on the right of the whole number, over a line, and the divisor under it for the fractional part; and both together constitute the mixt number required.

EXAMPLES.

1. Reduce $\frac{72}{9}$ to an equivalent whole number.
$\frac{72}{9} = 8$, p. 1.
2. Reduce $\frac{56}{5}$ to an equivalent mixt number.
$\frac{56}{5} = 11\frac{1}{5}$, p. 1. and 2.
3. Reduce $\frac{119}{9}$ to an equivalent mixt number.
Answer, $13\frac{2}{9}$.
4. Reduce $\frac{361}{12}$ to an equivalent mixt number.
Answer, $30\frac{1}{12}$.
5. Reduce $\frac{735}{12}$ to a mixt number.
Answer, $61\frac{3}{12}$.
6. Reduce $\frac{458}{16}$ to a mixt number.
Answer, $28\frac{5}{8}$.
7. Reduce $\frac{637}{20}$ to a mixt number.
Answer, $31\frac{17}{20}$.
8. Reduce $\frac{1728}{24}$ to a whole number.
Answer, 72.
9. Reduce $\frac{1788}{19}$ to a mixt number.
Answer, $94\frac{2}{19}$.

10. Reduce $\frac{577}{36}$ to a mixt number.

Answer, $16\frac{1}{36}$.

Case III. *To reduce a mixt number to an improper fraction.*

1. Multiply the integral part by the denominator of the fraction, and to that product add the numerator; the sum is the numerator required.

2. Under this new numerator place the old denominator, which will complete the fraction.

EXAMPLES.

1. Reduce $11\frac{1}{5}$ to an equivalent improper fraction

$$5 + 11 \times 1 = \frac{56}{5} \quad \begin{matrix} \text{p. 1.} \\ \text{p. 2.} \end{matrix}$$

2. Reduce $13\frac{2}{9}$ to an improper fraction.
Answer, $\frac{119}{9}$.
3. Reduce $30\frac{11}{12}$ to an improper fraction.
Answer, $\frac{371}{12}$.
4. Reduce $69\frac{3}{4}$ to an improper fraction.
Answer, $\frac{279}{4}$.
5. Reduce $28\frac{5}{8}$ to an improper fraction.
Answer, $\frac{229}{8}$.
6. Reduce $31\frac{17}{20}$ to an improper fraction.
Answer, $\frac{637}{20}$.
7. Reduce $941\frac{9}{19}$ to an improper fraction.
Answer, $\frac{17888}{19}$.
8. Reduce $16\frac{1}{36}$ to an improper fraction.
Answer, $\frac{577}{36}$.

Note, It is obvious, that the second case is the converse of the first and third, as they are also the contrary of it, and that the one is a proof of the other.

Case IV. *To reduce a compound fraction to a simple one.*

1. Multiply the numerators continually, and the last product will be a new numerator.

2. Multiply the denominators continually, and the last product will be a new denominator.

EXAMPLES.

1. Reduce $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{5}{8}$ to a simple fraction of the same value.

$$\frac{3 \times 1 \times 5 = 15, \text{ p. } 1.}{4 \times 2 \times 8 = 64, \text{ p. } 2.}$$

Sometimes a little of this process may be saved, by neglecting such figures above and below the line as are the same in both.

2. Reduce $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{8}{9}$ to a simple fraction.

$$\left.\frac{4 \times 7 = 28}{6 \times 9 = 54}\right\} \textit{Answer.}$$

3. Reduce $\frac{1}{2}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{2}{3}$ to a simple fraction.
Answer, $\frac{5}{21}$.
4. Reduce $\frac{4}{5}$ of $\frac{5}{8}$ of $\frac{3}{4}$ of $\frac{2}{3}$ to a simple fraction.
Answer, $\frac{2}{8}$.
5. Reduce $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{2}{3}$ to a simple fraction.
Answer, $\frac{2}{5}$.
6. Reduce $\frac{12}{18}$ of $\frac{18}{26}$ of $\frac{26}{35}$ of $\frac{35}{39}$ to a simple fraction.
Answer, $\frac{12}{39}$.
7. Reduce $\frac{5}{8}$ of $\frac{8}{9}$ of $\frac{3}{7}$ of $\frac{1}{3}$ to a simple fraction.
Answer, $\frac{5}{63}$.
8. Reduce $\frac{27}{53}$ of $\frac{5}{9}$ of $\frac{53}{63}$ of $\frac{9}{27}$ to a simple fraction.
Answer, $\frac{5}{63}$.

Note, The continued multiplication of all the denominators brings them down to the lowest name, and the continued multiplication of all the numerators ascertains the number of that name in the product.

Case V. *To reduce a fraction of a lower denomination to a fraction of a higher.*

1. Form a compound fraction, by comparing the given fraction with the superior denominations.
2. Reduce the compound fraction so formed to a simple one.

EXAMPLES.

1. Reduce $\frac{1}{2}$d. to the fraction of shilling.

$\frac{1}{2}$d. is $\frac{1}{2}$ of $\frac{1}{12} = \frac{1}{24}$s.

2. Reduce $\frac{1}{2}$d. to the fraction of one pound.
$\frac{1}{2}$d. is $\frac{1}{2}$ of $\frac{1}{12}$ of $\frac{1}{20} = \frac{1}{480}$.
3. Reduce 15 grains Troy to the fraction of 1 ℔.
Answer, $\frac{15}{5760}$.
4. Reduce 24 ℔. to the fraction of 1 Cwt. avoirdupoise.
Answer, $\frac{24}{112}$.
5. Reduce 18 ℔. wool to the fraction of a last.
Answer, $\frac{18}{4368}$.
6. Reduce 15 pecks to the fraction of 1 chalder.
Answer, $\frac{15}{256}$.
7. Reduce 20 gallons of wine to the fraction of a tun.
Answer, $\frac{20}{252}$.
8. Reduce 37 minutes to the fraction of a year.
Answer, $\frac{37}{525949}$.

Case VI. *To reduce a fraction of a greater denomination to a fraction of a lesser.*

1. Reduce the numerator by multiplication to the denomination assigned for a numerator.
2. Under the numerator so found, place the given denominator, which completes the fraction.

EXAMPLES.

1. Reduce L. $\frac{7}{11}$ to the fraction of one farthing.
$$\frac{7 \times 20 \times 12 \times 4 = 6720, \text{ p. } 1.}{11, \text{ p. } 2.}$$
2. Reduce $\frac{8}{9}$ yard to the fraction of a nail.
Answer, $\frac{128}{9}$.
3. Reduce $\frac{7}{8}$ Cwt. to the fraction of a pound.
Answer, $\frac{784}{8}$.
4. Reduce $\frac{7}{8}$ of a year to the fraction of an hour.
Answer, $\frac{61362}{8}$.
5. Reduce $\frac{4}{5}$ of a tun to the fraction of a pint.
Answer, $\frac{8064}{5}$.
6. Reduce $\frac{15}{256}$ of a chalder to the fraction of a boll.
Answer, $\frac{240}{256}$.

Case VII. *To find the value of a fraction.*

1. Reduce the numerator to the next inferior denomination, and divide by the denominator for parts of that denomination.

2. If there be a remainder, reduce it in the like manner to the next inferior denomination; and thus proceed, as in reduction by division, till all the known parts are quoted.

EXAMPLES.

1. Value L. $\frac{3}{4}$ $\left\{\frac{3 \times 20}{4}\right\} = 15s.$

2. Value L. $\frac{9}{16}$ $\left\{\frac{9 \times 20}{16}\right\} = 11\frac{4}{16}$, and $\left\{\frac{4 \times 12}{16}\right\} = 3$: consequently, $\frac{9}{16} = 11s. 3d.$

3. Value $\frac{53}{94}$ tun, *Answer*, 11 Cwt. 1 qr. $2\frac{92}{94}$ lb.

4. Value $\frac{41}{50}$ of a year. *Answer*, 299 days 7 hours 12 minutes.

5. Value $\frac{18}{4368}$ last of wool. *Answer*, 18 lb.

6. Value $\frac{15}{156}$ of a chalder. *Answer*, $24\frac{8}{13}$ pecks.

7. Value $\frac{20}{252}$ of a tun of wine. *Answer*, 20 gallons.

8. Value $\frac{19}{45}$ of a Scotch acre, *Answer*, 3 roods 18 falls 24 ells.

Case VIII. *To reduce a fraction to the lowest terms.*

1. Divide the denominator by the numerator, and the last divisor by the remainder, till nothing remain.
2. Divide both numerator and denominator by the last divisor in the former operation, which is their common measure, and the two quotients are the members of the new fraction.

EXAMPLES.

1. Reduce $\frac{144}{1728}$ to the lowest terms.

$$\begin{array}{r} 144)\ 1728\ (\ 12 \\ 144 \\ \hline 288 \\ 288 \\ \hline 0 \end{array}$$

Again, $\left\{\begin{matrix}144\\144\end{matrix}\right\} \frac{144}{1728} \left[\frac{1}{12}\right\}$ *Answer.*

2. Reduce $\frac{784}{952}$ to their lowest terms.

```
784 ) 952 ( 1
      784
      ---
      168 ) 784 ( 4
            672
            ---
            112 ) 168 ( 1
                  112
                  ---
                   56 ) 112 ( 2
                        112
                        ---
                          0
```

$$\text{Then } \begin{Bmatrix} 56 \\ 56 \end{Bmatrix} \frac{784}{952} \left[\frac{14}{17} \right\} \textit{Answer.}$$

3. Reduce $\frac{468}{845}$ to the lowest terms. *Answer*, $\frac{36}{65}$.

4. Reduce $\frac{2548}{2912}$ to the lowest terms. *Answer*, $\frac{7}{8}$.

Note, It will be shorter to divide both factors by any number at pleasure that will measure both without a remainder; and if it can be brought lower still, do the same by the quotients continually, till no common measure can be found: The last quotients give the fraction in the lowest terms.

For the ready invention of a divisor, observe,

1. If both factors terminate in even numbers, 2, or perhaps some multiple of 2, will be the common measure.
2. If they terminate in 5, or in 5 in one and 0 in another, 5 is the divisor.
3. If they terminate in ciphers, cut off the same number from both factors, and divide the figures to the left by any common measure.

EXAMPLES.

$\frac{355}{710} = \frac{71}{142} = \frac{1}{2}$, and $\frac{375}{1125} = \frac{1}{3}$, and $\frac{192}{576} = \frac{1}{3}$, and $\frac{60,0}{72,0} = \frac{5}{6}$.

1. Reduce $\frac{72}{81}$ to their lowest terms.
   Answer, $\frac{8}{9}$.
2. Reduce $\frac{144}{156}$ to their lowest terms.
   Answer, $\frac{12}{13}$.
3. Reduce $\frac{390}{480}$ to their lowest terms.
   Answer, $\frac{13}{16}$.

4. Reduce $\frac{225}{315}$ to their lowest terms.

Answer, $\frac{5}{7}$.

Case IX. *To reduce fractions having different denominators to other fractions of the same value, having one common denominator.*

1. Multiply each numerator, one after another, into all the given denominators, its own only excepted, for new numerators.

2. Multiply all the denominators continually for a common denominator.

EXAMPLES.

1. Reduce $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{8}$, to fractions having the same denominator.

$$\left.\begin{array}{l} 3 \times 5 \times 8 = 120 \\ 4 \times 4 \times 8 = 128 \\ 5 \times 5 \times 4 = 100 \end{array}\right\} \text{wherefore} \left\{\begin{array}{l} \frac{120}{160} = \frac{3}{4}. \\ \frac{128}{160} = \frac{4}{5}. \\ \frac{100}{160} = \frac{5}{8}. \end{array}\right.$$

$$4 \times 5 \times 8 = 160$$

2. Reduce $\frac{1}{2}$, $\frac{3}{5}$, $\frac{6}{7}$, $\frac{5}{8}$ to fractions having a common denominator.

Answer, $\frac{280}{560} = \frac{1}{2}$, $\frac{336}{560} = \frac{3}{5}$, $\frac{480}{560} = \frac{6}{7}$, $\frac{350}{560} = \frac{5}{8}$.

3. Reduce $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$ to fractions having one common denominator.

Answer, $\frac{720}{960} = \frac{3}{4}$, $\frac{768}{960} = \frac{4}{5}$, $\frac{800}{960} = \frac{5}{6}$, $\frac{840}{960} = \frac{7}{8}$.

4. Reduce $\frac{1}{3}$, $\frac{4}{9}$, $\frac{5}{6}$, $\frac{2}{5}$ to fractions having one common denominator.

Answer, $\frac{270}{810} = \frac{1}{3}$, $\frac{360}{810} = \frac{4}{9}$, $\frac{675}{810} = \frac{5}{6}$, $\frac{324}{810} = \frac{2}{5}$.

5. Reduce $\frac{4}{7}$, $\frac{5}{8}$, $\frac{3}{5}$, $\frac{2}{3}$ to fractions having one common denominator.

Answer, $\frac{4}{7} = \frac{480}{840}$, $\frac{5}{8} = \frac{525}{840}$, $\frac{3}{5} = \frac{504}{840}$, and $\frac{2}{3} = \frac{560}{840}$.

6. Reduce $3\frac{3}{4}$, $4\frac{1}{2}$, $5\frac{1}{5}$, $\frac{5}{8}$ to fractions of one common denominator.

Answer, $\frac{1200}{320}$, $\frac{1440}{320}$, $\frac{1664}{320}$, and $\frac{200}{320}$ in the order given.

Case X. *To reduce a given fraction to another of the same value, when possible, having the denominator assigned.*

1. Multiply the numerator by the assigned denominator.

2. Divide the product by the old denominator, and the quotient, if there is no remainder, gives the new numerator. If there is a remainder, it cannot be done.

EXAMPLES.

1. Reduce $\frac{5}{6}$ to a fraction whose denominator shall be 12.

$$\left.\frac{12 \times 5}{}\right\} = \left\{\begin{array}{l} 10 \text{ p. 1.} \\ \overline{} \\ 12 \text{ p. 2.} \end{array}\right.$$

2. Reduce $\frac{3}{4}$ to a fraction whose denominator shall be 20.
   Answer, $\frac{15}{20}$.
3. Reduce $\frac{1}{2}$ to a fraction whose denominator shall be 6.
   Answer, $\frac{3}{6}$.
4. Reduce $\frac{4}{15}$ to a fraction whose denominator shall be 60.
   Answer, $\frac{16}{60}$.

SECT. II. ADDITION OF VULGAR FRACTIONS.

In adding fractions, observe,

1. That compound fractions must be reduced to simple ones.
2. That mixt numbers must be reduced to improper fractions.
3. Fractions of different denominations must be reduced to others of the same integers.
4. Fractions of different denominators must be reduced to fractions of one common denominator.
5. Then the sum of the several numerators, set over the common denominator, gives a fraction equivalent to all those proposed to be added, which may be valued, or reduced to a mixt number, or to lower terms, as occasion requires.

EXAMPLES.

1. Add $\frac{3}{8}+\frac{5}{8}+\frac{7}{8}$. $7+5+3=\frac{15}{8}=1\frac{7}{8}$.

Here the denominator being common, no other reduction was necessary.

2. Add $\frac{4}{7}+\frac{5}{8}+\frac{3}{5}+\frac{2}{3}$. *Answer*, $2\frac{389}{840}$.
3. Add $\frac{3}{5}$ of $\frac{1}{2}+\frac{5}{6}$ of $\frac{1}{3}$.

$$\left.\begin{matrix} 3\times 1=3 \\ \overline{} \\ 5\times 2=10 \end{matrix}\right\}\text{ and }\left\{\begin{matrix} 5\times 1=5 \\ \overline{} \\ 6\times 3=18 \end{matrix}\right\}\text{ Then }\left\{\begin{matrix} 3\times 18=54 \\ 5\times 10=50 \end{matrix}\right.$$

$$\frac{104}{10\times 18=180}=\frac{26}{45}.$$

4. Add $4\frac{1}{2}+3\frac{1}{3}$. $4\frac{1}{2}=\frac{9}{2}$, and $3\frac{1}{3}=\frac{10}{3}$.

Then
$$\begin{aligned} 9\times 3&=27 \\ 10\times 2&=20 \end{aligned}$$
$$\frac{47}{2\times 3=6}=7\frac{5}{6}$$

5. Add L.$\frac{3}{7}$+$\frac{3}{7}$s.+$\frac{3}{7}$d.

$\frac{3}{7}$s.$=\frac{3}{140}$, and $\frac{3}{7}$d.$=\frac{3}{1680}$.

Then
$$\begin{array}{rcrcrcr} 3 & \times & 140 & \times & 1680 & = & 705600 \\ 3 & \times & 7 & \times & 1680 & = & 35280 \\ 3 & \times & 7 & \times & 140 & = & 2940 \end{array}$$

$$\left.\frac{743820}{7\times 140\times 1680=1646400}\right\}=9\text{s. }\tfrac{3}{7}\text{d.}$$

$$\left.\begin{array}{lcr} \text{Or, L.}\frac{3}{7} & = & 8\text{s. }6\frac{6}{7}\text{d.} \\ \frac{3}{7}\text{s.} & = & 5\frac{1}{7}\text{d.} \\ \frac{3}{7}\text{d.} & = & \frac{3}{7}\text{d.} \end{array}\right\}=9\text{s. }\tfrac{3}{7}\text{d.}$$

6. What is the sum of $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}$?
Answer, $2\frac{1}{12}$.

7. What is the sum of $\frac{3}{4}$ of $\frac{3}{5}$ + $\frac{9}{10}$ of $\frac{6}{7}$ + $\frac{5}{8}$?
Answer, $1\frac{237}{280}$.

8. What is the sum of L.$\frac{3}{4}$, + L.$\frac{5}{8}$, + $\frac{3}{4}$s. + $\frac{3}{4}$d.
Answer, $\frac{3624}{2560}$ = L. 1 : 8 : $3\frac{3}{4}$.

9. What is the sum of $\frac{1}{2}$ of $\frac{1}{3}$ + $13\frac{3}{4}$ + $24\frac{5}{6}$?
Answer, $38\frac{9}{12}$.

10. What is the sum of $3\frac{1}{4}$ Cwt. $4\frac{1}{2}$ ditto, $5\frac{1}{3}$ ditto, and $\frac{5}{8}$ ditto?
Answer, 14 Cwt. 0 qr. $8\frac{2}{3}$ lb.

SEC. III. SUBTRACTION OF VULGAR FRACTIONS.

In ſubtracting fractions, obſerve the firſt four directions laid down in addition, then will the difference of the numerators ſet over the common denominator be the remainder required.

EXAMPLES.

1 What is the difference between $\frac{7}{12}$ and $\frac{5}{12}$?
Anſwer, $7-5=\frac{2}{12}$.

2. What is the difference between $\frac{7}{8}$ and $\frac{5}{6}$?

$$\left.\begin{array}{r} 42 - 40 = 2 \\ \overline{} \\ 8 \times 6 = 48 \end{array}\right\} \textit{Anſwer.}$$

3. What is the difference between $3\frac{3}{4}$ and $2\frac{5}{6}$?
Anſwer, $\frac{11}{12}$.

4. What is the difference between $\frac{3}{4}$ of $\frac{1}{2}$ and $\frac{5}{6}$ of $\frac{1}{3}$?
Anſwer, $\frac{54}{144}-\frac{40}{144}=\frac{14}{144}$, or $\frac{7}{72}$.

5. What is the difference between L.$\frac{7}{9}$ and $\frac{7}{8}$s.?
Anſwer, L. $\frac{1120}{1440}-\frac{63}{1440}=\frac{1057}{1440}=$ 14s. $8\frac{1}{6}$d.

6. What is the difference between $\frac{4}{5}$ and $\frac{2}{3}$ of $\frac{3}{4}$?
Anſwer, $\frac{3}{10}$.

7. What is the difference between $13\frac{3}{8}$ and $\frac{5}{9}$?
Anſwer, $12\frac{59}{72}$.

8. What is the difference between $\frac{3}{7}$ of $\frac{5}{6}$ and $\frac{1}{3}$ of $\frac{1}{2}$?
Anſwer, $\frac{4}{21}$.

SEC. IV. MULTIPLICATION OF VULGAR FRACTIONS.

In Multiplication obſerve,

1. If any of the factors, or each of them, be a mixt number, it muſt be reduced to an improper fraction.
2. If one or both be compound fractions, they muſt be reduced to ſimple ones.

3. If they belong to different denominations, they must be brought to fractions of the same integer.
4. Then multiply the numerators, for a numerator to the product.
5. Multiply the denominators, for a denominator to the product.

EXAMPLES.

1. Multiply $\frac{5}{6}$ into $\frac{7}{8}$. *Answer*, $\frac{5}{6}+\frac{7}{8}=\frac{35}{48}$.

2. Multiply $3\frac{3}{4}$ into 5, $\frac{15}{4}\times\frac{5}{1}=\frac{75}{4}$, or $18\frac{3}{4}$.

3. Multiply $\frac{3}{4}$ of $\frac{5}{6}$ into $\frac{4}{5}$. $\frac{15}{24}\times\frac{4}{5}=\frac{60}{120}$, or $\frac{1}{2}$.

4. Multiply L. $\frac{3}{7}$ into $\frac{3}{7}$s. $\frac{3}{7}\times\frac{1}{20}\times\frac{3}{7}=\frac{9}{980}$.

5. Multiply $5\frac{5}{6}$ into $3\frac{3}{7}$. 20.

6. What is the price of $15\frac{3}{4}$ yards of cloth, at 15s. 6d. per yard? *Answer*, L. 12 : 4 : $1\frac{1}{2}$.

7. What is the square content of a room, 16 feet 8 inches, by 14 feet 4 inches? *Answer*, $238\frac{8}{9}$ feet.

8. What is the price of 15 Cwt. 3 qrs. of sugar, at 52s. 6d. per Cwt? *Answer*, L. 41 : 6 : $10\frac{1}{2}$.

SECT. V. DIVISION OF VULGAR FRACTIONS.

Here observe the three first directions laid down in the last section: Then,

1. Multiply the numerator of the dividend into the denominator of the divisor for a numerator.
2. Multiply the numerator of the divisor into the denominator of the divided for a denominator to the quotient.

EXAMPLES.

1. Divide $\frac{7}{8}$ by $\frac{5}{6}$. *Answer*, $\frac{5}{6})\frac{7}{8}(\frac{42}{40}$.

2. Divide $3\frac{3}{4}$ by $1\frac{1}{2}$. $\frac{3}{2})\frac{15}{4}(\frac{30}{12}$,

3. Divide $\frac{3}{7}$ by $\frac{3}{4}$ of $\frac{1}{3}$. $\frac{1}{4})\frac{3}{7}(\frac{12}{7}$.

4. Divide 5 by $\frac{5}{8}$. $\frac{5}{8})\frac{5}{1}(\frac{40}{5}$,

5. Bought $15\frac{3}{4}$ yards of cloth for L. 12 : 4 : $1\frac{1}{2}$, what was that per yard? *Answer*, 15s. 6d.

6. The ſquare content of a room is $238\frac{8}{9}$ feet, and one of its ſides $14\frac{1}{3}$ feet; what is the other ſide? *Anſwer*, $16\frac{2}{3}$.

7. What number, multiplied by $\frac{3}{5}$, will produce $25\frac{2}{3}$? *Anſwer*, $42\frac{7}{9}$.

8. Divide L. 276 : 16 : 8 among three men, each to have an equal ſhare, and a boy to have $\frac{2}{3}$ of one of the men's ſhare. *Anſwer*, L. 75 : 10 to each of the men, and L. 50 : 6 : 8 to the boy.

QUERIES.

1. Why is a mixt number reduced to an improper fraction by adding to the product of the denominator and integral part, the numerator of the fractional part for a numerator, and then ſubjoining the given denominator for a new one?

Sol. The product of the integer into the denominator increaſed by the numerator, expreſſes the ſum of the parts contained in both, and the ſame denominator being ſtill applied, the quality of thoſe parts is retained.

2. How comes it that the products of the numerators and denominators of a compound fraction, give a ſimple fraction of the ſame value with the compound one given?

Sol. The continued multiplication of the numerators and denominators reſpectively, brings each to the quality of the loweſt name, ſo that the numerator expreſſeth the number, and the denominator the quality of the parts contained in the fraction.

3. How comes it that fractions when abridged by a common diviſor ſtill retain the ſame value?

Sol. The ſame difference ſtill exiſts between any two numbers, whether multiplied or divided by the ſame number, for $\frac{2}{3}$ of a ſhilling = 8d. = $\frac{8}{12}$ = $\frac{16}{24}$ = 16 halfpence = $\frac{32}{48}$ = 32 farthings, &c. and the ſame reaſon is applicable, to any two numbers whatever, of which the one may be conſidered as a numerator, and the other as a denominator.

4. In what manner are fractions having different denominators reduced to others having one common denominator, without deſtroying the equality?

Sol. By multiplying each numerator into all the different given denominators continually for the numerator, and the denominator continually for a common denominator. Hence, ſince the numerator and denominator of each fraction is equally multiplied, the fractions produced muſt be equivalent.

5. In multiplying by a proper fraction, why is the product leſs than the multiplicand?

Sol. Becauſe the multiplier is leſs than 1.

6. What is the uſe of the laſt proceſs in addition and ſubtraction of fractions?

Sol. The fractions to be added or ſubtracted muſt either be in that ſtate, or reduced to that ſtate, that all the numerators muſt repreſent things of the ſame denomination, both abſolute and relative, ſo that their ſum may be a number of ſuch parts as the common denominator expreſſeth of the ſame common integer. Hence, when the denominators are different, till the fractions are reduced to one common denominator, addition and ſubtraction are impracticable.

7. How comes it that the product of proper fractions is leſs than their ſum?

Sol. The more any whole number is increaſed, the farther will its value be removed from its relative unit, and the farther any fraction is removed in value from its relative unit, the greater always will its denominator be. Therefore $\frac{1}{2} + \frac{1}{2} = 1$ but $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

8. How comes it that the product of the dividend's numerator and diviſor's denominator, gives a numerator to the quotient in diviſion, and the product of the other numerator and denominator, a denominator to the quotient?

Sol. The operation is the ſame as that by which fractions are reduced to one common denominator. Therefore, the dividend contains the diviſor as often as its numerator did that of the diviſor, for having one common denominator, though no mention is made of it in the rule, the factors are in the ſame ſtate with reſpect to each other, as whole numbers: For $\frac{4}{2} = \frac{4}{1} \div \frac{2}{1}$.

CHAP. IV.

OF DECIMAL FRACTIONS.

INTRODUCTION.

DECIMAL FRACTIONS, the invention of Joannes Regiomontanus, continue the decreaſe of an unit, which is the common boundary in the numeration of integers, to as many places towards the right, as the increaſe of numbers can be carried towards the left; the flux downwards in the one being in the ſame proportion as the flux upwards in the other.

1. From the foregoing definition, it will be eaſy to conceive, that in decimal fractions an unit is ſuppoſed to be divided into 10 equal parts, each of theſe into 10 other equal parts, each of theſe laſt into 10 other equal parts, *ad infinitum*.

2. Hence alſo it will be obvious, ſince equidiſtant places, on the left and right of the place of units, come under ſimilar names, it will not be neceſſary to write the denominator of a decimal fraction, it being ſufficient to diſtinguiſh the integers from the decimal places by a point or comma: as, $5.5 = 5\frac{5}{10}$. $67.25 = 67\frac{25}{100}$. $945.125 = 945\frac{125}{1000}$. $87.05 = 87\frac{5}{100}$.

3. As the flux of increaſe is to the left of the units place, and of decreaſe towards the right, it is plain, that a figure ſtanding in the ſecond place after the point will be only $\frac{1}{10}$ of the value of the ſame figure in the firſt place after the point, in the ſame ſenſe that a figure in the units place is only $\frac{1}{10}$ of the ſame figure in the place of tens. For this reaſon, ciphers ſtanding between ſignificant figures and the point decreaſe the value of theſe figures, in the ſame proportion the ſame number of ciphers on the left of the point would raiſe the value of the figures preceding them. Hence alſo ciphers on the right of ſignificant decimal places have the ſame effect as ciphers on the left of integers, which in both amount to nothing. Hence $.50 = .5$, for $\frac{50}{100} = \frac{5}{10}$: But $.05 = 5 \div 10$.

4. Since the decreaſe in decimals is carried down in one uninterrupted chain from the units place, it muſt certainly follow, that decimal places are added, ſubtracted, multiplied, and divided as integers, unleſs in ſome inſtances the decimal expreſſion fall ſhort of the vulgar

5 When the decimal expreffion is exactly equal to the vulgar, the decimal is, in that cafe, faid to be *finite*; but when there is a repetition of the fame figure or figures, it is, in that cafe, called *infinite* or *interminate*; which figures alfo acquire other names from the manner of their repetition.

From this brief account of the nature of decimals, their excellency, in computation will be pretty evident; but ftill more when we come to the actual application.

SECT. I. REDUCTION OF DECIMALS.

Cafe I. *To reduce a Vulgar Fraction to a Decimal.*

1. Annex ciphers, or fuppofe them annexed to the numerator, for a dividend.
2. Divide by the denominator; and if the quotient has as many places as the dividend had ciphers, and nothing remain, it will be equal to the fraction given, and be a finite decimal.

Hence will arife the following varieties.

1. When the denominator is found in the numerator, having one cipher annexed.

EXAMPLES.

Reduce $\frac{5}{8}$ to a decimal. 8) 50.

.625

Here the divifor is found in 50, and the quotient terminates in three places; wherefore .625 is a finite decimal, and exactly equal to $\frac{5}{8}$.

Reduce $\frac{3}{4}$ to a decimal. *Anfwer*, .75.

Reduce $\frac{1}{8}$ to a decimal. *Anfwer*, .125.

Reduce $\frac{7}{8}$ to a decimal. *Anfwer*, .875.

2. When the divifor cannot be found in the numerator increafed with a cipher, o is the firft place in the quotient.

EXAMPLES.

Reduce $\frac{5}{80}$ to a decimal. 80) 50.
 .0625

Reduce $\frac{3}{40}$ to a decimal. *Anſwer*, .075.

Reduce $\frac{1}{80}$ to a decimal. *Anſwer*, .0125.

Reduce $\frac{7}{80}$ to a decimal. *Anſwer*, .0875.

3. When the remainder repetes the ſame figure or figures ſucceſſively, the quotient is called a *ſingle repetend*, and the repeting figure is marked with a *point* above it, for the ſake of diſtinction.

EXAMPLES.

Reduce $\frac{1}{3}$ to a decimal. 3) 10
 $.\dot{3}$

Here the remainder 1 and the quotient figure 3 would repete *ad infinitum*; wherefore, when the quotient $.\dot{3}$ is diſtinguiſhed with a point above it, the calculator knows how far to continue it.

Reduce $\frac{5}{6}$ to a decimal. *Anſwer*, $.8\dot{3}$.
Reduce $\frac{11}{12}$ to a decimal. *Anſwer*, $.91\dot{6}$.
Reduce $\frac{119}{120}$ to a decimal. *Anſwer*, $.991\dot{6}$.

In the firſt example of this variety $.\dot{3}$ is a pure repetend.

In the ſecond, $.8\dot{3}$ is called a mixt repetend; the firſt figure being finite, and the ſecond interminate.

In the third example, the firſt two places are finite, and the third interminate.

In the fourth, the firſt three places are finite, and the fourth interminate.

4. When the same figure or figures repete after a certain rotation, the quotient will be a pure or mixt circulate, and the first and last figure of the circulation may be marked as repetends.

EXAMPLES.

Reduce $\frac{1}{11}$ to a decimal. 11) 100.. $.\dot{0}\dot{9}09$ a pure circulate.

Reduce $\frac{1}{7}$ to a decimal. *Answer*, $.\dot{1}4285\dot{7}$ pure.

Reduce $\frac{9}{28}$ to a decimal. *Answer*, $.32\dot{1}42857$ mixt.

Reduce $\frac{3}{22}$ to a decimal. *Answer*, $.1\dot{3}\dot{6}$ mixt.

Reduce $\frac{18254}{20500}$ to a decimal. *Answer*, $.89\dot{0}439\dot{0}2$ mixt.

The reason of this reduction will be abundantly plain, if it be considered, that, by the addition of ciphers to the numerator, it is multiplied by 1, and as many ciphers annexed to 1, as the quotient consists of places; and therefore the numerator and denominator are increased in the same proportion; and when there is no remainder, the decimal expression is exactly equal to the vulgar.

If there is a remainder which either repetes or circulates, the quotient will be less than the fraction from which it was reduced by a fraction, of which the remainder is the numerator, and the divisor the denominator, in the last place of the quotient. Thus the remainder 1, in the first example of variety 4th, is $\frac{1}{11}$ of $\frac{1}{10000}$. But in this case also, if the repetend be multiplied by the denominator of the given fraction, by carrying at 9 on the right hand, it will reproduce the numerator, with all the ciphers supposed to have been annexed.

More examples to facilitate practice.

1. Reduce $\frac{1}{25}$ to a decimal. *Answer*, .04.
2. Reduce $\frac{3}{39}$ to a decimal. *Answer*, $.\dot{0}7692\dot{3}$.
3. Reduce $\frac{4}{41}$ to a decimal. *Answer*, $.\dot{0}975\dot{6}$.
4. Reduce $\frac{5}{19}$ to a decimal. *Answer*, $.\dot{2}6315789473684210\dot{5}$.
5. Reduce $\frac{1}{13}$ to a decimal. *Answer*, .076923.
6. Reduce $\frac{1}{31}$ to a decimal. *Answer*, $.\dot{0}32258064516\dot{1}$.
7. Reduce $\frac{5}{7}$ to a decimal. *Answer*, $.\dot{7}1428\dot{5}$.
8. Reduce $\frac{4}{9}$ to a decimal. *Answer*, $\dot{4}$.
9. Reduce $\frac{3}{11}$ to a decimal. *Answer*, $.\dot{2}\dot{7}$.
10. Reduce $\frac{1}{69}$ to a decimal. *Answer* .0144927, &c.

Case II. *To reduce the decimal expression back to the vulgar.*

1. Finite decimals are reduced to the vulgar expression, by bringing numerator and denominator to their lowest terms.

EXAMPLES.

1. Reduce .4 to its equivalent vulgar expression. *Answer*, $\frac{2}{5}$.
2. Reduce .85 to its equivalent expression. *Answer*, $\frac{17}{20}$.
3. Reduce .675 to an equivalent vulgar expression. *Answer*, $\frac{27}{40}$.
4. Reduce .5375 to an equivalent vulgar fraction. *Answer*, $\frac{43}{80}$.

2. Decimal fractions terminating in repetends are reduced to the vulgar expression, by multiplying numerator and denominator by any digit that will exterminate the repetend; remembering always to carry for 9s in the repetends lowest place, and then finding the lowest terms.

EXAMPLES.

1. Reduce $.83\dot{3}$ to its original vulgar expression. *Answer*, $\frac{5}{6}$.
2. Reduce $.22708\dot{3}$ to its original fraction. *Answer*, $\frac{109}{480}$.
3. Reduce $.5347\dot{2}$ to the vulgar expression. *Answer*, $\frac{77}{144}$.
4. Reduce $.99930\dot{5}$ to the vulgar expression. *Answer*, $\frac{1439}{1440}$.

3. If the decimal expression terminate in a pure circle, place the circulating figures for a numerator, and as many 9s for a denominator, then reduce to lower terms.

EXAMPLES.

1. Reduce $.\dot{0}\dot{9}$ to the vulgar expression. *Answer*, $\frac{1}{11}$.
2. Reduce $.\dot{1}\dot{8}$ to the vulgar expression. *Answer*, $\frac{2}{11}$.
3. Reduce $.\dot{3}\dot{6}$ to the vulgar expression. *Answer*, $\frac{4}{11}$.
4. Reduce $.\dot{8}\dot{1}$ to the vulgar expression. *Answer*, $\frac{9}{11}$.

5. If the decimal expression terminate in a mixt circulate, subtract the finite part from the whole decimal, the remainder will be

the numerator; and as many ciphers as were places in the finite part, annexed to as many 9s as the circle consisted of places, will be the denominator; both which may be reduced to lower terms when possible.

EXAMPLES.

1. Reduce $.890\dot{4}391\dot{4}$ to the vulgar expression. *Answer*, $\frac{3710126}{4166625}$.
2. Reduce $.910\dot{7}1428\dot{5}$ to the vulgar expression. *Answer*, $\frac{51}{56}$.
3. Reduce $.410\dot{7}1428\dot{5}$ to the vulgar expression. *Answer*, $\frac{207}{504}$.

Case III. *To reduce the common divisions and subdivisions of any relative unit to the decimal of that, or any unit superior to the denomination given.*

1. Reduce the given denominations to the vulgar expression.
2. Reduce as directed in Case I.

EXAMPLES.

Reduce 9d. to the decimal of a shilling.

9d. $= \frac{9}{12}$s. wherefore 12) 90

.75 *Answer.*

Reduce 9d. to the decimal of a pound.

9d. $=$ L. $\frac{9}{240}$: wherefore 240) 900

.0375 *Answer.*

Reduce 18s. 4d. to the decimal of a pound.

18s. 4d. $= \frac{220}{240}$ or $\frac{11}{12}$: wherefore 12) 110

$.91\dot{6}$ *Answer.*

Reduce 19s. $11\frac{3}{4}$d. to a decimal.

19s. $11\frac{3}{4}$d. $= \frac{959}{960}$: wherefore 960) 9590

$.998958\dot{3}$ *Answer.*

Reduce 15s. $6\frac{1}{4}$d. to a decimal.

15s. $6\frac{1}{4}$d. $= \frac{745}{960}$ or $\frac{149}{192}$; wherefore 192) 1490

$.776041\dot{6}$ *Answer.*

It is much more expeditious to divide by the component parts of the divisor, thus.

1. To the given number of the lowest denomination affix ciphers, and divide it by the number of that which makes an unit of the next superior denomination.

2. To this quotient prefix the given number of units of the next higher denomination, or a cipher for them, if no units are given, and divide by the number of units of that denomination contained in the next superior.

3. Proceed thus prefixing and dividing throughout the whole, and the last quotient will be the decimal required.

EXAMPLES.

Reduce 17s. 6d. to a decimal.

12) 60

2) 175

.875 *Answer.*

Here it was unnecessary, in dividing by 20, to use the cipher, as ciphers must be annexed to the remainders to bring out the quotient.

Reduce 15s. $5\frac{1}{2}$d. to a decimal.

2) 10

12) 55

2) $15458\dot{3}$

$.77291\dot{6}$ *Answer.*

Here the division by 12 terminates in a repetend, which is affixed to the last remainder in the next division, and not a cipher, to bring out the quotient.

Reduce 17s. $\frac{1}{4}$d. to a decimal.

4) 10

12) 025

2) $170208\dot{3}$

$.851041\dot{6}$ *Answer.*

Here a cipher was prefixed for pence.

Reduce 1s. $4\frac{1}{2}$d. to a decimal.

$$\begin{array}{r|l} 2 & 10 \\ \hline 12 & 45 \\ \hline 2 & 1375 \\ \hline & .06875 \end{array}$$

Answer.

Here a cipher was prefixed to the last quotient, because 2 was not contained in the shillings, 1.

Reduce $4\frac{1}{2}$d. to the decimal of a pound.

$$\begin{array}{r|l} 2 & 10 \\ \hline 12 & 45 \\ \hline 2 & 375 \\ \hline & .01875 \end{array}$$

Answer.

Here a cipher was prefixed to the last quotient, because there were no shillings.

Reduce 3 qrs. 18 lb. to the decimal of 1 Cwt.

$$\begin{array}{r|l} 4 & 18 \\ \hline 7 & 45 \\ \hline 4 & 3\dot{6}42857\dot{1} \\ \hline & .910\dot{7}1428\dot{5} \end{array}$$

Answer.

Here the division is made by the component parts of 28; and, in dividing by 4, the figures of the circle are annexed, not ciphers, to bring out the quotient.

More examples to facilitate practice.

1. Reduce 3 qrs. 15 lb. to the decimal of 1 Cwt.
   Answer, $.883\dot{9}2857\dot{1}4$.
2. Reduce 11 oz. 19 dwt. 23 gr. to the decimal of 1 lb. troy.
   Answer, .99982638.
3. Reduce 25 gallons 7 pints to the decimal of a hogshead.
   Answer, $.410\dot{7}1428\dot{5}$.
4. Reduce 1 foot 8 inches to the decimal of a yard.
   Answer, $.\dot{5}$.
5. Reduce 2 feet 11 inches to the decimal of a yard.
   Answer, $.97\dot{2}$.

6. Reduce 23 hours 59 minutes to the decimal of 1 day.
Answer, .999305.

7. Reduce 47 gals. 3 quarts to the decimal of a butt ale measure.
Answer, .9947916.

8. Reduce 11 sacks 1 wey 6 todds to the decimal of a last of wool.
Answer, .996794871.

Case IV. *To find the value of a decimal.*

This case admits of three varieties.

I. To find the value of a finite decimal.

1. Multiply the given decimal by the number of parts of the next inferior denomination contained in an unit of the integer.

2. Point off from the product so many figures to the right hand as there are figures in the given decimal; then on the left of the point are parts, and on the right a decimal of those parts.

3. Reduce this new decimal in the same manner to the next inferior denomination, point off as before, reduce again, &c. till the decimal places become ciphers, or there be no denomination left for a multiplier.

EXAMPLES.

1. Value L. .875.

```
 .875
   20
------
17.500
   12
------
 6.000
```

Answer, 17s. 6d.

2. Value L. .06875

```
.06875
    20
-------
1.37500
    12
-------
4.50000
     4
-------
2.00000
```

Answer, 1s. $4\frac{1}{2}$d.

3. Value L. .375. *Anſwer*, 7s. 6d.
4. Value .01875 of L. 1 Sterling. *Anſwer*, $4\frac{1}{2}$d.
5. Value .99375 of a tun. *Anſwer*, 19 cwt. 3 qrs. 14 lb.
6. Value .475 of a lb. troy. *Anſwer*, 5 oz. 14 dwt.
7. Value .875 of a chalder. *Anſwer*, 14 bolls.
8. Value .945 of a day. *Anſwer*, 22 hours 40 min. 48 ſec.
9. Value .975 of a yard. *Anſwer*, 2 feet $11\frac{1}{10}$ inches.

II. To value a decimal terminating in a *repetend*.

Carry at nine in the product of the repetend, and where a cipher occurs on the right of the multiplier, annex the repetend for it in the product.

EXAMPLES.

1. Value L. $.851041\dot{6}$

$$\begin{array}{r} .851041\dot{6} \\ 20 \\ \hline 17.020833\dot{3} \\ 12 \\ \hline .2500000 \\ 4 \\ \hline 1.0000000 \end{array}$$

2. Value L. $.77291\dot{6}$.

$$\begin{array}{r} .77291\dot{6} \\ 20 \\ \hline 15.45833\dot{3} \\ 12 \\ \hline 5.5\ldots\ldots \\ 2 \\ \hline 1. \end{array}$$

H. Cormack
Watchmaker
2 Sinclair Terrace
Pulteneytown
Wick

3. Value $.99930\dot{5}$ of a day. *Answer*, 23 hours 59 minutes.

4. Value $.9998263\dot{8}$ of a ℔. troy. *Answer*, 11 oz. 19 dwt. 23 grains.

5. Value $.97\dot{2}$ of a yard. *Answer*, 2 feet 11 inches.

6. Value $.8\dot{3}3$ of a Cwt. *Answer*, 3 qrs. $9\frac{1}{3}$ ℔.

7. Value $.16\dot{6}$ of a year. *Answer*, 60 days 20 hours.

III. To value a decimal terminating in a *circle*.

To the product of the last figure of the circle, add the tens in the product of the first figure of the circle into the same multiplier.

EXAMPLES.

1. Value $.910\dot{7}1428\dot{5}$ Cwt.

$$
\begin{array}{lr}
 & .910\dot{7}1428\dot{5} \\
 & 4 \\
\hline
\text{Qrs.} & 3.642857142 \\
 & 7 \\
\hline
 & 4499999\dot{9} \\
 & 4 \\
\hline
\text{Lib.} & 18\ldots\ldots\ldots
\end{array}
$$

Here the tens in 4×7 are added to 4×5.

Here we begin with the last figure in the circle 1, and add the tens in 7×4 to 7×1, and the product ends in a repetend.

2. Value $.60\dot{7}1428\dot{5}$ quarters of 1 Cwt.

$$
\begin{array}{lr}
 & .60\dot{7}1428\dot{5} \\
 & 7 \\
\hline
 & 424999999 \\
 & 4 \\
\hline
\text{Lib.} & 17\ldots\ldots\ldots
\end{array}
$$

3. Value $.\dot{4}2857\dot{1}$ of a pound Sterling. *Answer*, 8s. $6\frac{6}{7}$d.

4. Value $.67\dot{8}5714\dot{2}$ L. *Answer*, 13s. $6\frac{6}{7}$d.

5. Value $.\dot{8}5714\dot{2}$ of a lb troy. *Answer*, 10 oz. 5 dwt. $17\frac{1}{7}$ gr.

6. Value $.785714\dot{2}85$ Cwt. *Answer*, 3 qrs. 4 lb.

SECT. II. ADDITION OF DECIMAL FRACTIONS.

Place the given numbers, whether pure decimals or mixt, so as places of the like kind may stand under each other successively, then will all the points be in one and the same column, the blanks on the left of the integers and on the right of the decimals. In the addition there will arise three varieties.

I. If all the decimals are finite, add as in whole numbers, and from the sum point off as many places to the right as the longest of the given decimals consisted of, and the separating point of the sum will be in the same column with those of the numbers to be added.

EXAMPLES

74.1525	.375	84.2
87.4125	.41535	67.25
93.5	.0125	347.125
6.0375	.3475	694.0375
8.15	.4	69.00875
	.06875	6.95
269.2525	1.61910	1268.57125

Reduce and add as follows,

L. 196	17	6	=	L. 196.875
194	18	9	=	194.9375
67	14	3	=	67.7125
45	0	9	=	45.0375
76	16	6	=	76.825
9	10	6	=	9.525
4	12	$8\frac{1}{4}$	=	4.634375
595	10	$11\frac{1}{4}$	=	595.546875
				20
				10.937500
				12
				11.2500
				4
				1.00

Reduce and add as follows,

L. 156	19	3	=	
674	11	9	=	
419	10	$11\frac{1}{4}$	=	
674	4	$8\frac{1}{2}$	=	
894	3	6	=	
97	0	3	=	
6	0	$0\frac{1}{4}$	=	
L. 2922	10	$5\frac{1}{2}$	=	$2922.52291\dot{6}$

II. When all or any of the given decimals repete, give every repetend the ſame number of places, and one place more than the largeſt finite, and for every 9 in the right hand column carry 1 to the ſecond column, but in every other column carry at 10.

EXAMPLES.

$74.35208\dot{3}$	$26.08\dot{3}$	$342.2541\dot{6}$
27.45625	$35.\dot{3}33$	$238.\dot{3}3333$
$18.908\dot{3}33$	$98.41\dot{6}$	$316.41\dot{6}66$
$39.50\dot{3}333$	84.25	$45.208\dot{3}3$
65.0375	77.75	$35.291\dot{6}6$
$9.04791\dot{6}$	$33.91\dot{6}$	40.125
$234.30541\dot{6}$	355.75	$1017.6291\dot{6}$

Reduce and add as follows.

L. 76	13	4	=	$76.\dot{6}666666$
97	6	8	=	$97.\dot{3}333333$
14	3	4	=	$14.1\dot{6}66666$
19	14	$3\frac{1}{2}$	=	$19.71458\dot{3}3$
14	0	7	=	$14.0291\dot{6}66$
18	0	$0\frac{1}{4}$	=	$18.001041\dot{6}$
17	19	$11\frac{3}{4}$	=	$17.998958\dot{3}$
15	15	$6\frac{1}{4}$	=	$15.776041\dot{6}$
273	13	$8\frac{3}{4}$	=	$273.686458\dot{3}$
				20
				$13.7291\dot{6}66$
				12
				8.75 .....
				4
				3...

Reduce and add as follows.

L. 159	7	$7\frac{3}{4}$	=	
89	17	5	=	
346	13	4	=	
414	18	8	=	
97	0	$5\frac{1}{4}$	=	
89	19	11	=	
67	4	7	=	
59	9	4	=	
1324	11	4	=	$1324.5\dot{6}$

III. If all or any of the given decimals terminate in a circle, make all the circles similar and conterminous, find the number of tens to be carried from the left hand column of circles, and add them with the right hand column; then proceed as in finite decimals.

Note 1. Similar circles are such as consist of an equal number of places; and unlike circles may be made similar thus:

Find the least multiple of the several numbers of the places in the given circles, and extend each of the given circles to as many places as there are units in the least multiple. Thus, $.\dot{0}\dot{9}$ and $\dot{4}8\dot{1}$ are dissimilar, because the one contains two places, and the other three; wherefore, since 6 is the least multiple of 2 and 3, if each of them is extended to six places, they will be similar.

$$.\dot{0}\dot{9} = .\dot{0}9090\dot{9}$$
$$\dot{4}8\dot{1} = \dot{4}8\dot{1}481$$

Note 2. Conterminous circles are such as begin and end at the same distance from the separating point; and, since any one of the circulating figures may be made the first of the circle without changing its value, circles may be made conterminous thus:

Set aside, by a circulating point on the left, a number of places equal to that of the longest finite part, and then prolong the several circles to as many places as will make them similar. Hence,

$$.54\dot{6}\dot{3} = 54\dot{6}\dot{3}6363$$
$$.9\dot{1}4\dot{8} = 91\dot{4}81481$$

EXAMPLES.

$\frac{3}{7} = .428571 = .428571$

$\frac{6}{7} = .857142 = .857142$

$\frac{5}{11} = .45 \quad = .454545$

$\frac{10}{27} = .370 \quad = .370370$

$\qquad\qquad\qquad 2.110630$

$\frac{5}{14} = .35714285$

$\frac{1}{26} = .03846153$

$\frac{19}{28} = .67857142$

$\frac{3}{11} = .27272727$

$\frac{4}{27} = .14814814$

$\qquad 1.49505124$

Reduce and add as follows.

L. $18\frac{11}{14} = 18.785714285$

$25\frac{9}{11} = 25.818181818$

$14\frac{1}{2} = 14.5$

$29\frac{3}{4} = 29.75$

$35\frac{7}{8} = 35.875$

$24\frac{1}{75} = 24.013333333$

$32\frac{7}{9} = 32.777777777$

$\qquad 181.520007215$

L. $35\frac{7}{11} =$

$96\frac{3}{22} =$

$78\frac{9}{44} =$

$57\frac{1}{90} =$

$68\frac{5}{7} =$

In all cases of practice, it will be sufficient to carry the decimals to five places; and if they do not become finite then, to add one to the sum of the right hand column for half of the number of decimal lines given that were not complete.

EXAMPLES.

Reduce and add as follows.

Lib.	oz.	dwt.	gr.		
714	11	19	13	=	714.99809
85	9	14	11	=	85.81024
67	8	15	9	=	67.73072
87	7	7	7	=	87.61371
59	3	14	8	=	59.30972
1015	5	11	0	=	1015.46250
					12
					5.55...
					20
					11...

Cwt.	Q.	L.	
574	3	19	=
647	1	17	=
785	2	23	=
945	2	$19\frac{1}{2}$	=
675	1	$11\frac{1}{3}$	=
974	2	$18\frac{1}{7}$	=

SECT. III. SUBTRACTION OF DECIMALS.

Place the minor under the major, so as the points may stand in the same column; then, in finding the remainder, there will be three varieties.

I. When both major and minor are finite decimals, subtract as in integers.

EXAMPLES.

From	410.0375	978.0125	6415.00325
Take	56.1275	89.875	963.013625
Rem.	353.91	888.1375	5451.989625

Reduce and subtract as follows.

L. 1741	13	6	=		L. 917	16	5	=	
987	15	9	=		578	17	$4\frac{1}{4}$	=	
753	17	9	=	753.8875					

II. If either or both terminate in repetends, carry the repeting figure one place farther than the finite factor in the first instance, and make the repetends conterminous in the second; and in both cases borrow 9 in the right hand place, but no where else.

EXAMPLES.

From	$7415.33\dot{3}$	$875.\dot{0}375$	$810.\dot{3}33$
Take	568.125	597.77777	579.666
Rem.	$6847.20\dot{7}$	277.25972	$230.\dot{6}66$

From	L. 714 13 4	L. 119 11 2	L. 54 1 6
Take	335 16 8	17 18 6	25 11 11
Rem.	$378.8\dot{3}$	$101.6\dot{3}33$	$28.4791\dot{6}$ decimally.

III. When either or both factors terminate in a circle, prepare the factors as in addition; and if the figure in the left hand column of the circle in the minor be greater than the figure in the same place of the major, add 1 to the right hand figure of the minor before subtraction.

EXAMPLES.

From	$\frac{25}{27}$ = .925925	$\frac{9}{14}$ = .64285714	$\frac{13}{14}$ = .9285714
Take	$\frac{4}{7}$ = .571428	$\frac{5}{28}$ = .17857142	$\frac{2}{3}$ = .6666666
Rem.	.354497		

Sect. IV. Multiplication of Decimals.

Point off so many decimal places in the product as are in both factors; and if the product hath not so many figures, prefix ciphers for the defect. Here there are six varieties.

I. If both factors are finite, or considered as such, the product is found as in integers.

EXAMPLES.

74567.8	.785	87.65	.3685
.345	.075	.125	.0275
25725.8910	.058875	10.95625	.01013375

II. If only one of the factors be finite, and the other terminate in a repetend, multiply by the finite factor, and carry at nine on the right hand: Before the products are added, they must be made conterminous; and, in adding, carry at nine in the right hand column: If the multiplier has ciphers on the right, continue the repetend for them in the product.

EXAMPLES.

27.883	.27083	.53824
8.75	500	.0345
139416	135.41666	269122
1951833		2152977
22306666		16147333
243.97916		.018569433

III. If one of the factors be finite, and the other circulate, multiply by the finite, and to the product of the right hand figure of the circle add 1 for every ten in the product of the left hand figure, then multiply as in finite decimals; but prepare and add the products as in addition of circulates. If there are ciphers on the right of the multiplier, continue figures in the circle for them.

EXAMPLES.

46.02439	865.3571428	47.857142
.742	587.5	80500
9204878	43267857142	239285714
184097560	605749999999	38285714285
3221707317	6922857142857	38525000
34.15009756	43267857142857	
	508397.32142857	

IV. When both factors repete, find the product as in finite decimals, and afterwards multiply it by 10, and divide by 9 for the true product: or, reduce the repetend in the multiplier to a vulgar fraction, and take parts of the multiplicand for it.

EXAMPLES.

$$\begin{array}{r} 345.8\dot{3} \\ 3 \\ \hline 103.750 \\ 10 \\ \hline 9\ )\ 1037.500 \\ \hline 115.27\dot{7} \end{array} \qquad \begin{array}{r} 3\ )\ 345.8\dot{3} \\ \hline 115.27\dot{7} \end{array}$$

$$\begin{array}{r} 574.\dot{6} \\ 3.8\dot{3} \\ \hline 9\ )\ 1724.0 \\ \hline 1915 \\ 45973 \\ 17240 \\ \hline 2202.88 \end{array} \qquad \begin{array}{rl} 2\ )\ 574.\dot{6} & \\ 3\tfrac{5}{6} & \\ \hline 1724.0 & \\ 3\ )\ 287.33 & \text{for } \tfrac{3}{6} \\ 95.77 & \text{for } \tfrac{1}{6} \\ 95.77 & \text{for } \tfrac{1}{6} \\ \hline 2202.\dot{8}8 & \end{array}$$

Multiply 876.8$\dot{3}$ into 5.$\dot{6}$. *Answer*, 4968.7$\dot{2}$.
Multiply 597.$\dot{6}$ into 8.8$\dot{3}$. *Answer*, 5279.38.
Multiply 845.5 into 77.2. *Answer*, 65295.679012345.
Multiply 973.8 into 4.7. *Answer*, 4653.024691357.

V. When one of the factors is a circulate, and the other a repetend, make the circulate the multiplier, subtract the finite part from the circulating, the remainder will be a new multiplier, whose product, divided by as many nines as the circle contained figures, will quote the true product.

EXAMPLES.

$$\begin{array}{r} 18.78341\dot{6} \\ 432 \\ \hline 99\ )\ 8114.436000 \\ \hline 81.964 \end{array} \times \begin{array}{r} 4.\dot{3}\dot{6} \\ 4 \\ \hline 4.32 \\ \hline \end{array} \qquad \begin{array}{r} 8.0208\dot{3} \\ 72 \\ \hline 99\ )\ 577.50000 \\ \hline 5.8\dot{3} \end{array} \times .\dot{7}\dot{2}$$

Multiply 235.01 into 3.26. *Answer*, 766.9031.
Multiply 32.7 into 45. *Answer*, 14.89.
Multiply 24.013 into 7.18. *Answer*, 172.4593.
Multiply 257.8 into 45.63. *Answer*, 11769.1.

If the finite part of the decimal had been 0, a cipher would have been annexed to the divisor.

VI. If both factors circulate, prepare the multiplier as in the last variety; multiply as directed in variety 3.; the product, divided by so many nines as were figures in the circle of the multiplier, will quote the true product.

EXAMPLES.

Multiply .32142857 into 6.81	53.57428 × 2.18
681—6=675	218—2=216
160714285	32144571
2249999999	53574285
19285714285	1071485714
99) 21.696428571	99) 11572.04571
.219155844	116.8893506

Note, The easiest method in this and the last case is to take parts of the multiplicand for the vulgar expression of the fractional part of the multiplier.

EXAMPLES.

784.53428
5.45 = $5\frac{5}{11}$

11) 3922.671422 Product of 5.
356.606492 Product of $\frac{5}{11}$.

4279.277914

Multiply $89.910\dot{7}1428\dot{5}$ into $4.\dot{3}\dot{6}$.

$$\begin{array}{r} 4 \\ \hline 11\ )\ 359.64\dot{2}85714\dot{2} \\ 32.694\dot{8}0519\dot{4} \\ \hline 392.337\dot{6}6233\dot{7} \end{array}$$

Multiply $674.\dot{5}74\dot{8}$ into $8.\dot{7}\dot{2}$. *Answer*, $5887.\dot{1}98\dot{7}$.

SECT. V. DIVISION OF DECIMALS.

When the quotient is found, there must be so many places pointed off from it, for decimal places, as, with the decimal places in the divisor, will exactly equal the number of decimal places in the dividend.

The application of this rule is rendered extremely easy by the following directions.

1. When there is an equal number of decimals in the divisor and dividend, the quotient will be a whole number.

2. When there are more decimal places in the divisor than in the dividend, annex ciphers to the dividend till the places become equal, and then the quotient will still be a whole number.

3. When there are more decimal places in the dividend than in the divisor, point off so many places from the quotient as make up the difference.

4. When the whole quotient will not make up the excess of the number of decimal places in the dividend above the divisor, prefix ciphers to the quotient to make up the defect.

EXAMPLES.

$$\begin{array}{r} .75\ )\ 588,75\ (\ 785 \\ 525 \\ \hline 637 \\ 600 \\ \hline 375 \\ 375 \\ \hline 0 \end{array} \quad \text{p. 1.}$$

$$\begin{array}{r} .75\ )\ 58875.00 \\ 78500 \\ \hline \end{array} \quad \text{p. 2.}$$

$$\begin{array}{r} .75\ )\ .58875 \\ \hline .785 \end{array} \quad \text{p. 3.}$$

$$\begin{array}{r} 75\ )\ .58875 \\ \hline .00785 \end{array} \quad \text{p. 4.}$$

3,65) 974678,95

4.85) 87438975

56.5) 3797.6785

89.5) 1.7346875

In finding the quotient there will arise six varieties.

I. When both factors are finite, divide as in integers; and, if there be a remainder, after all the dividend is exhausted, the division may be continued by annexing ciphers, till there be no remainder, or till the quotient terminate in a repetend or circle, or till you think proper to limit it.

EXAMPLES.

.95) 74157.675 (78060.7105263157894736842 terminating in a [circle.

```
665
---
 765
 760
 ---
  576
  570
  ---
   675
   665
   ---
    100
     95
    ---
    500
    475
    ---
     250
     190
     ---
      60, &c.
```

3.25) 76.75 (23.61538 limited at 5 places.

```
650
----
1175
 975
----
2000
1950
----
 500
 325
----
1750
1625
----
1250
 975
----
2750
2600
----
 150
```

1.45) 8347.975 (

9.85) 37418.75 (

6.75) 3987,125 (

1.875) 976.8465 (

II. If the divisor be finite, and the dividend repete, divide as before; but after the dividend is exhausted, continue the division, by annexing the repetend to the remainder.

EXAMPLES.

$.37$) $4.1966\dot{2}$ ($11.34\dot{2}$ $\qquad$ 12.5) $78.8\dot{3}$ ($6.30\dot{6}$

```
 37              750
 ---             ---
  49              383
  37              375
  ---             ---
  126             833
  111             750
  ---             ---
   156             83
   148
   ---
    82
    74
    --
     8
```

3) $7459.30\dot{6} \div 15$

5) $2486.43\dot{5}$

$497.287\dot{1}$

III. If the divisor be finite, and the dividend a circulate, divide as before, but continue the division, by annexing the figures in the circle successively to the remainders.

EXAMPLES.

5) $3.\dot{7}59\dot{2}$ $\qquad$ 6.5) $25.0\dot{7}1428\dot{5}$ $\qquad$ 7.42) $341.50\dot{0}975\dot{6}$

$7.\dot{5}1\dot{8}5$ $\qquad$ $3.\dot{8}5714\dot{2}8$ $\qquad$ $46.\dot{0}243\dot{9}02$

IV. If the divisor is a single repetend, multiply the dividend by 9, and point off in the product the given number of integers for a new dividend.

EXAMPLES.

$$5478.45 \div .\dot{6} \qquad 5149.7\dot{1}6 \div .\dot{3} \qquad 67.\dot{7}4\dot{8} \div .\dot{7}$$

$$\begin{array}{r} 5478.45 \\ 9 \\ \hline .\dot{6}\ )\ 4930.605 \\ \hline 8217.675 \end{array} \qquad \begin{array}{r} 5149.7\dot{1}6 \\ 9 \\ \hline .\dot{3}\ )\ 4\dot{6}347.450 \\ \hline 15449.15 \end{array} \qquad \begin{array}{r} 67.\dot{7}4\dot{8} \\ 9 \\ \hline .\dot{7}\ )\ 60.\dot{9}73\dot{8} \\ \hline 87.\dot{1}0553\dot{4} \end{array}$$

$$59879.75 \div .\dot{3} \qquad 87645.6 \div .\dot{7}$$

$$84198.845 \div .\dot{8} \qquad 3275940 \div .\dot{5}$$

V. If the divisor only terminate in a repetend, from the whole divisor subtract the terminate part, prepare the dividend as in the last variety, and the quotient arising from these new factors will be that required: or, multiply both factors by any digit that will exterminate the repetend.

EXAMPLES.

$$\left.\begin{array}{r} 6.8\dot{3} \\ 68 \\ \hline 6.15 \end{array}\right\} \begin{array}{r} 7588.16 \\ 9 \\ \hline 6829.344 \\ \hline 111.04\dot{6}2439\dot{0} \end{array} \qquad \left.\begin{array}{r} 2.3\dot{7} \\ 23 \\ \hline 2.14 \end{array}\right\} \begin{array}{r} 759.011562 \\ 9 \\ \hline 683.1104058 \\ \hline 319.21047 \end{array}$$

$$\left.\begin{array}{r} .23\dot{7} \\ 9 \\ \hline 2.14 \end{array}\right\} \begin{array}{r} 759.011562 \\ 9 \\ \hline 6831.104058 \\ \hline 319.21047 \end{array}$$

$87697.83 \div 8.1\dot{6}$ $974.7146 \div .\dot{5}1\dot{5}$

$97685.\dot{6} \div 4.8\dot{3}$ $57684.\dot{5} \div 9.\dot{2}$

VI. When both factors, or the divisor only, terminate in a circle, set the divisor under itself, so many places; do the same by the dividend, and the remainders respectively, when the minors are taken from the majors, will be new factors Or divide by the mixt number as in problem II. of division of integers.

EXAMPLES.

$$\left.\begin{array}{r} 4.\dot{3}\dot{6} \\ 4 \\ \hline 4.32 \end{array}\right\} \begin{array}{r} 81.964 \\ 81964 \\ \hline 81.14436 \\ \hline 18.78341\dot{6} \end{array} \qquad \left.\begin{array}{r} 555.23\dot{1}2195 \\ 55523 \\ \hline 555.2256672 \end{array}\right\} \begin{array}{r} 25293.8\dot{6}66666 \\ 252938\dot{6} \\ \hline 25293.6137280 \\ \hline 45.\dot{5} \end{array}$$

$$\left.\begin{array}{r} 4.\frac{4}{11} \\ \hline 48 \end{array}\right\} \begin{array}{r} 81.964 \\ \hline 901.604 \\ \hline 18.78341\dot{6} \end{array}$$

Divide $3748.8\dot{3}$ by $5.\dot{1}\dot{8}$. *Answer*, 723.459, &c.

Divide 59478.6 by $8.\dot{0}\dot{9}$. *Answer*, 7351.2876, &c.

SECT. VI. APPLICATION OF DECIMALS TO PRACTICAL QUESTIONS.

1. There is a room 18 feet 8 inches, by 16 feet 9 inches; how many yards of flooring doth it contain?

Feet. Inch.	2)			
18 : 8 =	18.666			
	4			
	74.666			
	4		Feet.	Inch.
	298.666	for	16	0
	9.333	for		6
	4.666	for		3
9)	312.666	for	16	9
	34.740	*Answer.*		

2. There is a piece of mahogany 19 feet 10 inches long, 5 feet 3 inches broad, and 4 feet 4 inches deep; how many solid feet doth it contain, and what it is worth at 9d. per solid foot?

	19.833	
	$5\frac{1}{4}$	
	99.1666	
	4.9583	
3)	104.125.	
	$4\frac{1}{3}$	
	416.5..	
	34.7083	
	451.2083	solid feet
	.0375	
	22560416	
	315845833	
	1353625...	
	16.9203125	= L. 16 : 18 : $4\frac{7}{8}$ *Answer.*

3. There is a box 6 feet 6 inches long, at one end, 4 feet 5 inches broad, and at the other 3 feet 3 inches; what is the tunnage, supposing the depth 4 feet 9 inches, and what will the freight amount to at L. 3 : 7 : 6 per tun?

```
        Feet. Inch.
           4    5   breadth at one end,
           3    3   breadth at the other,
          ------
           7    8
          ------
           3   10   mean breadth.

Feet. Inch.
 3    10    =     3.83
                   6½
                ---------
                23...
                  1.916
                ---------
                 24.916
                     4½
                ---------
                99.66666
                12.45833 for ½
                 6.22916 for ¼
                ---------
        4,0 ) 11,8.35416 solid feet.
               ----------
               2.958885416 tunnage
                     3.375
               ----------
               14794270833
              207119791666
              887656250
              887656250
              -------------
              9.9861328125 = L. 9 : 19 : 8½ freight.
                        20
              -------------
              19.72265625
                       12
              -------------
               8.671875..
                        4
              -------------
               2.6875..
```

4. A gentleman ordered his estate, valued at L. 7418 : 19 : 6 to be divided among his four daughters, A, B, C, and D, in such proportion that A should draw $\frac{1}{2}$ of it, B. $\frac{1}{3}$ of it, C. $\frac{1}{4}$ of it, and D. $\frac{1}{5}$.

of it, without prejudice of the one to the other; required a state, and the proportion to be drawn by each.

$$\text{L. } 7418 : 19 : 6 = \text{L. } 7418.975$$

$$\tfrac{1}{2} = .5 + \tfrac{1}{3} = .\dot{3}33 + \tfrac{1}{4} = .25 + \tfrac{1}{5} = .2 = 1.28\dot{3}$$

$$\begin{array}{r|l}
1.28\dot{3} & 7418.975 \\
3 & 3 \\
\hline
5\ )\ 3.850 & 22256.925 \\
\hline
7\ )\ .77 & 4451.385 \\
.11 & 635.9121429 \\
\cline{2-2}
 & 5781.01948 \div \left\{ \begin{array}{c|l} 2 & = 2890.50972 \\ 3 & = 1927.00648 \\ 4 & = 1445.25487 \\ 5 & = 1156.20389 \\ \cline{2-2} & 7418.975 \textit{ fere.} \end{array} \right.
\end{array}$$

5. Bought a ship for L. 745 : 13 : 4, what should I charge for $\frac{3}{7}$ of $\frac{1}{3}$ of that ship to gain L. 100 on the whole?

Answer, L. 80 : 10 : $9\frac{1}{4}$.

6. Bought $57\frac{5}{8}$ yards of cloth for L. 47 : 18 : 6, carriage and other charges came to 19s. 8d. what did it cost per yard?

Answer, L. .8487 = 16s. $11\frac{1}{3}$.

7. In $375\frac{1}{2}$ ducats, at 4s. 6d. each, how many French crowns at 2s. $7\frac{1}{2}$d? *Answer*, $643\frac{5}{7}$.

8. Divide L. 765 : 6 : $8\frac{1}{2}$ among nine men, so as eight may have equal shares, and the ninth only $\frac{2}{3}$ of a share.

Answer, { To each man of the eight L. 88 : 6 : $1\frac{1}{4}$
{ To the ninth, 58 : 17 : $5\frac{1}{4}$

9. What is the value of 13 Cwt. 3 qrs. 19 lb of sugar, at L. 2, 9s. $11\frac{1}{2}$d. per Cwt.?

Answer, L. 34 : 15 : 5.

10. Divide L. 875 : 14 : 6 among three men, A. B. C. so as A may have $\frac{3}{4}$, B. $\frac{3}{8}$, and C. $\frac{3}{5}$?

Answer, A. 380.75, B. 190.375, and C. 304.6.

11. What will 46 lb. 8 oz. 16 dwt. of cochineal cost at L. 7, 8s. 6d. per lb.?

Answer, L. 346 : 19 : 10¼.

12. What is the price of 16 ingots of silver, each weighing 7 lb. 8 oz. 10 dwt. at 5s. per oz.

Answer, L. 370.

13. A capital stock consists of 13 shares, whereof A. has 1, B. 3, C. 4, and D. 5 : a dividend is to be made of L. 1799 : 11 : 10¾ profits; what will fall to each?

Answer, A. receives L. 138 : 8 : 7½.

14. What is the value of 5473 gross of incle, at L. 2 : 11 : 11⅓ per gross?

Answer, L. 14214 : 11 : 11½.

15. A bankrupt's estate is computed to pay 8s. 6⅔d. per pound; how much will a debt of L. 375 : 19 : 7 recover?

Answer, L. 160 : 16 : 8½.

SECT. VII. THE REDUCTION AND VALUATION OF DECIMALS BY INSPECTION.

That the learner might not be perplexed with too many things at one time, we thought proper to introduce *reduction by inspection*, when the computation by decimals otherwise was well understood, and to allow it an entire section.

I. To find the decimal answering to shillings, pence, and farthings, by inspection.

In business, three places of decimals are reckoned exact enough, as they are sufficient to bring out the lowest denomination for which we have any coin.——The rule is,

Half the greatest even number of shillings gives the figure next the point; the number of farthings contained in the given pence and farthings, increased by 1, when their number amounts to 24, or exceeds 24, gives two figures more for seconds and thirds: if the number of farthings in the denominations under shillings does not exceed a single digit, it must possess the third place, and a cipher the second, unless there be an odd shilling, for which, in either case, 5 must be added to the second place.

EXAMPLES.

12s. 6d. = .625	for $6 \times 4 + 1 = .025$ $12 \div 2 = .6$..	.625
15s. 4d. = .766	for $4 \times 4 = .016$ $14 \div 2 = .7$ 2s. = .05	.766
14s. $1\frac{1}{4}$d. = .705	for $1\frac{1}{4}$d. = 5 frs. = .005 14s. $\div$ 2 = .7	.705

II. To value the decimal of a pound by inspection.

Double the figure nearest the point for shillings, and if the figure in the second place be 5, or above 5, reckon one shilling more, and take 5 from the second place for it; then will the remainder in the second place, or figure; if no deduction was made, together with the figure in the third place, be farthings, abating 1 for every 25.

EXAMPLES.

.625 = 12s. 6d.	for 2×6 = 12s. od. and $25 - 1 = \frac{24}{4}$ = 6d.	12s. 6d.
.766 = 15s. 4d.	for 7×2 = 14s. od. for 5 in the 2 add 1s. od. 16 frs = os, 4d.	15s. 4d.

.166 =	.375 =
.05 =	.999 =
.333 =	.666 =
.0375 =	.003125 =
.833 =	.625 =

QUERIES.

1. In reduction to decimals, why are ciphers annexed to the numerator, before division?

Sol. To annex ciphers to any number is to multiply by 10, 100, 1000, &c. consequently the dividend becomes so many 10s, 100s, 1000s, &c. and the quotient will consequently be of the same kind as both numerator and denominator have been increased in the same proportion. This also accounts for the manner of reducing the relative parts of any integer to a decimal.

2. In adding and multiplying repetends, why, in the first column, do we carry at 9?

Sol. A series of 9s infinitely continued, is equal to 1 in the left hand place; for, were the series carried to infinity, the difference between that series and an unit, would be equal to unity divided by infinity. Hence one line converging to another may be infinitely extended, and yet never reach it.

3. Why are repetends made conterminous before addition?

Sol. The finite value of a pure circulate is a fraction whose numerator is the repetend, and denominator a number of as many places of 9s, with a number of ciphers on the right, as there are places between the point and repetend. Hence, when conterminous repetends are added, their sum is a numerator to the common denominator, and if 1 for every 9 in the sum is added thereto, it will thereby become equal to the finite expression.

4. In addition of circulates, why are the 10s in the first column of the circle, carried, to be added with the last column?

Sol. It would be the same thing to annex to the conterminous circles, the first figure of every circle before addition: For, since any repetend whatever is to the same number complete, as 10 to 9, if of one place, and 100 to 99, if of two, &c.; any number multiplied by 1, with as many ciphers, as it contains figures, and the product divided by as many 9s, will give the same number perpetually circulating; but to add 1 for every 10 contained in the sum, is the same thing as to multiply that sum by 10 and divide by 9. This also accounts for the same mode of proceeding in multiplication, &c.

5. In multiplication, why ought the decimal places in the product to equal the decimal places in both factors?

Sol. Since the given decimals must be conceived as fractions, it will be evident that the numerators and denominators are multiplied together respectively, since as many places are taken from the product, as there are in the denominators of both factors. Hence also the necessity of prefixing ciphers, when the whole product falls short of the decimal places in both factors counted together. Hence also the rule for qualifying the quotient in division is accounted for, since the number of places in the divisor and quotient must always be equal to those of the dividend.

CHAP. V.

PROPORTION.

PROPORTION is that rule, by which, from a comparison of circumstances, arising from certain conditions or stipulations, certain conclusions are drawn, and consequences deduced and ascertained.

In simple proportion, three numbers are always given to find a fourth; of which the first two are always conditional, and the third implies a demand, and in consequence moves the question. In all direct processes the answer, or fourth proportional, bears the same ratio to the third as the second bears to the first; wherefore, the greater the second term is in respect to the first, the greater will the fourth term be in respect to the third; and the less the second term is in respect to the first, the less will the fourth term be in respect to the third. Hence, in all direct proportions, the product of the extremes will always be equal to the product of the means. On the other hand, if the terms are in reciprocal proportion, the fourth proportional must always bear the same ratio to the second as the third does to the first; consequently, the greater the third term is in respect of the first, the less must the fourth be in respect to the second; and the less the third is, compared with the first, the greater will the fourth be, compared with the second. Hence again, in reciprocal proportion, the product of the first and second terms will always be equal to the product of the third and fourth.

To state the question.

Rule I. Write down that number or term which is of the kind, whether money, weight, measure, time, &c. with the answer, for the middle term.

Rule II. On the right of the middle term already wrote down, place the term upon which the demand lies.

Rule III. On the left of the middle term, place that term of the two conditional ones which is of the same kind with the term on the right.—Then will the terms be placed in a proportional order.

Or, write down that term on which the demand lies for the middle term, and the term homogeneal with it on the left, and the term homogeneal with the answer on the right.

To find a fourth proportional.

Rule, If, upon comparing the first and third, more be found to require more, or less to require less, then will the terms be in di-

rect proportion, and the product of the two last, divided by the first, will quote the answer; but if less require more, or more require less, then will the product of the two first, divided by the last, quote the answer.

Note, Similar terms, that is, the first and third, or second and fourth, must always be of the same denomination, and the preparation may be made as in Reduction, Vulgar or Decimal Fractions.

EXAMPLES.

Bought $175\frac{1}{2}$ yards of cloth, and paid L. 105 : 12 : 0; what did it cost me per piece of 25 yards?

Y.		L.		Y.
$175\frac{1}{2}$	:	105.6	::	25
		50		2
351				50

$$
\begin{array}{r l}
351\,)\ 5280.0 & (\ 15 : 0 : 10\tfrac{30}{117}. \\
351 & \\
\hline
1770 & \\
1755 & \\
\hline
15 & \\
20 & \\
\hline
300 & \\
12 & \\
\hline
3600 & \\
351 & \\
\hline
90 &
\end{array}
$$

Illustration.

Because the answer is to be money, the price is put for the middle term, on the right of which stands the term which implies a demand, 25 yards, and on the left $175\frac{1}{2}$ yards, being the term which is of the same kind with 25: Now, as 25 yards, when compared with $175\frac{1}{2}$, reckon less, and of consequence must bring less, the two last terms are multiplied together, and their product divided by the first. Previous to any multiplication or division, be-

cause in the first term there is a fraction, the first and third terms are reduced into parts expressed by the denominator; and since there are shillings in the second, as well as pounds, the shillings are reduced to a decimal.

The same answer may be effected decimally, thus:

175.5 : 105.6 :: 25 : 15 0 $10\frac{1}{3}$ nearly.

25

5280

2112

175.5) 26400 (15.042

1755

8850

8775

7500

7020

4800

3510

290

Or by reduction, thus:

$175\frac{1}{2}$: 105 12 :: 25

20

351 : 2112 :: 50

50

105600 (300s. $10\frac{30}{117}$d. = L. 15 0 $10\frac{1}{3}$.

1053

300

12

3600

351

90

Or by vulgar fractions, thus:

$$\frac{351}{2} : \frac{2112}{20} :: \frac{25}{1} : \frac{105600}{7020} = 15\tfrac{30}{702} = \text{L. } 15\ 0\ 10\tfrac{1}{3}.$$

Or thus:

```
        105   12
              10
        ----------
        1056
           5
        ------
  351 ) 5280
        ------
          15   0   10⅓ as before.
```

This question, by the second arrangement, would stand,

Yds.		Yds.		L.		L.
175.5	:	25	::	105.6	:	15.042

But the work, as well as the answer, will be the same as before.

The work of all questions in a direct proportion may be readily proved by multiplying extremes and means, the products of which, when the work is right, will be respectively equal; but in reciprocal proportion, the product of the first and second terms will always be equal to that of the third and fourth.

2. Lent my neighbour, upon an emergency, L. 217 : 10 : 0 for 112 days; how long may I retain L. 870 of his money to be indemnified?

```
        L.              D.          L.
        217 10    :    112    ::   870    state.
        217.5     :    112    ::   870    preparation.
        ------                     ---
         43.5                      174    abridged by 5.
         112
        ------
          870
         4785
        ------
  174 ) 48720 ( 28
        348
        ------
         1392
         1392
        ------
           0
```

Illustration.

By comparing the first and third terms together, we find that more requires less, because it would not be fair to keep L. 870 the same time that its owner had L. 217 : 10 : 0; wherefore the product of the two first, divided by the last, quotes the answer.

3. The height of my staff from the ground is 5 feet 9 inches, and it casts a shadow of 6 feet 3 inches; what should be the height of a steeple which casts a shadow of 217 feet 6 inches?

6.25 :	5.75 ::	217.5	
1.25	1.15	43.5	abridged by 5 continually.
		.23	
.25	.23		
		1305	
.05		870	
	.05)	10.005	
		200.1	*Answer.*

4. When flour sold at 26s. 6d. per bag, the 6d. loaf weighed 4 lb. 9 oz. what should be the assize of bread when flour rises to 37s. 6d.?

Sixpences.	Lib.	Sixpences.	
53 :	4.75 ::	75	
	.95	15	abridged by 5.
	.19	3	abridged by 5.
	53		
3)	10·07		

3·356 = 3 lb. 4 oz. $5\frac{3}{5}$ dwt.

5. Suppose an acre of land, 4 falls broad and 40 falls long, is to be excambed with another acre, 9 falls in breadth; required the length sufficient to complete the acre?

B.		L.		B.		L.	
4	:	40	::	9	:	$17\frac{7}{9}$	*Answer.*

6. Bought $\frac{2}{7}$ of $\frac{2}{9}$ of a ship for L. 217 : 10 : 10; what was the ship worth at that rate?

$$\frac{2 \times 2 = 4}{9 \times 7 = 63}$$

	Num.		L.		Den.		L.
Then	4	:	$217.541\dot{6}$	::	63	:	3426 5 $7\frac{1}{2}$

7. Bought $\frac{2}{7}$ of a ſhip for L. 217 : 10 : 10; what will $\frac{2}{9}$ coſt at that rate?

$2\times9=18$ and $2\times7=14$ L. s.

— — Then 18 : 217.5416 :: 14 : 169 4 nearly.

$7\times9=63$ 63 Or 9 : 217.5416 : 7 : 169 4

8. A man had a ninety-nine years leaſe, and being aſked how much of it was run, anſwered, that $\frac{2}{3}$ of the time paſt was equal to $\frac{4}{5}$ of the time to come? what is the time paſt and to come?

$2 \times 5 = 10$
$4 \times 3 = 12$ Then 22 : 99 :: 10 : 45 time to come.
—
22 22 : 99 :: 12 : 54 time paſt.

9. The ſhorter end of the beam of a balance is 27 inches, and the longer end 36 inches; how much ſuſpended on the longer end will equiponderate 20 lb. on the ſhorter?

In.	lb.	In.	lb.
27 :	20 ::	36 :	15

$$\text{For } \frac{3 \times 20}{4} = 15$$

10. The arms of the beam of a balance meaſure from end to end 63 inches, and 20 lb. on the one end equiponderates 15 lb. on the other; required the length of the arms?

	Wt.	L.	Wt.	L.
20 + 15 =	35 :	63 ::	20 :	36

$$\text{For } \frac{4 \times 63}{7} = 36 \text{ longer end;}$$

And 35 : 63 :: 15 : 27

$$\text{For } \frac{3 \times 63}{7} = 27 \text{ ſhorter end.}$$

11. There is a floor, 42 feet 9 inches long, and 33 feet 6 inches wide, to be laid with deals uniformly 10 feet 3 inches by $8\frac{1}{2}$ inches; how many will it require, allowing the odd half inch in breadth to go for ſeaming?

42.75	3) 10.25
$33\frac{1}{2}$	
21.375	$3.41\dot{6}$ for 4 inches,
128.25	$3.41\dot{6}$ for 4 inches,
1282.5	$6.83\dot{3}$ for 8 inches, area of a deal.
1432.125 area of the floor.	

	Area.		Deal.		Area.	
Then	$6.8\dot{3}$	:	1	::	1432.125	
	3				3	
	20.5				) 4296.375 (	209.579 *Answer.*

410
1963
1845
1187
1025
1625
1435
1900
1845
55

12. There is a room 24 feet 6 inches by 18 feet 9 inches, whose height is 10 feet 3 inches, to be lined with paper $\frac{5}{8}$ yard broad; how many yards will it require?

Yd.F.	24 6
$\frac{5}{8}=1.875$	18 9
3	43 3
5.625 area of 1 yard.	2
	86 6 $=86.5\times10\frac{1}{4}=886.625$ area of the walls.

	Area.		Yd.		Area.		Yd.
Then	5.625	:	1	::	886.625	:	158 nearly.

13. If 6 the third of 20 be, what's the fourth of 33?

$$\frac{20}{3} : \frac{6}{1} :: \frac{33}{4} : \frac{504}{80} \text{ or } 7\frac{17}{40}.$$

$$\text{For } \frac{6}{1} \times \frac{33}{4} \div \frac{20}{3} = \frac{504}{80}.$$

14. 560 men are besieged in a garrison, and have provisions only for 3 months; how many men must evacuate the garrison, that the remainder may be supplied for 5 months longer?

Mo.		Men.		Mo.		Men.
3	:	560	::	8	:	210 to stay in,

and 560 — 210 = 350 to be turned out.

15. If in travelling 12 hours a-day, I finish a journey in 24 days; in how many days ought I to finish the same journey, when I travel 16 hours a-day, at the same rate?

H.		D.		H.		D.
12	:	24	::	16	:	18

$$\text{For } \frac{24 \times 3}{4} = 18 \textit{ Answer.}$$

16. The valued rent of a whole parish is L. 6754 : 10, and the assessment for poor's rates L. 134 : 15; with how much ought a valuation of L. 475 : 16 to be assessed?

V. R.		Assess.		V. R.		
6754.5	:	134.75	::	475.8	:	L. 9 : 9 : $10\frac{1}{4}$.

17. The diameter of a pipe which empties a cistern in 24 hours is $9\frac{1}{2}$ inches; in what time will a pipe do it whose diameter is $6\frac{1}{3}$ inches?

$$6\tfrac{2}{6} = 38 \quad \text{But} \quad 38 \times 38 = 1444$$
$$9\tfrac{3}{6} = 57 \quad \text{and} \quad 57 \times 57 = 3249$$

	Sq.Diam.		H.		Sq.Diam.		H.
Then	3249	:	24	::	1444	:	54

$$\text{For } \frac{3249 \times 6}{361} = 54 \text{ hours. } \textit{Answer.}$$

18. A. can do a piece of work in 30 days, B. can do it in 40; in what time will they do it working together?

	D.		W.		D.		
First,	30	:	1	::	40	:	$1\frac{1}{3}$

Secondly, $2\frac{1}{3}=7$: 40 :: $1=\frac{3}{3}$: $17\frac{1}{7}$ *Answer.*

19. In the mint of England, 1 lb. of gold, including 1 oz. of alloy, is coined into $44\frac{1}{2}$ guineas; at how much ought 1 lb. of pure gold to be valued, at that rate?

Oz.		L.				L.
11	:	46.725	::	12	:	$50:19:5\frac{3}{4}$

20. Three men or four women can cut down a field of corn in 48 hours; in what time will four men and two women do it?

	M.		W.		M.		
First,	3	:	4	::	4	:	$5\frac{1}{3}$

	W.		H.		W.		H.
Secondly,	4	:	48	::	$7\frac{1}{3}$	:	$26\frac{2}{11}$.

21. An officer's salary in the year is L. 48 : 2 : 6; how much ought he to draw for 6 weeks, or 42 days?

D.		L.		D.		L.
365	:	48.125	::	42	::	5.537 or L. 5 : 10 : 9

22. If 231 solid inches make a wine gallon, and 282 a gallon of ale; how many gallons of wine will a cask contain which was made to contain just 38 gallons of ale?

S. I.		A. G.		S. I.		W. G.
282	:	38	::	231	:	46.389

23. A room, $19\frac{1}{2}$ by $18\frac{1}{2}$, is to be covered with Holland duck $\frac{3}{4}$ yard wide; how much will it require?

2.25	19.5
3	$18\frac{1}{2}$
6.75	9.75
	156.0
	195
	360.75 area of the room.

Area.		Y.		Area.		
6.75	:	1	::	360.75	:	53.4

24. A. and B. set out from the same point in a circular island, 134 miles round, to travel back to back, so as to meet at a certain point; A. travels 11 miles in 2 days, and B. 17 miles in 3 days; in how many days, and after how many miles travel to each, will they meet?

First, Find how many miles each travelled in 3 days.

D.		M.		D.		M.	
2	:	11	::	3	:	16.5.	A. travels in 3 days.
						17.	B. travels in 3 days.
						33.5	travelled by both.

	M.		D.		M.		D.
Then,	33.5	:	3	::	134	:	12
Secondly,	2	:	11	::	12	:	66 miles travelled by A.
Thirdly,	3	:	17	::	12	:	68 miles travelled by B.

25. The distance between London and Edinburgh is 360 miles? from London A. set out, running at the rate of 10 miles an hour, and from Edinburgh, at the same instant, B. at the rate of 8 miles an hour: how many hours and how many miles will each have travelled before they meet?

Answer, A. travels 200 miles, B. 160, in 20 hours.

26. A privateer, sailing at the rate of 10 miles an hour, discovers a ship a-head making way at the rate of 8 miles an hour; required how long and how many miles the ship can hold out, before she be overtaken by the privateer?

Difference of sailing, 2 miles; for 10 — 8 = 2,

Ship a-head of the privateer, 18 miles.

	M.		H.		M.		H.
Therefore	2	:	1	::	18	:	9 the time,

	H.		M.		H.		
and	1	:	8	::	9	:	72 miles sailed by the ship,
and	1	:	10	::	9	:	90 miles sailed by the privateer.

27. One sets out from a certain place, and travels at the rate of 7 miles in 5 hours; and, 8 hours after, another posts after him at the rate of 5 miles in 3 hours; how long and how far can the first travel before he be overtaken by the second?

Answer, The first must travel 50 hours and 70 miles.
The second 42 hours, and also 70 miles.

28. The orbit in which the earth performs its annual revolution is reckoned to contain 518,222,400 miles; how much does the earth move in a day?

365.25 : 518222400 :: 1 : 1418815.6 miles. *Answer*.

29. Bought a pack of cloth containing $855\frac{1}{4}$ yards, which cost at Leeds, L. 611 : 10, the carriage and other charges came to L. 12 19s. 6d.; what is the value of a piece of $28\frac{1}{2}$ yards, at that rate?
Answer, L. 20 : 15 : $11\frac{1}{4}$.

30. Lent my neighbour, upon an occasion L. 547 : 10 for 93 days; how many days ought I to retain L. 476 : 15 of his money to requite the obligation? *Answer*, 106.8 days.

31. To discover the height of a steeple, I measured its shadow at 6 o'clock in the evening $379\frac{1}{3}$ feet; my staff $5\frac{1}{2}$ feet in length, being perpendicular to the plane, cast a shadow of $11\frac{7}{8}$ feet; what was the height of the steeple? *Answer*, 175.6 feet.

32. When I bought my flour at 29s. 8d. per bag, the 6d. loaf weighed 4 lb $9\frac{1}{2}$ oz.; what should be the weight of the 6d. loaf when the bag of flour costs 34s. $7\frac{1}{4}$d.? *Answer*, 4 lb 1 oz. 5 dwt.

33 When I bought flour at 26s. 6d. per bag, the 6d. loaf weighed 4 lb 9 oz.; it only weighs at present, 3 lb 4 oz. $5\frac{1}{7}$ dwt.; what is the price of flour? *Answer*, 37s. 6d.

34. A piece of ground 18 chains 30 links by 6 chains 40 links, for the accommodation of a neighbour, is to be excambed for another piece 8 chains 60 links in breadth; what length will be necessary to make the areas equal? *Answer*, 13 chains 61.8 links.

35. Bought $\frac{3}{8}$ of $\frac{2}{3}$ of a ship for L. 540; what was the value of the ship at that rate? *Answer*, L. 2160.

36. Bought $\frac{3}{8}$ of a ship for L. 725 : 10; what is $\frac{2}{3}$ of the ship worth at that rate? *Answer*, L. 1289 : 15 : $6\frac{1}{4}$.

37. A man had a lease for three 19 years, and being asked how much of it was to run, answered that $\frac{3}{5}$ of the time past is equal to $\frac{3}{4}$ of the time to come; required the particulars? *Answer*, time past $31\frac{2}{3}$ years, time to come $25\frac{1}{3}$.

38. The shorter end of the beam of a balance is 24 inches, and the longer 36; how much suspended on the longer end will equiponderate 15 ℔ on the shorter? *Answer*, 10 ℔.

39. The arms of the beam of a balance measure together 60 inches, and 10 ℔ on the longer end equiponderate 15 on the shorter; required the length of the arms respectively? *Answer*, 24 and 36.

40. There is a floor $56\frac{3}{4}$ feet long, and $33\frac{2}{3}$ feet broad, to be laid with deals uniformly 10 feet 10 inches by 9 inches; how many will it require allowing $\frac{1}{2}$ inch in breadth for seaming? *Answer*, 248.98.

41. A room 33 feet 9 inches by 24 feet 8 inches, and from the belt to the cornice 9 feet 9 inches in height is to be lined with paper $\frac{3}{8}$ yard wide; how many yards will it require? *Answer*, 337.518 yards.

42. The diameter of a pipe which conveys water to serve 40,000 people is 15 inches, the inhabitants have increased to 100,000; what should the diameter of that pipe be which is to convey water to supply the additional 60,000? *Answer*, 18.37 inches.

43. How many gallons of ale will a cask contain that was made to contain 126 gallons of wine? *Answer*, 115.5 gallons.

44. Bought 125 lbs. of silk which cost me 42s. 6d. per lb. avoirdupoise, and sold it by the lb. troy for the same money, what did I gain or loss by the bargain? *Answer*, L. 57 : 3 : $2\frac{1}{2}$ gained.

45. Bought 375 quarters of wheat, and sold it again by the Scots firlot; how many bolls did it measure? *Answer*, 757 bolls 1 firlot.

Note, The Scots wheat firlot is to the English corn bushel as 100 to $99\frac{21}{100}$.

46. A. and B. together, usually drink a gallon of rum in 12 days; A. does it alone in 20 days; in what time will B. do it? *Answer*, 30 days.

47. Bought 278 Flemish ells of cambric and paid L. 72 : 10 : 6,

I retailed it again by the English yard at 8s. 6d. per yard; whether did I gain or loss, and how much? *Answer*, L. 16 : 1 : 9 gained.

48. There is a room $109\frac{1}{4}$ feet in perimeter, and $9\frac{1}{4}$ feet in height, which, deducing two windows, each $6\frac{1}{2}$ feet by $5\frac{3}{4}$, is to be hung with tapestry; how much will it require, supposing the tapestry ell wide? *Answer*, 83.594 yards, if the tapestry be an English ell in breadth; but if a Scots ell of 37 inches, it will require 101.668 yards.

QUERIES.

1. Required the reason, why, in direct proportion, the product of the 2d and 3d terms, divided by the first, will always quote the 4th, or answer?

Sol. Suppose it required to find the value of 36 yards of cloth, when 12 yards cost L. 8; the state would be,

Yds.	L.	Yds.
12 :	8 ::	36

Now, instead of 12, let us suppose 1 yard costs L. 8, then it is evident that 36 yards would cost $8 \times 36 = 288$; but it was not 1 yard, but 12 yards, that cost L. 8, therefore $\frac{1}{12}$ of L. 288 is the price of 36 yards: Therefore $\frac{288}{12} =$ L. 24, the price required.

2. In reciprocal proportion, why is the product of the first two terms divided by the last?

Sol. Suppose the question put, If 12 men finish a work in 15 days, in what time will 20 do it? Now, it is plain, that whatever work could be executed by 12 men in 15 days, the same work would require 1 man 12 times as many, viz. $12 \times 15 = 180$; but, if it would require 1 man 180 days, 20 men will do it in $\frac{1}{20}$ of the time: Therefore, $\frac{180}{20} = 9$, the 4th term, or answer.

CHAP. VI.

COMPOUND PROPORTION.

WHEN questions in this rule require two or more operations before the answer can be obtained, as some of the foregoing, the process will be less perplexed if the terms are arranged in a successive order, and so reduced to one simple operation; for which observe the following rules.

I. Of the five given terms, three are conditional, and two imply a demand. Of the conditional terms, let that which is the principal cause of gain or loss, action or passion, increase or decrease, appear as the first term; let that which includes in it time or distance appear in the second place; and the remaining conditional term in the third place. The other two terms, which imply a demand, take the order of their arrangement from the other three.

II. Consider to which of the three conditional terms, as antecedents, the answer, or sixth proportional, is to be a consequent; or, by comparing the antecedents and consequents on the five terms, find which of the antecedents wants a consequent.

III. Then, if the term sought be of the same kind with the third, the continued product of the three last terms, divided by the product of the two first, will quote the answer; but if the term sought be of the same kind with the first or second, the continued product of the first, second, and fifth terms, divided by the product of the third and fourth terms, will quote the answer.

EXAMPLES.

1. If 12 roods of grass be cut down by 2 men in 6 days; how many roods will be cut down by 8 men in 24 days?

Men.	Days.	Roods.	Men.	
2 :	6 ::	12 :	8 :	24 days.

$$\frac{12 \times 8 \times 24}{12} = 192 \text{ roods.}$$

Or by abridging the terms:

$$8 \times 24 = 192$$

Illustration.

As men are the cause of action, 2 stands in the first place; as days imply time, 6 possesses the second place; and roods being the action, 12 possesses the third place; in which order also the other two terms fall to be placed, the men first, and then the days: Then, because a term of the same kind with the third is required, the continued product of the three last is divided by the product of the first two.

2. Suppose L. 100 would defray the expence of a certain work for 22 weeks 6 days, when 5 men were employed in it; in what time would 12 men employed in the same work draw L. 150?

	M.		D.		L.		M.		L.
	5	:	160	::	100	:	12	:	150
Abridged,	5	:	16	::	1	:	12	:	15

$$\frac{5 \times 16 \times 15}{12} = 100.$$

Because a term of the same kind with the second is required, the continued product of the first, second, and last, divided by the product of the third and fourth, quotes the answer.

3. What principal sum will gain L. 20 in 3 months, at 5 per cent. per annum?

L.		M.		L.		M.		L.
100	:	12	::	5	:	3	:	20
20	:	4	::	1	:	1	:	20

$$20 \times 4 \times 20 = 1600$$

Here the required term is of the same kind with the first.

4. A court, 33 yards square, is paved with 285 stones, 15 inches by 10; how many stones, 24 inches by 18, will pave a court 40 yards square?

$$15 \times 10 = 150$$
$$18 \times 24 = 452$$

Then $285 : 150 : 33 :: 432 : 40$

$$\frac{285 \times 150 \times 40}{33 \times 432} = 119\tfrac{94}{99}, \text{ or } 120 \text{ nearly.}$$

5. A farmer computed that 14 ſhearers would complete his harveſt in 20 days; but finding his corn ripe, and dreading an alteration of weather, reſolves, after 8 days work, to have the remainder completed in 7 days; how many hands muſt he add for that purpoſe?

Sh.		D.		Harveſt.		D.		H.
14	:	8	:	.4	::	7	:	.6

$$\frac{14 \times 8 \times .6}{2.8} = 24, \text{ conſequently he muſt add 10.}$$

6. On 25th March 1774, there was lent on a mortgage L. 760, and, 25th May 1780, the intereſt came to L. 281 : 4; at what rate per cent. per annum was the money lent?

L.		D.		L.		L.		D.		
760	:	2253	:	281.2	::	100	:	365	:	6 per cent. nearly.

7. If 60 Cwt. be carried 20 miles for L. 14 : 10, how many miles ought 15 Cwt. to be carried for L. 5 : 8 : 9?

Cwt.		M.		L.		Cwt.		L.		M.	
60	:	20	:	14.5	::	15	:	5.4375	:	30	*Anſwer.*

8. Four maſons were employed in building a wall, 20 yards long, which they completed in 10 days, by working 6 hours a day: in what time will 8 maſons continue the ſame wall 100 yards farther, when they work 12 hours a day?

M.		H.		Y.		M.		Y.		H.		D.	H.
4	:	60	:	20	::	8	:	100	:	150	or	12	6

9. 18 men built a wall, 40 feet long, 3 feet thick, and 16 feet high, in 12 days; how many men must be employed to build a wall, 360 feet long, 8 feet thick, and 10 feet high, in 60 days?

$$40 \times 3 \times 16 = 1920 \text{ solidity.}$$
$$360 \times 8 \times 10 = 28800 \text{ solidity.}$$

	M.		D.		Sol.Ft.		D.		Sol.Ft.		M.
Then	18	:	12	:	1920	::	60	:	28800	:	54 *Answer.*

10. If 24 men dig a ditch, 724 yards long, 7 feet deep, 3 feet wide at the bottom, and 5 feet wide at the top, in 236 days, when the day is 13 hours 24 minutes long; in how many days, 12 hours 36 minutes long, may 16 men dig a ditch, 675 yards long, 9 feet deep, 5 feet wide at the bottom, and 7 feet wide at the top?

$$724 \times 3 \times 7 \times 4 = 60816 \text{ solid feet,}$$
$$\text{and } 675 \times 3 \times 9 \times 6 = 109350 \text{ solid feet.}$$

236 days, 13 hours 24 minutes long, = 189744 minutes,
and 1 day, 12 hours 36 minutes long, = 756 minutes.

Therefore,

M.		Min.		Sol.Ft.		M.		Sol. Ft.		Min.		D.	H.	M.
$\frac{24}{1}$	:	189744	:	60816	::	$\frac{16}{1}$	:	109350	:	$\frac{511752}{756}$	=	676	11	36

Note, Every compound question may be resolved into as many simple questions as there are terms on the left of the middle term, thus:

Resolve the question into all its simple terms, so as the middle term of the compound question may be the third in every simple one; then multiply all the antecedents of the first ratios continually for a new antecedent, and their consequents for the consequent of a new ratio; then will the common third term be to its consequent, or answer, as the new antecedent to its consequent. Hence the rationale in compound processes is the same as in those of simple proportion.

11. It was found that 15 men consumed the value of 13s. in bread in 6 days, when wheat sold at 12s. per boll; what value will 30 men consume in bread in 12 days, when wheat falls to 10s. per boll, supposing the consumpt at the same rate?

M.	D.	s.		s.		M.	D.	s.
15 :	6 :	12 :		13	::	30 :	12 :	10

15 : 30
6 : 12 :: 13 Then $15 \times 6 \times 12 = 1080$
12 : 10 and $30 \times 12 \times 10 = 3600$

Then $\frac{1080}{9}$: $\frac{3600}{30}$ :: 13 : $43\frac{1}{3}$s.

12. It was found that 18 roods of ditching was done by 3 men in 16 days, by working 15 hours a-day; how much may be done by 8 men in 4 days, by working 9 hours a-day.

Anſwer, $7\frac{1}{5}$ roods.

13. 10 Men had finiſhed a wall 30 yards in length $2\frac{1}{2}$ yards in height and $\frac{1}{2}$ yard thick, in the ſpace of 12 days, when 6 men were added to their number, and the whole employed in building a wall 96 yards in length, 7 feet in height, and 2 in thickneſs; in what time will they accompliſh it?

Anſwer, $29\frac{11}{15}$ days.

14. When money was at 5 per cent. I lent my friend L. 300, and ſuffered him to retain it 40 days; how long may I retain L. 400 of his money to be indemnified, when intereſt falls to $4\frac{1}{2}$ per cent?

Anſwer, $33\frac{1}{3}$ days.

15. A ſhip of 220 tuns burthen was chartered for 4 months at L. 120; what ought to be paid for a ſhip of 450 tuns in 9 months, at that rate?

Anſwer, L. 525 : 5 : $7\frac{1}{4}$.

16. If 48 men in $5\frac{1}{2}$ days, dig a trench $23\frac{1}{2}$ yards long, $2\frac{1}{2}$ deep, and $3\frac{2}{3}$ broad; what length of a trench $3\frac{1}{2}$ yards deep, and $5\frac{1}{3}$ wide, can be dug by 24 men in 189 days, at that rate?

Anſwer, $188\frac{4}{5}$ yards.

CHAP. VII.

PRACTICE.

WHEN an unit is one of the given terms in proportion, the formal process is laid aside, and the following simple contractions substituted, which have been occasionally adopted for expediting the computation of invoices, bills of parcels, accounts of sales, &c.

Table of the aliquot parts of a pound.

s.	d.			d.				
10	0	=	L. $\frac{1}{2}$	8	=	L. $\frac{1}{30}$	or	$\frac{2}{3}$ s.
6	8	=	$\frac{1}{3}$	6	=	$\frac{1}{40}$	or	$\frac{1}{2}$
5	0	=	$\frac{1}{4}$	4	=	$\frac{1}{60}$	or	$\frac{1}{3}$
4	0	=	$\frac{1}{5}$	3	=	$\frac{1}{80}$	or	$\frac{1}{4}$
3	4	=	$\frac{1}{6}$	2	=	$\frac{1}{120}$	or	$\frac{1}{6}$
2	6	=	$\frac{1}{8}$	$1\frac{1}{2}$	=	$\frac{1}{160}$	or	$\frac{1}{8}$
2	0	=	$\frac{1}{10}$	1	=	$\frac{1}{240}$	or	$\frac{1}{12}$
1	8	=	$\frac{1}{12}$	$\frac{1}{2}$	=	$\frac{1}{480}$	or	$\frac{1}{24}$
1	0	=	$\frac{1}{20}$					

CASE I.

When the price of an unit is any aliquot part of a pound contained in the table above, divide the given quantity as directed in the table, and the quotient gives the answer, which, for certainty of calculation, ought also to be found another way.

EXAMPLES.

1. At 10s. what cost 4597 yards?

2) 4597	2,0) 4597,0
2298 : 10 *Answer*, or	2298 : 10

Note, Whatever remains is so many units of the given quantity, and must be rated at the given price.

2. At 6s. 8d. what cost 9375 yards?

3) 9375	3,0) 9375,0
3125 *Answer.*	3125

3 At 5s. what cost 4197 yards?	*Answer*, L. 1049	5	0
4. At 4s. what cost 3133 yards?	626	12	0
5. At 3s. 4d. what cost 5211 yards?	868	10	0
6. At 2s. 6d. what cost 4171 yards?	521	7	6
7. At 2s. what cost 3333 yards?	333	6	0
8. At 1s. 8d. what cost 4131 yards?	344	5	0
9. At 1s. what cost 3171 yards?	158	11	0

10. At 8d. what cost 3051 yards?

$\dfrac{3051}{30}$ = L. 101 14; or,

3) 3051
1017
1017
20) 2034 (101 14

In dividing by 30 for 8d. in the last example, there is a remainder, after the pounds, of 21, = 21 yards, at 8d. so mentally 21 sixpences = 10s. 6d. and 21 twopences = 3s. 6d.

11. At 6d. what cost 4537 yards?	*Answer*, L. 113	8	6
12. At 4d. what cost 3133 yards?	52	4	4
13. At 3d. what cost 4111 yards?	51	7	9
14. At 2d. what cost 1211 yards?	10	1	10
15. At $1\frac{1}{2}$d. what cost 9713 yards?	60	14	$1\frac{1}{2}$

CASE II.

When the given price is any even number of shillings, multiply the quantity by one half the price, or, which is the same thing, the decimal of the price, and double the right hand figure of the product for shillings, the rest of the product is pounds.

EXAMPLES.

1. At 8s. what cost 5733 yards?

5733 ·4 L. 2293 4	or	5) 5733 1146 12 1146 12 L. 2293 4

2. At 14s. what cost 4171 yards?	*Answer,* L. 2919	14
3. At 18s. what cost 1173 yards?	1055	14
4. At 12s. what cost 5111 yards?	3066	12
5. At 6s. what cost 7945 yards?	2383	10
6. At 16s. what cost 917 yards?	733	12
7. At 2s. what cost 4578 yards?	457	16

CASE III.

If the given price be no aliquot part of a pound, nor even number of shillings, it must be disposed into aliquot parts, and these divided for severally; or, when any lesser part can be taken out of a greater preceding it, divide that part for it, and the sum of these partial quotients will give the answer.

EXAMPLES.

1. At 3s. 9d. what cost 4151 yards?

6) 4151						10) 4151					
8) 691	16	8	for 3s. 4d.			2) 415	2	0	for 2s. 0d.		
86	9	7	for 0 5		or	2) 207	11	0	for 1 0		
L. 778	6	3	*Answer.*			2) 103	15	6	for 0 6		
						51	17	9	for 0 3		
						L. 778	6	3			

2. At 5s. 8½d. what cost 3727 yards?

4) 3727					
10) 931	15	0	for	5s.	0d.
3) 93	3	6	for	0	6
4) 31	1	2	for	0	2
7	15	3¼	for	0	0¼
L. 1063	14	11½			

12) 3727			
2			
745	8	0	at 4s.
40) 310	11	8	
7	15	3¼	
L. 1063	14	11½	

							Answer,		
3. At	7s.	4¾d.	what cost	5471	yards?	L.	2023	2	7¼
4. At	15	$9\frac{5}{8}$,	what cost	1741	yards?		1375	11	5
5. At	11	10,	what cost	3213	yards?		1901	0	6
6. At	2	$7\frac{3}{5}$,	what cost	5411	yards?		712	8	11½
7. At	8	5¾,	what cost	3119	yards?		1322	6	6¼
8. At	18	4,	what cost	5119	yards?		4692	8	4
9. At		$6\frac{2}{3}$,	what cost	7113	yards?		197	11	8
10. At	15	$5\frac{1}{11}$,	what cost	5315	yards?		4098	16	6
11. At	9	9¾,	what cost	2118	yards?		1039	2	10½
12. At		2¾,	what cost	3471	yards?		39	15	5¼
13. At		$1\frac{5}{8}$,	what cost	1197	yards?		8	2	1
14. At		3¾,	what cost	9371	yards?		146	8	5¼

Note, To discover whether any part of the price not divided for be any aliquot part of the price already found, set the price found and that to be divided for in the form of a mixt number; multiply the integral part by the denominator of the fractional, and divide the product by the numerator: the quotient, if nothing remain, will express the aliquot part, For instance,

To discover what part of 3d. is $\frac{3}{4}$, $\frac{3 \times 4}{3} = \frac{1}{4}$.

To discover what part of 6d. is $\frac{2}{3}$, $\frac{3 \times 6}{2} = \frac{1}{9}$.

To discover what part of 8d. is $\frac{4}{5}$, $\frac{5 \times 8}{4} = \frac{1}{10}$.

To discover what part of 10d. is $\frac{5}{6}$ $\frac{6 \times 10}{5} = \frac{1}{12}$.

And so of any other.

CASE IV.

If there is a fraction in the quantity, either reduce it to a decimal, and continue your divisions decimally, or take parts of the given price for the fractional part of the quantity.

EXAMPLES.

1. At L. 2 : 8 : 8, what cost $597\frac{3}{4}$ yards?

	597.75				
	2				
5)	1195.5	for L.	2	0	0
12)	239.1	for	0	8	0
	19.925	for	0	0	8
	1454.525	= L.	1454	10	6

	597			
	73 × 8d.	L. 2 : 8 : 8		
	1791			
	4179			
3,0)	4358,1			
	1452	14	0	
	1	4	4	= $\frac{1}{2}$
	0	12	2	= $\frac{1}{4}$
L.	1454	10	6	*Answer.*

	L.	s.	d.				*Answer.*		
2. At	3	15	$9\frac{1}{2}$,	what cost	$5741\frac{1}{3}$	pieces?	L. 21757	5	$2\frac{1}{2}$
3. At	1	13	4,	what cost	$2711\frac{5}{8}$	pieces?	4519	7	6
4. At	1	18	4,	what cost	$1171\frac{8}{9}$	pieces?	2246	2	5
5. At	2	11	11,	what cost	$1415\frac{5}{12}$	pieces?	3674	3	$8\frac{1}{2}$
6. At	5	5	5,	what cost	$2171\frac{7}{8}$	pieces?	11447	11	$9\frac{1}{2}$
7. At	2	4	4,	what cost	$1111\frac{7}{16}$	pieces?	2463	13	9
8. At	3	3	$3\frac{1}{4}$,	what cost	$976\frac{11}{16}$	pieces?	3091	16	6
9. At	4	8	$8\frac{8}{9}$,	what cost	$874\frac{3}{5}$	pieces?	3880	12	$7\frac{1}{4}$
10. At	7	14	$10\frac{1}{2}$,	what cost	$497\frac{5}{6}$	pieces?	3855	1	11

CASE V.

If the given price be near any number of pounds, or any aliquot part of a pound, find the price for pounds, or nearest aliquot part above the given price: find also the price for the difference, subtract the one from the other, and the remainder gives the answer.

EXAMPLES.

1. At L. 1 : 18 : 4, what cost $574\frac{5}{8}$ yards?

12) 574.625				
2				
1149.250	for L.	2	0	0
47.885	for	0	1	8
1101.364 *Answer.*				

L. 574	0	0	for L.	1	0	0
287	0	0	for	0	10	0
143	10	0	for	0	5	0
95	13	4	for	0	3	4
0	19	2	for	0	0	$0\frac{1}{2}$
0	4	$9\frac{1}{2}$	for	0	0	$0\frac{1}{8}$
L. 1101	7	$3\frac{1}{2}$ *Answer.*				

	L.	s.	d.			*Answer.*		
2. At	1	19	11,	what cost 3174 yards?		L. 6334	15	6
3. At	3	16	8,	what cost $1171\frac{1}{2}$ yards?		4490	15	0
4. At	2	17	6,	what cost $847\frac{5}{8}$ yards?		2436	18	$5\frac{1}{2}$
5. At	0	9	11,	what cost $3714\frac{1}{2}$ yards?		1841	15	$5\frac{3}{4}$
6. At	0	11	11,	what cost $1976\frac{7}{8}$ yards?		1177	17	9

CASE VI.

When a troublesome fraction is given in the price, it may be exterminated by multiplying the whole price by the denominator; divide for the product as if it were the price given, but divide the answer found by the same number or denominator by which you multiplied for the true answer.

EXAMPLES.

1. At $7\frac{4}{7}$d. what cost 3147 yards?

$7\frac{4}{7}$	3147				
7	2				40) 314.7
4s. 5d.	12) 629	8	0	for 4s.	
	4) 52	9	0	for 4d.	6) 78.675
	13	2	3	for 1d.	2) 13.1125
					7) 6.55625
	7) 694	19	3		.9366
	L. 99	5	$7\frac{1}{7}$	*Answer,* =	99.2803

	s.	d.			*Answer.*		
2. At	4	$5\frac{5}{12}$,	what cost 5174 yards?	L.	1151	11	5
3. At	6	$9\frac{7}{8}$,	what cost 4593 yards?		1566	17	$7\frac{1}{2}$
4. At	0	$6\frac{4}{20}$,	and $\frac{2}{3}$ of $\frac{1}{20}$, what is the duty on 77859 lb. of tobacco?		2022	3	5
5. At	6	$3\frac{13}{20}$,	what is the duty of 784 Cwt. sugar?		247	2	$5\frac{3}{4}$
6. At	11	$9\frac{2}{3}$,	what cost 5914 yards?		3490	18	0

Bills of parcels, invoices, accounts of sale, &c. are always computed and checked by these rules. Specimens, to be computed, follow.

In order to ascertain accuracy, every article ought to be computed by different processes, till the results agree, and then there can be no error but in the general summation; whereas otherwise every article must be computed anew before the error be discovered, or the calculator pronounce that he is right.

BILLS OF PARCELS. No. 1.

Glasgow, 15th May 1788.

Mess. Macleans, Mackay, and Co.

Bot. of Russel, Baird, & Co.

28 pss Osnaburghs, 2315 yards, at $7\frac{2}{3}$d. per yard,	L. 73	19	$0\frac{1}{4}$
48 pss Irish linen, 1007 yards, at 1s. $7\frac{5}{8}$d. per ditto,	82	6	$10\frac{1}{4}$
47 pss checks, 1421 yards, at $11\frac{7}{8}$d. per ditto,	70	6	$2\frac{1}{4}$
54 pss striped linen, 1635 yards, at 3s. $5\frac{5}{12}$d. per ditto,	282	3	$0\frac{1}{4}$
17 pss fine holland, 408 yards, at 3s. $7\frac{3}{8}$d. per ditto,	73	14	9
29 pss sheeting, 2175 yards, at 1s. $1\frac{5}{8}$d. per ditto,	123	9	$6\frac{1}{4}$
39 pss figured cloth, 1170 yards, at 4s. $7\frac{5}{7}$d. per ditto,	271	12	$1\frac{3}{4}$
	L. 977	11	6

No. 2.

Glasgow, 20th May 1788.

Mr William Glen

Bot. of Hardy, Shirvah, & Co.

50 pss plain lawns, 500 yards, at 2s. $9\frac{7}{8}$d. per yd.	L. 70	11	$5\frac{1}{2}$
64 — striped ditto, 640 yds. at 2s. $3\frac{1}{3}$d. per do.	72	17	$9\frac{1}{4}$
85 — flowered do. 858 yds. at 3s. $5\frac{1}{8}$ per do.	147	0	$5\frac{1}{2}$
33 — sprigged do. 330 yds. at 3s. $2\frac{5}{6}$d. per do.	53	7	11
57 — ditto ditto, 570 yds. at 2s. $11\frac{1}{6}$d. per do.	83	10	5
94 — silk gauzes, 940 yds. at 3s. $1\frac{1}{3}$d. per do.	147	5	4
	L. 574	13	$4\frac{1}{4}$

No. 3.

Glasgow, 25th May 1788.

Mr Daniel Campbell

Bot. of John Yuill and Co.

			L.	s.	d.
78 dozen pairs mens shoes,	at 45s. $5\frac{1}{2}$d.	per doz.	L. 177	5	9
84 dozen ditto,	at 51s. $7\frac{3}{4}$d.	per doz.	216	18	3
73 dozen womens shoes,	at 43s. $1\frac{1}{3}$d.	per doz.	157	7	$1\frac{1}{4}$
67 ditto ditto pumps,	at 47s. $3\frac{5}{8}$d.	per doz.	158	9	8
95 ditto childrens shoes,	at 23s. $5\frac{5}{8}$d.	per doz.	111	9	$6\frac{1}{4}$
97 ditto ditto pumps,	at 25s. $7\frac{3}{4}$d.	per doz.	124	7	$7\frac{3}{4}$
			L. 945	17	$6\frac{1}{4}$

INVOICES.

No. 1.

Glasgow, 25th Aug. 1788.

Invoice of sundries shipped on board the Glasgow, Captain Fairweather, for Jamaica, by Samuel Adventurer, on the proper account and risk of the shipper, consigned to Thomas Trustee and Co. merchants in Kingston, for sale and returns; contents, cost, and charges, as under, viz.

	L.	s.	d.
20 kegs white lead, containing 1190 lb. at 43s. $7\frac{3}{5}$d. per Cwt.	23	3	$7\frac{1}{4}$
8 casks nails, cont. 240 M. at 2s. $3\frac{7}{8}$d. per M.	27	17	6
6 ditto ditto, cont. 120 M. at 2s. $9\frac{1}{3}$d. per M.	16	13	4
6 ditto ditto, cont. 169 M. at 3s. $11\frac{3}{7}$d. per M.	33	7	$11\frac{1}{2}$
4 ditto ditto, cont. $60\frac{1}{2}$ M. at 5s. $7\frac{4}{5}$d. per M.	17	1	10
4 casks double-edged bills, cont. 38 doz. and 7, at 12s. $7\frac{5}{8}$d. per dozen,	24	7	6
2 casks single-edged ditto, cont. 15 doz. and 11, at 11s. $11\frac{3}{4}$d. per doz.	9	10	8
2 casks, cont. 5 Cwt. 3 qrs. 18 lb. turtle twine, at L. 4 : 17 : $5\frac{5}{8}$ per Cwt.	28	16	$1\frac{1}{4}$
Charges till on board,	3	9	8
	L. 184	8	2

Errors excepted,

SAMUEL ADVENTURER.

No. 2.

Glasgow, 30th Aug. 1788.

Invoice of merchandise, per the Lady Margaret, for Virginia, Captain Kippen, by order and for account of Mr Robert Carmichael merchant in Petersburg; contents, cost, and charges as under, shipped by Russel, Black, and Co.

R. C. 5 boxes.

	L.	s.	d.
No. 1. qt. 14 doz. mens turned pumps, at 57s. $5\frac{5}{8}$d.	40	4	$6\frac{1}{4}$
2. 19 doz. boys ditto, at 43s. $7\frac{1}{4}$d.	41	9	$3\frac{1}{4}$
3. 16 doz. mens shoes, at 56s. $9\frac{7}{8}$d.	45	9	2
4. 15 doz. boys ditto, at 44s. $2\frac{4}{7}$d.	33	3	6
5. $19\frac{1}{2}$ doz. womens ditto, at 55s. $8\frac{1}{2}$d.	54	6	$3\frac{1}{4}$

2 bales sail cloth.

	L.	s.	d.
No. 1. qt. 21 pss. 824 yards, at $11\frac{8}{9}$d.	40	16	$4\frac{1}{2}$
2. 19 pss. 715 yards, at $12\frac{4}{7}$d.	38	2	8
1 copper still and head, wt. 6 Cwt. 2 qrs. 15 ℔. at 1s. $4\frac{5}{7}$d. per ℔.	51	14	$10\frac{1}{4}$
1 pewter worm for ditto, 4 Cwt. 1 qr. at 1s. $1\frac{2}{3}$d. per ℔.	27	2	$1\frac{1}{4}$
Charges,	3	7	6
Commission, at $2\frac{1}{2}$ per cent.	9	7	$10\frac{1}{4}$
L.	385	4	3

Errors excepted,

Russel, Black, & Co.

We shall add, as an Appendix to this chapter, some few practical methods of calculation, suited to particular cases.

I. Practical rules for finding the value of the short hundred, or five score.

CASE I.

If the rate of 1 be ſhillings, multiply it by 5 for the anſwer in pounds.

EXAMPLES.

1. What is the value of 100 gallons of rum, at 8s. per gallon?

$5 \times 8 = $ L. 40. *Anſwer.*

2. What is the value of 100 dozen pairs of ſhoes, at 47s. per dozen?
3. What is the value of 100 moidores, at 27s. each?
4. What is the value of 100 ſets of buckles, at 24s. per ſet?
5. What coſt 500 pieces of 36s.?
6. What coſt 1000 yards of ſail-cloth, at 2s. per yard?

CASE II.

If the rate be pence, multiply by 5, and divide by 12 for the anſwer; and if the rate be ſhillings and pence, reduce it to pence before multiplication.

EXAMPLES.

1. At 7d. what coſt 1 piece Oſnaburghs, containing 100 yards?

$$\frac{7 \times 5}{12} = \text{L. } 2:18:4 \textit{ Anſwer.}$$

Note, The remainder is ſo many times 1s. 8d. or 12ths of a pound.

2. At 1s. 7d. what coſt 100 yards of linen?

$$\frac{19 \times 5}{12} = \text{L. } 7:18:4 \textit{ Anſwer.}$$

3. At 17s. 6d. what coſt 100 dozen of claret?
4. At 11s. 11d. what coſt 500 yards of broad cloth?
5. At 19s. 8d. what coſt 1000 barrels of herrings?
6. At 15s. 9d. what coſt 1200 ſtones of flax?

CASE III.

If the rate be farthings, multiply by 5, and the product will be 5 pences.

EXAMPLES.

1. At $\frac{3}{4}$d. what cost 100 oranges?
$3 \times 5 = 15 = $ 6s. 3d. *Answer.*
2. At $1\frac{1}{4}$d. what cost 100 yards of incle?
$5 \times 5 = 25 = $ 10s. 5d. *Answer.*
3. At $3\frac{3}{4}$d. what cost 100 yards of ribbon?
4. At $1\frac{1}{2}$d. what cost 1000 yards of tape?
5. At $\frac{1}{4}$d. what cost 1500 yards of twine?
6. At $4\frac{1}{2}$d. what cost 2000 yards of ribbon?

II. For finding the value of the long hundred, or six score.

Reckon 1 pound for every penny in the rate, and for every farthing the part of a pound it is of a penny, and the half of the amount will be the answer.

EXAMPLES.

1. What is the value of 120 yards, at $9\frac{1}{2}$d. per yard?
$$\frac{9 : 10}{2} = \text{L. } 4:15:0 \textit{ Answer.}$$
2. What is the value of 120 yards, at 1s. $8\frac{1}{4}$d. per yard?
$$\frac{20 : 5}{2} = \text{L. } 10:2:6 \textit{ Answer.}$$
3. What is the value of 120 yards, at 3s. $3\frac{3}{4}$d. per yard?
4. What is the value of 240 yards, at 2s. $9\frac{1}{2}$d. per yard?
5. What is the value of 360 yards, at 1s. $7\frac{1}{4}$d. per yard?
6. What is the value of 1200 yards, at 1s. $4\frac{3}{4}$d. per yard?

III. At any rate per ℔. to find the value of 1 Cwt. or 112 ℔.

For every penny in the rate, reckon 9s. 4d. and for every farthing $\frac{1}{4}$ of 9s. 4d. or 2s. 4d.

EXAMPLES.

1. At 1s. 8d. per lb. what cost 112 lb.?

$$\begin{array}{rrr} & 9 & 4 \\ & & 10 \\ \hline 4 & 13 & 4 \\ & & 2 \\ \hline \text{L. } 9 & 6 & 8 \end{array}$$

2. At $7\frac{1}{2}$d. per lb. what cost 112 lb.?
3. At $8\frac{3}{4}$d. per lb. what cost 112 lb.?
4. At $9\frac{1}{4}$d. per lb. what cost 3 Cwt.?
5. At $10\frac{1}{2}$d. per lb. what cost 56 lb.?
6. At 1s. $3\frac{3}{4}$d. per lb. what cost 5 Cwt.?

IV. At any rate per dozen, to find the value of the great grofs.

Multiply the rate of 1 dozen in pence by 3, and divide the product by 5, and the quotient gives the anfwer in pounds.

EXAMPLES.

1. At 9d. per dozen, what is the value of the great grofs?

$$\frac{9 \times 3}{5} = \text{L. } 5:8:0 \textit{ Anfwer.}$$

2. At 1s. 3d. per doz. what is the value of the great grofs?
3. At 2s. 6d. per doz. what is the value of the great grofs?
4. At 3s. 4d. per doz. what is the value of the great grofs?

CHAP. VIII.

TARE AND TRETT.

Tare is an allowance on weighable goods made at the custom-house to the importer, and by the seller to the buyer, for the outside package, whether cask, chest, wrapper, &c. whence arise the following distinctions.

1. *Gross weight* is the weight of the commodity and package together.

2. *Tare* is an allowance or deduction from the gross weight for the package; and if it is weighed before the goods are packed, the tare is inserted in the invoice. By a particular stipulation, it may be a certain deduction on the whole; it may also be at a certain allowance per cask, per chest, &c. or at a certain allowance per Cwt.

3. *Trett* is an allowance of 4 lb. on the 104 lb. certain, granted for break, waste, or dust mixed with such goods as are sold by the pound weight.

4. *Clough* is an allowance made to the citizens of London on some weighable goods, generally of 2 lb. per 336 lb. to turn the scale, or make good the weight, in case of shrinkage, when the goods are re-weighed.

5. *Suttle* is what remains after deduction of the tare, when trett is also allowed; in which case alone it is called *suttle*.

The computations relative to these distinctions admit of several varieties; to each of which we shall assign a distinct case, and to each variety two different methods of computing, the one to be used as a proof of the other.

CASE I.

When a certain allowance is ingrossed in the invoice, deduct the tare from the gross; and this may be done in lbs. or Cwts.

EXAMPLES.

1. What is the net of five hogsheads of sugar, contents, gross, and tare, as under?

	Cwt.	Q.	lb.		Tare.	Cwt.	Q.	lb.		Tare.
No. 1.	15	3	17	—	111 lb.	15	3	17	—	111 lb.
2.	14	2	15	—	114	14	2	15	—	114
3.	16	1	19	—	110	16	1	19	—	110
4.	13	2	22	—	112	13	2	22	—	112
5.	15	1	11	—	119	15	1	11	—	119
	76	0	0		566	76	-	-	112)	566 (5
	76					5	0	6		560
	76									
	76					70	3	22		6
						70				
	8512 grofs.					70				
	566 tare.					7106				
	7946 lbs. net.					7946 net.				

2. Received from on board the Bell 55 hogfheads tobacco, weighing grofs, per invoice, 575 Cwt. 3 qrs. 14 lb.; the allowance upon the whole for tare is 6225 lbs.: required the net weight?

Anfwer, 58273 lb. = 520 Cwt. 1 qr. 5 lb.

3. Received 54 chefts, weighing each 4 Cwt. 2 qrs. 14 lb. grofs; allowance on each for tare $45\frac{4}{9}$ lbs.: required the net?

Anfwer, 227 Cwt. 3 qrs. 10 lb. = 25518 lbs.

4. Received 7 frails of raifins, weighing each 3 Cwt. 3 qrs. 10 lb. grofs; tare upon the whole 168 lb.: required the net?

Anfwer, 25 Cwt. 1 qr. 14 lb.

5. Received 14 bags of cotton, weighing each 2 Cwt. 2 qrs. 7 lb.; tare at 9 lb. per bag: required the net?

Anfwer, 3892 lb.

6. Sold as under,

		Cwt.	Q.	lb.	Tare.
Hhds. of fugar, No.	1.	14	3	15	109 lb.
	2.	15	2	17	113
	3.	13	1	24	111
	4.	12	0	27	108
	5.	16	1	11	117
	6.	15	2	18	107

at $4\frac{5}{8}$ per lb. net weight; what is the grofs, the tare, the net, and the price?

Anfwer, L. 177 : 13 : $1\frac{3}{4}$.

CASE II.

When the tare is at a certain rate per Cwt. take aliquot parts for the tare from these two standard rates, 16 lb. = $\frac{1}{7}$, and 14 lb. = $\frac{1}{8}$, and deduce from the gross; or deduce the tare from 112 for the net of 1 Cwt. and then the gross will be to the net as 112 to its net, without any subtraction.

EXAMPLES.

1. What is the net of 372 Cwt. 3 qrs. 14 lb. at 16 lb. tare per Cwt.?

	Cwt.	Q.	lb.	
7)	372	3	14	gross.
	53	1	2	tare.
	319	2	12	net.

8)						
112 : 112	::		372	3	14	
16						
96					6	
14 : 12		7)	2237	1	0	
7 : 6			319	2	12	net.

2. What is the net of 493 Cwt. 2 qrs. 24 lb. gross, tare 14 per Cwt.? *Answer*, 432 Cwt.

3. What is the net of 573 Cwt. 2 qrs. 13 lb. gross, tare 17 per Cwt.?

8)	573	2	13	gross.
7)	71	2	$22\frac{1}{2}$	for 14
2)	10	0	27	for 2
	5	0	$13\frac{1}{2}$	for 1
	87	0	7	tare.
	486	2	6	net.

	112
	17
112 :	95

	573.616
	95
	2868.080
	51625.44
8)	54493.52
2)	6811.69
7)	3405.845
	486.549 net.

4. What is the net of 44 Cwt. 2 qrs. 12 lb. gross, tare 21 per Cwt.? *Answer*, 36 Cwt. $27\frac{1}{4}$ lb.

5. What is the net of 170 Cwt. 2 qrs. 14 lb. gross, tare 6 per Cwt.? *Answer*, 161 Cwt. 1 qr. $26\frac{1}{4}$ lb.

6. What is the net of 78 Cwt. 1 qr. 15 lb. groſs, tare 11 per Cwt.? *Anſwer*, 70 Cwt. 2 qrs. 21 lb.

7. What is the net of 227 Cwt. 3 qrs. 24 lb. groſs, tare 13 per Cwt.? *Anſwer*, 201 Cwt. 2 qrs.

8. What is the value of 545 Cwt. 3 qrs. 19 lb. of ſugar, at 51s. $7\frac{1}{2}$d. per Cwt. tare 9 per Cwt.? *Anſwer*, L. 1295 : 18 : 5.

9. What is the value of 374 Cwt. 3 qrs. 25 lb. of tobacco, at $7\frac{1}{8}$d. per lb. tare 22 per Cwt.? *Anſwer*, L. 1072 : 3 : $9\frac{1}{4}$.

10. What is the value of 515 Cwt. 2 qrs. 17 lb. of cotton, at L. 4 : 19 : $4\frac{3}{4}$d. per Cwt. tare 12 per Cwt.? *Anſw.* L. 2288 : 2 : 2.

CASE III.

When trett is allowed, divide the remainder, after tare is deduced, in this caſe ſuttle, by 26 for the trett; deduce the quotient from the ſuttle for the net: Or, multiply the ſuttle by $.9615 = \frac{100}{104}$, and the product will be the net, without any ſubtraction.

EXAMPLES.

1. What is the net of 836 Cwt. 2 qrs. 17 lb. groſs, tare 22 per Cwt. and trett 4 per 104?

7)	836	2	17	groſs.
4)	119	2	$2\frac{1}{4}$	
2)	29	3	$14\frac{1}{2}$	
	14	3	$21\frac{1}{4}$	
	164	1	10	tare.
26)	672	1	7	ſuttle.
	25	3	12	trett.
	646	1	23	net.

$$\begin{array}{r} 112 \\ 22 \\ \hline 112 : 90 \\ \hline 56 : 45 :: 836.65176 \\ 45 \\ \hline 418325880 \\ 334660704 \\ \hline 7\ )\ 37649.3292 \\ \hline 8\ )\ 5378.4756 \\ \hline 672.3094 \text{ ſuttle.} \\ .9615 \\ \hline 646.4255 \text{ \&c. net.} \end{array}$$

2. What is the net of 374 Cwt. 1 qr. 15 lb. tare 18 per Cwt. trett 4 per 104? *Answer*, 302 Cwt. 0 qrs. $14\frac{1}{2}$ lb.

3. What is the net of 118 Cwt. 2 qrs. 14 lb. gross, tare 12 per Cwt. trett 4 per 104? *Answer*, 101 Cwt. 3 qrs. 10 lb.

4. What is the net weight of 346 Cwt. 3 qrs. 12 lb. gross, tare 12 per Cwt. trett 4 per 104? *Answer*, 297 Cwt. 3 qrs. $3\frac{1}{2}$ lb.

CASE IV.

When clough is to be deduced, the remainder, after deduction of trett, is called *subsuttle*. For finding the clough, divide the subsuttle by $168=\frac{2}{336}$, or multiply by .00595.

EXAMPLES.

1. What is the net of 139 Cwt. 1 qr. 22 lb. tare 8 per Cwt. trett 4 per 104, clough 2 per 336? *Answer*, 123 Cwt. 3 qrs. $1\frac{1}{4}$ lb.

2. What is the net of six hogsheads tobacco, weighing gross 55 Cwt. 26 lb. tare 30 lb. per hhd. trett 4 per 104, and clough 2 per 336; and what will the net amount to at $9\frac{1}{2}$d. per lb.? *Answer*, 51 Cwt. 1 qr. $\frac{1}{4}$ lb. the net amount L. 227 : 4 : $11\frac{1}{2}$.

QUERIES.

1. How is the tare ascertained when the cask is full? *Answer*, Before the cask is filled it is weighed, and it is also weighed when full; this last weight is called the gross, the first is the tare, and the difference the net, or weight of the tobacco, sugar, &c.

2. Why is trett deduced, after the tare is allowed? *Answer*, Because it is only allowed on the goods, not on the package.

CHAP. IX.

COMMISSION IN FACTORAGE AND BROKERAGE.

WHEN one person acts for another, in buying or selling goods for his account, recovering or paying money, &c. a certain premium is allowed to him in name of commission, which is generally at some rate per cent. To compute which, take the following methods to prove each other.

1. The sum given will be to the commission due for it as 100 is to the assigned rate. Or, practically,

2. If the rate be any even part of 100, divide for it; and if it be not, divide for the nearest, and take parts of that for the remainder.

EXAMPLES.

1. My factor in Jamaica sends me an account of sales per L. 547, 17s. 6d.; what is his commission thereon at 8 per cent.?

100 : 8 :: 547.875	50) 537.875
8	
	10.9575 for 2 per cent.
43.83 . . . = L. 43 16 7	4
	43.83 . . .

2. Sold a consignment for my constituent, amounting to L. 896, 13s. 4d.; what commission am I entitled to at $2\frac{1}{2}$ per cent. and what are the net proceeds of his consignment, supposing the other charges L. 5 : 12 : 8?

100 : $2\frac{1}{2}$ :: L. 896 13 4	
2	
	4,0) 896 13 4
1793 6 8	
448 6 8	22 8 4 commission.
	5 12 8 charges.
22.41 13 4	
20	28 1 0
8.33	868 12 4 net proceeds
12	
4	

3. Negotiated bills for A. B. to the extent of L. 3745 : 18 : 6; what is my commission at $\frac{1}{2}$ per cent.?

Answer, L. 18 14 7.

4. Discounted bills last month to the amount of L. 23456 : 6 : 6; what is my commission on the whole, at $\frac{1}{4}$ per cent.?

Answer, L. 58 12 $9\frac{1}{2}$.

5. Bought for my employer goods to the amount of L. 1274, 19s. 10d, charges attending the purchase came to L. 15 : 11 : 11; for how much may I draw, including commission at 3 per cent.?

Answer, L. 1329 6 1.

6. A broker procures insurance and does other business to the extent of L. 57459 : 10; what is his premium at 3s. 4d. per L. 100?

Answer, L. 95 15 $3\frac{1}{2}$.

7. Bought the following goods for A. B., viz.

	L.	s.	d.
5 bales Osnaburghs, 60 pieces, 6020 yards, at $6\frac{2}{3}$d. per yard,	167	4	$5\frac{1}{3}$
19 pieces of broad cloth, 475 yards, at 13s. $8\frac{7}{8}$d. per yard,	326	6	$3\frac{1}{2}$
2 chests vermillion, 4834 ℔. at 15s. $7\frac{1}{2}$d. per ℔.	3781	11	11
5478 ounces of silver, at 4s. $10\frac{5}{8}$d. per oz.	1342	17	5
80 pieces of fine lawn, 800 yards, at 3s. $7\frac{7}{8}$d. per yard,	146	5	0
200 dozen 7 pairs stockings, at 33s. $4\frac{4}{7}$d.	334	19	$5\frac{1}{7}$
13 Cwt. 3 qrs. 19 ℔. refined sugar, at $8\frac{5}{8}$ per ℔.	57	7	$6\frac{1}{8}$
Package and other charges,	15	7	8
Required a complete invoice to be sent per the Betsey, adding commission at $3\frac{1}{2}$ per cent.?	216	6	$4\frac{3}{4}$
	L. 6388	6	$1\frac{1}{4}$

CHAP. X.

INSURANCE.

INSURANCE is a ſecurity or writ called a *Policy*, given by the inſurers, or underwriters, to indemnify the inſured from ſuch loſſes as are expreſſed in the policy, upon account of the premiums, and with the reſtrictions particulariſed in the writ.

In caſe of loſs, the inſured generally recover two per cent. ſhort of the ſum underwritten, *i. e.* L. 98 for L. 100, eſpecially in time of war, when the riſk is ſuppoſed greater; but, as an equivalent, the inſured may cover this 2 per cent, as well as the premium and value on board, and this is called covering his outſet.

In caſe of a partial loſs incurred, (for the preſervation of the whole) all concerned in ſhip, freight, and cargo, are to bear a proportional part of what was ſo ſacrificed for the common good, and muſt be made good by the inſurers in ſuch proportions as they have underwritten; and this is called an *average*.

From theſe different circumſtances ariſe the following caſes

CASE I.

When the rate per cent., premium, and ſum to be inſured, are given to find the premium, the computation is the ſame as in commiſſion, factorage, &c.

EXAMPLES.

1. At $7\frac{1}{2}$ per cent. what premium will pay for inſuring L. 785 : 15?

$100 : 7\frac{1}{2} :: 785.75$			
7		20) 785 15 0	
5500.25		2) 39 5 9	for 5 per cent.
392.875		19 12 $10\frac{1}{2}$	for $2\frac{1}{2}$.
58.93125	=	58 18 $7\frac{1}{2}$	*Anſwer.*

2. At 9 per cent. premium, what muſt be paid for inſuring L. 1745 : 19? *Anſwer*, L. 157 : 2 : $8\frac{1}{2}$.

3. At 15 guineas per cent. premium, what will insure L. 347, 12s. 6d.? *Answer*, L. 54 : 15.

4. At $13\frac{1}{2}$ per cent. premium, what must be paid for insuring L. 2860? *Answer*, L. 386 : 2.

CASE II.

The sum insured and discount per cent. given, to find the first recovery, observe the following

EXAMPLES.

1. There is a loss of L. 1978 insured, what is the discount and sum recovered, at 2 per cent. discount?

```
100 : 98 :: 1978
              98
         -------
           15824
          17802
         -------
         1938.44 short recovery.
         -------
           39.56
```

```
5.0 ) 197.8
     -------
      39.56 discount.
     -------
    1938.44 short recovery.
```

2. At $2\frac{1}{2}$ per cent. discount, what sum must be recovered for L. 1597 : 13 : 4. *Answer*, L. 1557 : 14 : 6.

3. At $2\frac{1}{4}$ per cent. discount, what sum must be recovered for L. 6745 : 19 : 4? *Answer*, L 6594 : 3 : $7\frac{5}{8}$.

4. At $1\frac{5}{8}$ per cent. discount, what sum must be recovered for L. 4937 : 11 : 11? *Answer*, L. 4857 : 7 : $2\frac{1}{2}$.

CASE III.

The rates per cent. premium and discount given, and the real value at risk, to find what sum must be insured to cover the whole outset in a single voyage.

Add the rates of the premium and discount together, subtract the sum from L 100; then the real value at risk, or the outset, will be to the sum to be insured as the remainder formerly found to 100. For a proof, take the premium and discount on the covered sum, and the remainder will be just equal to the first outset, or real value at risk.

EXAMPLES.

1. What ſum muſt I inſure to cover L. 570, premium 8 per cent. diſcount 2 per cent. in caſe of loſs?

$$\begin{array}{r} 100 \\ 8 + 2 = 10 \\ \hline 90 : 100 :: 570 \\ 100 \\ \hline 9\,)\,57000 \\ \hline 633.\dot{3}\ \textit{Anſwer.} \end{array} \qquad \begin{array}{rl} 10\,)\,633.\dot{3} & \text{covered property.} \\ 63.\dot{3} & \text{prem. and diſc.} \\ \hline 570. & \text{firſt outſet. Proof.} \end{array}$$

2. What ſum muſt I inſure to cover L. 547 : 19, premium 13 guineas per cent. 2 per cent. diſcount in caſe of loſs?

Anſwer, L. 649 : 12 : $3\frac{1}{2}$.

3. What ſum muſt I inſure to cover L. 1974 : 17 : 6, premium 20 guineas per cent. 2 per cent. diſcount in caſe of loſs?

Anſwer, L. 2564 : 15 : $5\frac{1}{2}$.

4. I want inſured in Amſterdam L. 574 : 18, premium 7 guineas per cent.; in Edinburgh L. 345 : 10, premium 8 guineas per cent.; in London L. 459 : 11 : 9, premium $7\frac{1}{2}$ guineas per cent.; the diſcount at Amſterdam will be $2\frac{1}{2}$ per cent. and in each of the other places 2 per cent.: required the covered outſet at each place?

Anſwer, At Amſterdam,	L.	637	14	$3\frac{1}{2}$.
At Edinburgh,		385	12	$0\frac{1}{2}$.
At London,		509	18	$10\frac{1}{2}$.

CASE IV.

Given the ſame things as before, to find the premium out and home, and what ſum muſt be inſured to cover the inſet.

Find the premium on the voyage out as before; deduct the premium from the ſhort recovery; then will the proportion be, as the remainder to the premium, ſo the ſum of the outſet and premium of the voyage out, to the premium of the voyage home, which, added to the premium of the voyage out, gives the the total premium. Laſtly, find what ſum inſured will cover the inſet by Caſe III.

EXAMPLES.

1. What ſum muſt I inſure to cover a voyage out and in per L. 5780, premium 8 per cent. diſcount 2 per cent. in caſe of loſs?

(1.) 90 : 100 :: 5780
100

9) 57800

50) 6422.22

Off 2 per cent. diſcount, 128.44

Premium and outſet, 6293.77

513.77 premium out.

(2) 98 — 8 = 90 : 8 :: 6293.66
8

9) 5034.933

559.437 prem. home,
513.6666 prem. out,

1073.1036 total prem,

(3.) 90 : 100 :: 6293.66
100

9) 62936.66

6992.96 *Anſwer.*

2. What ſum muſt I inſure to cover a voyage out and in per L. 978 : 18 : 0, premium 9 guineas per cent. diſcount 2 per cent. in caſe of loſs? *Anſwer*, L. 1223 : 9 : 0.

3. What ſum muſt I inſure to cover L. 3749 : 15 : 0 for a voyage out and in, premium 17 guineas per cent. 2 per cent. diſcount in caſe of loſs? *Anſwer*, L. 5720 : 6 : 7.

4. What ſum muſt I inſure to cover L. 1111 : 11 : 11, for a voyage out and in, premium 10½ guineas per cent. diſcount 2 per cent. in caſe of loſs? *Anſwer*, L. 1440 : 1 : 5.

CHAP. XI.

COMPUTATIONS IN THE PURCHASE OF STOCKS.

THE supplies to the British government, for defraying the expence of war, or answering other exigencies of state, are raised by borrowing money from the bank of England, East India company, &c. for payment of the interest of which the public taxes are appropriated. Although the principal is not paid up by the government, yet the creditor may have his money replaced to him at any time, by selling out his property or stock in Exchange Alley, sometimes at loss, and sometimes with profit, according to the rise or fall of stocks. The public funds or stocks come under various designations, as *Bank stock*, *India stock*, *India annuities*, *India bonds*, *South Sea annuities*, *Three per cent. reduced annuities*, *Three per cent. consolidated ditto*, *Three and a half per cent. ditto*, *Four per cent. ditto*, *Long ditto*, *Exchequer bills*, *Navy bills*, &c.

Stocks are always bought or sold at a certain rate per cent. and computed as in the following cases.

CASE I.

When the stock is any number of even hundreds, multiply the price by the number.

EXAMPLES.

1. What is the price of L. 700 bank stock, at L. 225½ per cent.?

225	10
	7
L. 1578	10 *Answer*.

2. What is the price of 800 three and a half per cent. annuities, L. 82 : 13 : 4 per cent.? *Answer*, L. 661 : 6 : 8.

3. What is the price of L. 900 bank annuities, at L. 91 : 19 : 9½ per cent.? *Answer*, L. 827 : 18 : 1½.

CASE II.

When the stock is no even number of hundreds, the price may be calculated as commission.

EXAMPLES.

1. What must be paid for L. 578½ India stock, at L. 155½ per cent.?

100 :	155.5 ::	578.5		
20	31.1	115.7	2) 578.5	for 100
		31.1	10) 289.25	for 50
4			10) 28.925	for 5
		12727	2.8925	for ½
		3471	L. 899.5675	
	4)	3598.27		

L. 899.5675 *Answer*, = L. 899 : 11 : 4⅕.

2. What must be paid for L. 497¼ consolidated annuities, at L. 84 : 17 : 6 per cent.? *Answer*, L. 422 : 9 : 3¾.

3. What must be paid for L. 999⅝ annuities 4, at L. 88 : 16 : 8 per cent.? *Answer*, L. 888.

4. What must be paid for L. 1976 annuities 3, at L. 73 : 11 : 6 per cent.? *Answer*, L. 1453 : 16 : 10.

5. What must be paid for L. 37¾ bank stock, at L. 215 : 7 : 7 per cent.? *Answer*, L. 81 : 6 : 1.

6. What must be paid for L. 1979⅝ consolidated annuities, at L. 69 : 14 : 5 per cent.? *Answer*, L. 1380 : 7 : 1½.

CHAP. XII.

PARTNERSHIP.

WHEN two or more merchants join their ſtocks to carry on ſome concerted branch of buſineſs, they are ſaid to be in *company* or *partnerſhip*; and, as the conditions of their contract may admit of ſome varieties, different caſes will ariſe in the diſtribution of the gain, loſs, or proceeds. We ſhall begin with the moſt ſimple.

CASE I.

When the partners are all equally concerned, the number of partners will always be a diviſor to the ſubject to be divided.

EXAMPLES.

1. Two merchants, A. and B. concerned in trade, had a dividend to make of L. 1575 : 15 : 6; what falls to each?

2) 1575 15 6

787 17 9 *Anſwer.*

2. Three partners were concerned in trade, in which they were equally in advance; but one had L. 100 per annum for his management: at balancing their books, they found there was a dividend of L. 1979 : 15 : 4; how much will be drawn by each?

Anſwer, L. 626 : 11 : 9.

CASE II.

If the company's capital be divided into a certain number of ſhares, the dividend will be made on the whole number of ſhares, as before, and the quotient multiplied by the number of ſhares any partner holds for his dividend.

EXAMPLES.

1. The capital of a trading company is L. 96745 : 16 : 8, which consists of 64 shares; what is A. B.'s stock in company, who holds 5 shares?

8)	96745	16	8	
8)	12093	4	7	
	1511	13	$0\frac{7}{8}$	= 1 share;
			5	
L.	7558	5	$4\frac{3}{8}$	*Answer.*

2. A trading company, consisting of 75 shares, have of capital L. 194570 : 15 : 0; what is A. B.'s stock in company, who holds 13 shares? *Answer*, L. 33725 : 11 : 11.

3. A trading company have of capital stock L. 236574 : 17 : 6, consisting of 156 shares; how much is C.'s stock in company, who holds 19 shares? *Answer*, L. 28813 : 12 : $1\frac{1}{2}$.

CASE III.

When each partner is concerned to a certain extent, and the conditions of the contract such, that all dividends shall be made upon the capital each partner has employed in the company trade, then it will be,

As the total stock to the total dividend,
so each partner's stock to his particular dividend; or,
As the total stock to the total dividend,
so is L. 100 to its dividend; or,
As the total stock to the total dividend,
so is L. 1 to its dividend.

Note, If either of the last proportions be used, the rest must be done by practice.

EXAMPLES.

1. A., B., and C. purchase a ship for L. 2500, of which A. paid down L. 850, B. L. 780, and C. the remainder; by the first voyage they netted L. 750, how much ought to be drawn by each?

2500	:	750	::	850	:	255	A.
				780	:	234	B.
				870	:	261	C.
						750	Proof.

Or thus: 2500 : 750 :: 100 : 30 per cent.

$30 \times 8.5 = 255$ A.
$30 \times 7.8 = 234$ B.
$30 \times 8.7 = 261$ C.

750 as before.

Or thus: 2500 : 750 :: 1 : ·3

$850 \times \cdot 3 = 255$ A.
$780 \times \cdot 3 = 234$ B.
$870 \times \cdot 3 = 261$ C.

750 as before.

2. A., B., and C. join in an adventure to Virginia, and ſhip off, on their joint credit, goods to the amount of L. 3000; to this cargo A. adds from his own warehouſe the value of L. 100, B. to the value of L. 200, and C. to the value of L. 300: they had returns in tobacco which cleared L. 6930, what ſhare of this belongs to each partner?

Anſwer, A. draws L.	2117	10	0
B.	2310	0	0
C.	2502	10	0

3. A., B., C., and D. freight a ſhip for Grenada with goods to the amount of L. 5768 : 19 : 6. charges of ſhipping and inſurance coſt them L. 174 : 0 : 6 further; they had returns in due time, by which they cleared L. 6847 : 16 : 0; how much will A. draw in proportion to L. 1267, how much will B. draw in proportion to L. 1459, how much will C. draw in proportion to L. 1578, and how much will D. draw in proportion to the remainder?

Anſwer, A. draws L.	1459	17	11
B.	1681	2	$6\frac{1}{2}$
C.	1818	4	$10\frac{1}{2}$
D.	1888	10	$7\frac{1}{2}$
L.	6847	15	$10\frac{1}{2}$

4. A., B., C., and D. buy a ſhip, of which A. had $\frac{2}{9}$, B. $\frac{2}{7}$, C. $\frac{5}{11}$, and D. paid L. 277 : 19 : 0; what was D.'s ſhare of the ſhip, what

did the ſhip coſt, and what did A., B., and C. each contribute towards the purchaſe?

Anſwer, A. paid	L.	819	4	$5\frac{1}{4}$	
B.		1053	5	$8\frac{1}{4}$	
C.		1536	2	$10\frac{3}{4}$	
D.		277	19	0	for $\frac{57}{156}$ parts.
	L.	3686	12	$0\frac{1}{4}$	price of the ſhip.

5. There was an average loſs of L. 1276 : 15 : 7; A. had on board the value of L. 2117 : 10 : 0, B. the value of L. 2347 : 11 : 0, C. the value of L. 2984 : 15 : 0, and D. the value of L. 3741 : 16 : 8, inſurance was made on the ſhip and freight for L. 647 : 19 : 0; what ſhare of the loſs muſt each of the freighters, and what ſhare of the loſs muſt the owners ſuſtain?

Anſwer, A. paid	L.	228	7	$2\frac{1}{4}$
B.		253	3	$4\frac{1}{2}$
C.		321	17	$8\frac{3}{4}$
D.		403	10	8
Owners of the ſhip		69	16	$6\frac{3}{4}$
	L.	1276	15	$6\frac{1}{4}$
Loſt with the remainders				$\frac{3}{4}$

6. A bankrupt owes to A. L. 347 : 10 : 0, to B. L. 549 : 11 : 0, to C. L. 697 : 14 : 0, to D. L. 419 : 18 : 6, and to E. L. 725 : 11 : 6; a dividend of his effects is to be made for L. 1897 : 14 : 8: what will each of the creditors draw?

Anſwer, A. draws	L.	240	13	$2\frac{1}{4}$
B.		380	11	$8\frac{1}{2}$
C.		483	3	$7\frac{3}{4}$
D.		290	16	$3\frac{3}{4}$
E.		502	9	$9\frac{1}{4}$
				$\frac{1}{2}$
	L.	1897	14	8

In caſes of this kind, when the creditors are numerous, find the proportional dividend for L. 1, and multiply it into all the 9 digits ſeverally, and then into 10; multiply this laſt product into all the 9 digits, and then into 10, which is eaſily effected by removing the decimal point of each of the 10 preceding products one ſtep further to the right, as in the following example, being the laſt queſtion reſumed.

Debt.		Subject.		Debt.		
$2740\frac{1}{4}$	:	1897.733	::	L. 1	:	.69255

Therefore	L. 1= .69255	10= 6.9255
	2=1.38510	20=13.8510
	3=2.07765	30=20.7765
	4=2.77020	40=27.7020
	5=3.46275	50=34.6275
	6=4.15530	60=41.5530
	7=4.84785	70=48.4785
	8=5.54040	80=55.4040
	9=6.23295	90=62.3295
	10=6.92550	100=69.2550

Now let A.'s dividend be required:

69.255 × 3 = 207.765	for L. 300	0	0
27.702 × 1 = 27.702	for 40	0	0
4.84785	for 7	0	0
.34627	for 0	10	0
240.66112	as before, &c.		

CASE IV.

If the dividend is also to be proportioned to the time the capital is to be employed, observe this proportion:

As the sum of the products of the capitals, into the times they are respectively employed, to the whole dividend, so is each particular product to its particular dividend; or, find the dividend to L. 100 or L. 1, and do the rest by practice.

EXAMPLES.

1. A. put into company L. 560 for 8 months, B. L. 279 for 10 months, and C. L. 735 for 6 months; the gain upon the whole was L. 1000; what is each partner entitled to draw?

(1.)

A. 560 × 8 = 4480
B. 279 × 10 = 2790
C. 735 × 6 = 4410

								L.	s.	d.	rem.
For A.	11680	:	1000	::	4480	:		383	11	$2\frac{3}{4}$	208
For B.	11680	:	1000	::	2790	:		238	17	$4\frac{3}{4}$	80
For C.	11680	:	1000	::	4410	:		377	11	$4\frac{1}{4}$	880
								1000	0	0	

(2.)

11680 | 1000					L.	s.	d.
292 | 250.0 |	.085616	×	4480	=	383	11	$2\frac{1}{4}$
			2790	=	238	17	$4\frac{1}{4}$
			4410	=	377	11	$4\frac{1}{4}$

2. A., B., and C., butchers, hire a paſture for L. 24 : A. had 40 cows on it for 4 months, B. 30 cows for 2 months, and C. 36 for 5 months ; what proportion of the rent falls to each ?

Anſwer,	A. pays L.	9	12	0
	B.	3	12	0
	C.	10	16	0

3. A. and B trade in company : A. put into company the firſt of January L. 1259, but B. could advance nothing till the firſt of May : *Queritur*, what muſt B. put in then to entitle him to an equal ſhare of the profits at the year's end ?

Anſwer, L. 1888 : 10 : 0.

4. A., B., and C., butchers, farm a large graſs incloſure for L. 119 : 10 : 0 per annum : A. kept in this incloſure 90 cows for 2 months, at the end of which he took out 70 for ſlaughter, and then put 40 in their room ; at the expiration of another month he withdrew 20, but ſuffered the remainder to continue 2 months longer : B. had 110 cows in the incloſure for $1\frac{1}{2}$ month, when he withdrew 50 ; and at the end of $1\frac{1}{2}$ month more he withdrew the remainder, but put 55 in their ſtead, which continued 3 months ; C. had 70 cows in for 3 months, when he added 15 to their number, and continued the whole for 2 months longer : required the proportion each butcher pays of the rent, according to the uſe he had of the incloſure ?

Anſwer,	A. pays L.	34	2	10
	B.	44	16	3
	C.	40	10	$10\frac{1}{2}$
				$\frac{1}{2}$
	L.	119	10	9

CHAP. XIII.

BARTER.

BARTER is that rule by which merchants proportion quantities of goods to be exchanged, the one to the other, according to the conditions of the bargain, that neither party may be a loſer.

Find the value of the commodity of which the quantity is aſſigned, and then, by another operation, find how much of the other commodity that value will purchaſe. Or, by one operation,

As the price by the yard, lb., Cwt., &c. of the known commodity to the aſſigned quantity, ſo is the price by the yard, &c. of the other commodity to the quantity required, in a reciprocal proportion.

EXAMPLES.

1. How many pounds of cotton, at 9d. per lb. may I have in barter for 13 Cwt. 3 qrs. 14 lb. of ſugar, at 56s. per Cwt.?

(1.)	(2.)			
13.875	.0375	: 1	::	38.85
2.8				
	.0075			7.7700
111000				
27750	.0015			1.5540
38.8500 ſugar value.	3			3108

Anſwer, 9 Cwt. 1 qr. = 1036 lbs.

By one operation:

$$
\begin{array}{rrrr}
672 & : & 13.875 & :: \; 9 \\
 & & 672 & \\
\hline
 & & 27750 & \\
 & & 97125 & \\
 & & 83250 & \\
\hline
 & 9\,) & 9324.000 & \\
\hline
\end{array}
$$

1036 lbs. *Anſwer*, as before.

2. How many yards of cloth, at 10s. per yard, may I have in barter for 189 gallons of rum, at 6s. 8d. per gallon?

Answer, 126.

3. A. takes 70 Cwt. of cotton of B., at 35s. 6d. per Cwt. and gives him 19 pieces of broad cloth, at L. 3 : 19 : 7½d. per piece; what is the balance, and to whom due?

Answer, To B., L. 48 : 12 : 1½.

4. A. hath tobacco at 7d. per lb. ready money, but in barter he rates it at 8d.: B. had nutmegs worth L. 5 : 12 : 0 per Cwt.; how much must he advance them to be on a footing with A., and what quantity of nutmegs must be given for 17 Cwt. 3 qrs. 25 lb. of tobacco?

Answer, 1174¼ lbs.

5. A. has 41 Cwt. hops, at 31s. 6d. per Cwt. for which B. gives him L. 21 : 10 : 0 in money, and a certain quantity of almonds at 5d. per lb.; what quantity of almonds did A. receive?

Answer, 2067⅗ lbs.

6. How many pieces silk, at L. 3 : 15 : 6, may I have in barter for 57¼ Cwt. of wool, at L. 6 : 7 : 10 per Cwt.?

Answer, 97.977 pieces.

7. A. bartered with B. a puncheon of rum, measuring 109½ gallons, at 6s. 8d. per gallon, for 41 yards 2 qrs. 2 nails of broad cloth; what was it valued at per yard?

Answer, 17s. 6¼d.

8. A. delivers to B., for a certain quantity of linen, 3¾ Cwt. of sugar, at 5½d. per lb. ready money, but in barter 7d. and is willing to lose 20 per cent. for ½ ready money; how much linen, at 2s. 3d. per yard, must be delivered by B., and what is the ready money price per yard, to equal the barter?

Answer, 43⅗ yards, at 1s. 9d. per yard, ready money.

CHAP. XIV.

PROFIT AND LOSS.

BY this rule merchants are enabled to make a proper eſtimate of the different articles of their trade, and to judge with preciſion where their profits lie, and whence come their loſſes. This rule admits of four varieties.

VARIETY I.

The buying and ſelling price being given, to diſcover the gain or loſs per cent. ſubtract the leſſer from the greater; then will the gain per cent. be to 100 as the leſſer price is to the difference.

EXAMPLES.

1. Bought cloth at 8s. 6d. and ſold it again for 10s. 6d.; what did I gain per cent.?

Sixpences.		Sixpences.		
17	:	4	::	100
				4
			17)	400
				$23\frac{9}{17}$

Had the profit on any number of yards been required, ſuppoſe 786, work by practice, thus:

10) 78.6 = L. 78 : 12 : 0.

2. Bought cloth at 1s. and ſold it again for 1s. 1d.; what had I per cent.? *Anſwer*, $8\frac{1}{3}$.

3. Bought 7964 lb. of tobacco, at $7\frac{1}{2}$d. and ſold it again for $9\frac{3}{4}$d.; what was the gain per cent. and how much was there on the whole? *Anſwer*, 30 per cent. and L. 74 : 13 : 3 on the whole.

4. If 2d. on the ſhilling be a dealer's profit, what does he make per cent. and how much in the year, ſuppoſing he deals to the extent of L. 25,000 per annum? *Anſwer*, L. 4166 : 13 : 4.

5. Bought a horſe for 15 guineas, and ſold him again for L. 20; how much had I per cent.? *Anſwer*, 27 per cent. nearly.

6. Bought 375 cows, at L. 2 : 15 : 0 a piece; their grazing, while I kept them on hand, came to L. 74 : 15 : 0, incidental charges L. 12 : 14 : 6: I ſold them again at L. 3 : 10 : 6 a piece; what did I gain or loſe upon the whole, and how much per cent.?

Anſwer, Gained L. 203 : 3 : 0, which is 18.1 per cent.

VARIETY II.

If a certain rate per cent. is propoſed to be gained, ſay, As 100 to 100 + the rate, ſo is the buying price to the advanced.

EXAMPLES.

1. Bought cloth at 8s. 6d. upon which I would have $23\frac{9}{17}$ per cent. profit; how muſt I charge the yard of it to compaſs my deſign?

Sixpences.		Sixpences.		
100	:	$123\frac{9}{17}$	::	17
		17		

21.00 = 10s. 6d. per yard. *Anſwer*.

2. Bought at 1s. and would ſell again at $12\frac{1}{2}$ per cent. profit; what muſt I charge? *Anſwer*, $13\frac{1}{2}$d.

3. Bought goods to the value of L. 374 : 15 : 0; but coming to a bad market, was obliged to ſell them 25 per cent. under prime coſt: what did I receive for them? *Anſwer*, L. 281 : 1 : 3.

Here it will be 100 : 100 — 25.

4. Bought goods to the value of L. 217 : 16 : 8, and ſold them again at $16\frac{1}{2}$ per cent. advance; what did I receive, and how much may I gain on the whole? *Anſwer*, L. 253 : 15 : 6.

5. Sent to Virginia a cargo of goods, coſt with charges L. 2197, 10s. 6d. which were ſold at 95 per cent. advance on the invoice; what does the bill of ſales amount to? *Anſwer*, L. 4285 : 3 : $5\frac{1}{4}$.

6. A grocer bought 7 Cwt. 1 qr. of pepper, for L. 21 : 17 : 0, which happening to be damnified, he is willing to loſe at the rate of $12\frac{1}{2}$ per cent.; how muſt he charge the lb. at that rate?

Anſwer, $5\frac{13}{16}$d.

VARIETY III.

To find the rate proportional on one advanced price, by having another and the rate on it given, obferve this proportion:

As the loweft price to the higheft, fo 100 + the given rate to 100 + the required rate.

EXAMPLES.

1. Sold cloth for 8s. per yard, upon which I had 15 per cent. but markets rifing, I fold what remained at 9s. per yard; what had I per cent. on laft fales?

$$8 : 9 :: 115$$
$$\times 9$$
$$8)\,1035$$
$$129\tfrac{3}{8} - 100 = 29\tfrac{3}{8} \text{ Anfwer.}$$

2. Sold tobacco for $6\frac{2}{3}$d. by which I had $5\frac{1}{2}$ per cent.; how much per cent. would I have got if I could have fold it for $7\frac{1}{4}$d. per lb.? *Anfwer*, 18.7.

3. Sold a parcel of goods for L. 457 : 9 : 0, by which I cleared 18 per cent.; the purchafer fold them immediately for L. 500; how much per cent. would they have brought in had I come to the fame market? *Anfwer*, 29 nearly.

4. Sold my cloth at 13d. by which I had $12\frac{1}{2}$ per cent.; in a neighbouring town the fame cloth is fold for 14d.; what is the feller's profit per cent. upon the fuppofition that he bought on the fame terms with me? *Anfwer*, 21 per cent.

VARIETY IV.

When the felling price and rate per cent. profit are given, to find the prime coft, the proportion will be,

As 100 + the rate per cent. profit to 100, fo is the advanced price to the prime coft.

EXAMPLES.

1. Sold cloth for 10s. 6d. per yard, on which I had $23\frac{9}{17}$ per cent.; what was the prime coft?

$$
\begin{array}{cccccc}
123\frac{9}{17} & : & 100 & :: & & 10.5 \\
\hline
 & & 17 & & & 17 \\
21.00 & & \overline{17.00} & & 3\,) & \overline{178.5} \\
 & & & & 7\,) & \overline{59.5} \\
 & & & & & \overline{8.5} = 8s.\ 6d.\ \textit{Answer.}
\end{array}
$$

2. Sold 500 yards linen for L. 135, by which I had 15 per cent.; what did it cost me per yard? *Answer*, 4s. $8\frac{8}{23}$d.

3. Sold 65734 lbs. of tobacco, at 9d. by which I had 25 per cent.; what was the prime cost of the whole and per lb.?

Answer, $7\frac{1}{5}$d. per lb. and L. 1972 : 0 : $4\frac{3}{4}$ total.

4. Sold 3 puncheons of rum, measuring by the gauge $347\frac{1}{2}$ gallons, for L. 167 : 13 : 4, by which I had $21\frac{3}{4}$ per cent.; what was the prime cost per gallon? *Answer*, 7s. $10\frac{1}{4}$d.

PROMISCUOUS EXAMPLES.

1. Bought 4 hogsheads tobacco, containing net 39 Cwt. 3 qrs. 14 lb. for which I paid L. 483 : 16 : 4; I manufactured it into snuff, which weighed 4370 lb.; I sold the snuff at 2s. 8d. per lb. and the manufacturing, by my account, cost me L. 11 : 17 : 0; what did I gain or lose, and how much per cent.?

Answer, Gained L. 86 : 19 : $10\frac{1}{2}$, which is 18 per cent. nearly.

2. Sold 4370 lb. snuff, made out of 4466 lb. of tobacco, at 2s. 8d. per lb. and had 18 per cent. profit; the expence of manufacture cost L. 11 : 17 : 0; what did the tobacco cost per lb.?

Answer, 2s. 2d.

3. Bought at Glasgow 20 hogsheads sugar, containing net 198 Cwt. 3 qrs. 15 lb. for which I paid at the rate of 55s. 6d. per Cwt. In the passage by the Canal one of the hogsheads was staved, and I lost thereby 7 Cwt. 3 qrs. of my sugar; what did I lose per cent.?

Answer, $3\frac{4}{5}$.

CHAP. XV.

ALLIGATION, OR RULE OF MIXTURES.

BY this rule, grocers, apothecaries, dealers in ſpirits, &c. diſcover the mean rate of a mixture compounded of divers ſimples given, and the quantity of ſimples neceſſary to make a mixture of an aſſigned rate or quality; for which reaſon this rule falls under two denominations, *Alligation Medial*, and *Alligation Alternate*.

SECT. I. ALLIGATION MEDIAL.

ALLIGATION MEDIAL, from ſeveral quantities and rates of divers ſimples given, diſcovers a mean rate of a mixture compounded out of them.

CASE I.

The quantity of the ingredients, and the prices of each given, to find the value of ſome part of the mixture,

Multiply the ingredients ſeverally by their reſpective prices, and divide the ſum of thoſe products by the ſum of the ingredients for the anſwer.

EXAMPLES.

1. A vintner mixeth 20 gallons of ſpirits at 5s. per gallon, 36 gallons at 3s. and 40 gallons at 2s.; what is a gallon of the mixture worth?

$$\begin{array}{rcccr} 20 & \times & 5 & = & 100 \\ 36 & \times & 3 & = & 108 \\ 40 & \times & 2 & = & 80 \\ \hline 96 & & &) & 288\ (\ 3s.\ \textit{Anſwer.} \\ & & & & 288 \\ \hline & & & & 0 \end{array}$$

2. A tobacconist mixeth snuff, viz. 30 lb. at 9d. 60 lb. at 1s. 2d. and $24\frac{1}{2}$ lb. at 2s. per lb.; what is a pound of the mixture worth? *Answer*, 1s. $2\frac{1}{4}$d.

3. A grocer mixeth 12 Cwt. 3 qrs. of sugar, at 55s. per Cwt. 15 Cwt. 2 qrs. 19 lb. at 57s. 6d. per Cwt. and 19 Cwt. 3 qrs. 24 lb. at 47s. 6d.; how may he sell 1 Cwt. of the mixture, to gain 20 per cent.? *Answer*, L. 3 : 3 : 3 per Cwt.

4. A grocer mixeth 112 lb. of raisins, at $3\frac{1}{2}$d. per lb. 78 lb. of almonds, at $6\frac{1}{4}$d. per lb. and 97 lb. of currants, at $5\frac{1}{2}$d. per lb.; what will 1 lb. of the mixture be sold at, allowing the grocer 45 per cent. profit? *Answer*, 7d. per lb.

CASE II.

To know how much of each simple is in any assigned portion of the mixture, observe this proportion:

As the quantity of the mixture to the several quantities of the mixtures given, so the quantity of the assigned portion to the quantities of the simples sought.

EXAMPLES.

1. A grocer mixeth 10 lb. of raisins with 30 lb. of almonds and 40 lb. of currants; it is required to know how many ounces of each sort is in a pound of the mixture?

					oz.		oz.	
40 + 30 + 10 =	80	:	10	::	16	:	2	raisins.
	80	:	30	::	16	:	6	almonds.
	80	:	40	::	16	:	8	currants.

2. A vintner mixeth 50 gallons of spirits with 60 of another kind, and 70 of another kind; required the number of gallons of each in 20 gallons of the mixture? *Answer*, $5\frac{5}{9}$, $6\frac{2}{3}$, $7\frac{5}{9}$ respectively.

3. A founder mixeth 20 lb. of brass with 30 lb. of copper and 4 lb. of silver; how much of each metal is contained in a pound of the mixture? *Answer*, $\frac{20}{27}$, $\frac{5}{9}$, $\frac{2}{27}$, respectively.

CASE III.

To increase or diminish the quantity of the mixture, this is the proportion:

As the ſum of the given quantities of the ſimples to the ſeveral quantities given, ſo the quantity of the mixture propoſed to the quantity of the ſimples ſought.

EXAMPLES.

1. A miller mixeth 20 buſhels of wheat at 5s. 36 buſhels at 3s. and 40 buſhels at 2s.; what is a buſhel of the mixture worth, and how much of each muſt be added to increaſe the quantity to 120 buſhels?

	s.		s.			buſh.	s.
20 at	5	=	100		96 : 20 :: 120 :	25 at	5
36 at	3	=	108	(2.)	96 : 36 :: 120 :	45 at	3
40 at	2	=	80		96 : 40 :: 120 :	50 at	2
96			288		Proof	120	
			3				

2. A grocer had an order for 15 Cwt. barley, 8 Cwt. ſplit peaſe, and 12 Cwt. ſplit beans; but before the order was made up, it was reſtricted to 20 Cwt. of the mixture: how much of each kind muſt be taken to fulfil the ſecond order, ſo as the proportionality in the mixture may be ſtill retained? *Anſwer*, $8\frac{4}{7}$ barley, $4\frac{4}{7}$ peaſe, $6\frac{6}{7}$ beans.

3. A refiner melted down 15 oz. of gold, No. 1. 18 oz. No. 2. and 24 oz. No. 3. for a certain purpoſe; at ſame time he receives an order for 100 ounces of the ſame mixture: how much muſt he take of each to complete the order? *Anſwer*, $26\frac{18}{57}$ No. 1. $31\frac{11}{19}$ No. 2. and $42\frac{2}{19}$ No. 3.

CASE IV.

Given the total of a mixture, with the whole value, and the value of the ſeveral ingredients, thence to find the ſeveral quantities mixed.

Multiply the total of the mixture by the leaſt value, ſubtract the product from the total value, and the remainder is the firſt dividend; ſubtract the value of the loweſt rated unit from the value of the higheſt rated unit, and the remainder is the firſt diviſor: the quotient ariſing from theſe two factors ſhews the quantity of the higheſt priced ingredient, and the other is the complement to the whole.

EXAMPLES.

1. Suppose sugar at 50s. per Cwt. and at 60s. per Cwt. were mixed together, so as 25 Cwt. of the mixture were worth L. 70; how much of each sort was adopted in the mixture?

	25	Cwt. in whole.
	2.5	price of 1 Cwt. of the cheapest.
Take	62.5	value at the lowest price.
From	70	
3 — 2.5 = .5	7.5	dividend.
	15	Cwt. of the highest kind.
Then 25 — 15 =	10	of the lower kind.

2. Suppose a mixture of 400 lb. weight of raisins at 4d. per lb. and almonds at 6d. per lb. were valued at L. 7 : 10 : 0 altogether; how many lb. of each were in the mixture? *Answer*, 100 lb. of almonds and 300 lb. of raisins.

3. Suppose 190 gallons of rum, No. 1. at 8s. and No. 2. at 9s. were mixed together, in such proportion as the whole should be worth L. 80 : 10 : 0; how much of each kind is employed in the composition? *Answer*, 100 No. 1. and 90 No. 2.

SECT. II. ALLIGATION ALTERNATE.

ALLIGATION ALTERNATE shews the due proportion of several ingredients, and counterchanges the places of such excesses or differences as arise between the mean price and the extremes, ascribing that to the greater extreme which proceeds from the lesser, *et vice versa*: But, as the whole of this section is rather curious than useful, we shall proceed to something more interesting.

CHAP. XVI.

OF EXCHANGE.

Exchange may be defined the trade of money carried on between one place and another, however diſtant, by paying money or value in the one, and receiving a bill upon the other, to entitle the giver to receive the value where it is payable.

Were the value of money the ſame at all times and in all places, and were the ſame mode of reckoning to prevail in all places, this rule would be quickly diſcuſſed; but the riſe and fall of exchange, ſometimes above and ſometimes below par, the difference of the unit in point of value, real or imaginary, in different places, and the various ways by which theſe units are divided, have rendered this rule an extenſive field both for the learner's practice and ſpeculation.

The exchange which Britain has occaſion to negotiate directly with other places may be reduced to three diſtinct heads, namely,

I. When Britain receives a variable number of pounds from other countries for L. 100 Britiſh Sterling.

II. When Britain receives a variable number of ſhillings, or of ſome other denomination, for the Britiſh pound.

III. When Britain pays a variable number of pence for the piece of exchange of another country.

SECT. I.

Of the British Exchanges on the L. 100.

I. *With Ireland.*

In Ireland accounts are kept as in Britain; but the par of L. 100 Sterling is L. $108\frac{1}{3}$ Iriſh, though the courſe or price of bills runs from 6 to 12 per cent.

To reduce Sterling money to Irish, the given Sterling will be to the equivalent Irish as 100 to its equivalent by the given course.

To reduce Irish to Sterling, the Irish will be to the Sterling as the equivalent of L. 100 Sterling by the course to L. 100; and the same rules hold good in all cases where Britain exchanges on the L. 100 Sterling.

EXAMPLES.

1. Britain remits to Ireland L. 756 : 10 : 0 Sterling; how much Irish will that amount to, the exchange at 12 per cent.?

$$100 : 112 :: 756.5$$
$$\begin{array}{r} 756.5 \\ 112 \\ \hline 847.28 \end{array} = \text{L. } 847 : 5 : 7\tfrac{1}{5}$$

Or practically

756.5	for L.	100
75.65	for	10
15.13	for	2
847.28		

2. Britain remits to Ireland L. 847 : 5 : $7\frac{1}{5}$ Irish; how much Sterling was paid for the bill, the exchange at 12 per cent.?

$$112 : 100 :: 847.28$$
$$\begin{array}{r} 847.28 \\ 100 \\ \hline 2\ )\ 84728 \\ \hline 7\ )\ 42364 \\ \hline 8\ )\ 6052 \\ \hline 756.5 \end{array}$$

3. Britain remits bills to Dublin to the amount of L. 586 : 10 : 0 Sterling; for how much Irish will Britain be credited at Dublin, the exchange at $9\frac{5}{8}$ per cent.?

4. Dublin is indebted to Britain in L. 642 : 19 : $0\frac{3}{10}$ Irish; for how much Sterling may Britain draw, the exchange at $9\frac{5}{8}$ per cent.?

5. Britain receives a bill of exchange from Ireland per L. 395, 18s. $8\frac{1}{4}$d. Irish; for how much Sterling will Ireland get credit, exchange at $7\frac{1}{4}$ per cent.?

6. Dublin receives a bill of exchange per L. 367 : 9 : 2 Sterling; for how much Irish will Dublin give credit, exchange at $7\frac{3}{4}$ per cent.?

Note, It will be proper to advertise the young exchange-negotiator here, that the fluctuation in the course of exchange is occasion-

ed by the balance of trade being in favour of one place more than another: For inſtance, if Britain owes to Ireland L. 100,000 for linen, and Ireland, on the other hand, owes to Britain L. 50,000 for broad cloth, the balance is in favour of Ireland L. 50,000; to pay which Britain muſt procure bills from other countries; and the ſcarcer any commodity is, it is always the dearer. Bankers generally avail themſelves of ſuch circumſtances, and remit when the exchange is high, that they may draw back with advantage when the exchange comes to fall; as will appear in the following

EXAMPLES,

Applied to drawing and remitting.

1. When the exchange was at 12 per cent. betwixt Britain and Ireland, a banker in Glaſgow laid out in purchaſing bills on Dublin L. 2000, which he remitted his correſpondent there; by and by the exchange fell to 6 per cent.; what did he receive for his bills on Dublin?

(1.)	(2.)
2000 for 100 per cent.	106 : 100 :: 2240
200 for 10 per cent.	100
40 for 2 per cent.	
———	224000
2240 at his credit in Dublin.	
	He draws back L. 2113 4 $1\frac{3}{4}$

The ſame anſwer can be effected ſpeedily thus:

$$112 : 2000 :: 106 : \text{L. } 2113\ 4\ 1\tfrac{3}{4}$$

$$\text{For } \frac{112 \times 2000}{106} = \text{L. } 2113\ 4\ 1\tfrac{3}{4}$$

2. When the exchange was at 5 per cent. Britain contracted a debt in Ireland, payable at 3 months, per L. 3745 : 15 : 0 Sterling; when the term of payment came, bills on Ireland were purchaſed at 10 per cent.; how much Sterling was laid out in purchaſing the remittance? *Anſwer*, L. 3575 : 9 : 9.

3. When the exchange was at 7 per cent. a merchant in Glaſgow drew for his convenience on his friend in Dublin per L. 2749 Sterling; when he came to replace his draughts, bills were ſold at $9\frac{1}{4}$; how much Sterling did he remit? *Anſwer*, L. 2680 : 2 : $4\frac{1}{2}$.

4. When bills on Dublin were ſold at Glaſgow for $10\frac{1}{2}$ per cent. a dealer put himſelf in caſh by drawing for L. 754 : 18 : 6 Ster-

ling; what will it cost him to replace his draughts, should the exchange fall to $6\frac{1}{2}$ per cent.? *Answer*, L. 783 : 5 : 7 nearly.

It frequently happens, that draughts on Ireland made in British Sterling are negotiated with bankers here; in which case, whatever be the current exchange, the intrinsic par, must be paid in Ireland; consequently, if the current exchange be above par, the banker will have a proportional discount on the bill, and, if below par, he can give a proportional premium: Hence the proportion will be, as the political par to the intrinsic, so the content of the bill to its value here.

EXAMPLES.

1. Presented a bill to a banker in Edinburgh per L. 100 British Sterling, payable in Dublin; what ought I to receive from the banker, the exchange being at $9\frac{1}{2}$ per cent.?

Pol. par.		Int. par.		Cont. of the bill.		
109.5	:	$108\frac{1}{3}$	::	100	:	98 : 18 : $8\frac{1}{4}$ value here.

2. I have a bill on Dublin per L. 100 Sterling; what is its value here, when the political par is $7\frac{1}{2}$ per cent.?
Answer, 107.5 :: $108\frac{1}{3}$ :: 100 : 100.775.

3. Sold a bill on Dublin per L. 376 : 18 : 0 British Sterling; what did I receive, when the exchange was at 6 per cent.?
Answer, L. 385 : 3 : 11.

4. Suppose all things as before, but that the exchange was at 10 per cent.: what ought I to have received?
Answer, L. 371 : 3 : $9\frac{1}{2}$.

II. *With America and the West Indies.*

EXAMPLES.

1. New York furnishes Britain with a bill of sales per L. 845, 17s. 6d. currency net; how much Sterling must be remitted to Britain, exchange at 80 per cent.?

	180	:	100					
Abridged	9	:	5	::	845	17	6	
							5	
				9)	4229	7	6	
				L.	469	18	$7\frac{1}{3}$	*Answer*.

2. New York remits a bill per L. 469 : 18 : $7\frac{1}{5}$ Sterling to Britain; how much currency did the bill cost at New York, exchange at 80 per cent.?

3. Boston is indebted to Britain in L. 1345 : 19 : 6 currency; how much Sterling must Boston remit, exchange at $65\frac{1}{2}$ per cent.?

Answer, L. 813 : 5 : $6\frac{3}{4}$.

4. A cargo of goods was consigned to Virginia, value per invoice L. 897 : 8 : 4, which was 5 per cent. above prime cost; the goods were sold in Virginia at $102\frac{1}{2}$ per cent. advance on the invoice, currency for Sterling; how much Sterling must be remitted home, deducing 8 per cent. for commission and charges, exchange at 35 per cent. and what does the British merchant clear per cent. by the adventure? *Answer*, L. 1238 : 8 : $8\frac{1}{4}$ to be remitted home, which, including 5 per cent. on the invoice, is 43 per cent. nearly.

5. A young fellow, to try his fortune, laid out L. 1000 upon goods in Glasgow, suited for the Jamaica market, ships them on board the Nancy at 5 per cent. freight, and goes himself supercargo, at 15 guineas passage; his debentures and drawbacks cleared the outset charges. When he arrived in Jamaica, he sold his cargo at 195 per cent. advance on the invoice, currency for Sterling. Being disposed to stay on the island, he remits home the proceeds in bills of exchange, at 40 per cent. after clearing the freight and passage, and retention of L. 130 currency for his pocket; what did the remittance amount to in Sterling?

Answer, L. 1948 : 10 : $8\frac{1}{2}$.

6. An ounce of gold, valued at L. 4 Sterling, is sold in Carolina for L. 18 : 19 : 10; what is the exchange at that rate?

Answer, $474\frac{19}{24}$ per cent.

7. At Philadelphia, a remittance to Britain of L. 563 : 4 : $1\frac{5}{7}$ Sterling, cost L. 985 : 12 : 3 currency; what was the exchange?

Answer, 75 per cent. nearly.

8. A merchant, in balancing his books, finds there is due him in Maryland L. 1575 : 14 : 0 currency, at 45 per cent. exchange; in Virginia L. 3479 : 10 : 0 currency, at $36\frac{1}{2}$ per cent. exchange; in Philadelphia L. 3419 : 19 : 0, at 27 per cent. exchange; and in Jamaica L. 4547 : 18 : 6, at $38\frac{3}{4}$ exchange: what money stands in the Sterling column of his ledger?

Answer, In Maryland	L.	1086	13	$9\frac{1}{4}$
In Virginia		2549	1	$8\frac{1}{4}$
In Philadelphia		2692	17	6
In Jamaica		3277	15	$8\frac{1}{2}$

9. When a mechanic receives 5s. per day at Philadelphia, and in Britain only 1s. 6d. what is the difference of their wages, exchange

80 per cent.? *Answer*, Better at Philadelphia by 1s. $3\frac{1}{3}$d. Sterling.

10. When L. 357 : 13 : 4 Sterling gave me credit in Maryland for L. 519 : 12 : 6 currency, what was the exchange, at that rate?

Answer, $45\frac{1}{4}$ per cent.

III. *With London and other trading places in Britain.*

As the balance of trade, and some other considerations, put the price of bills more or less above par, a premium is paid for bills at sight, or any short date, at a variable rate per cent. or the bill is taken so many days after date, to make it at par.

EXAMPLES.

1. Glasgow draws on London for L. 273 : 13 : 6, at 10 days date, premium $2\frac{1}{2}$ per cent.; how much is paid for the bill at Glasgow?

$$\begin{array}{rl} 4{,}0\,) & 27{,}3.675 \\ 10\,) & \;\;6.841875 \\ & \;\;\;\,6841875 \\ \hline & 281.2010625 = \text{L. } 281 : 4 : 0 \textit{ Answer.} \end{array}$$

2. Glasgow draws on London for L. 397 : 15 : 0, at 20 days date, premium $1\frac{3}{4}$; what is paid for the bill?

Answer, L. 404 : 14 : $2\frac{1}{2}$.

3. Glasgow draws on London, at 40 days date, for L. 447 : 17s. premium $\frac{7}{8}$ per cent.; what is paid for the bill?

Answer, L. 451 : 15 : 2.

4. London draws on Glasgow for L. 756 : 19 : 6, at one month's date; what was received for the bill in London, the re-exchange at $3\frac{1}{2}$ per cent.? *Answer*, L. 730 : 8 : $7\frac{1}{2}$.

5. Glasgow draws on London at par for L. 577 : 17 : 6, but the bill returns under protest; what will take up the bill at Glasgow, re-exchange being charged at $3\frac{1}{4}$ per cent. exclusive of other charges? *Answer*, L. 599 : 10 : 11.

SECT. II.

OF THE BRITISH EXCHANGES ON THE POUND STERLING.

BRITAIN exchanges on the pound Sterling with Holland and Hamburgh, whence it receives an uncertain number of Flemiſh ſchillings and pence in return.

I. *With Holland.*

The pound Flemiſh is divided as the Britiſh pound; but, when the integer is a guilder,

16 pennicks	=	1 ſtiver.
20 ſtivers	=	1 guilder.

The par of L. 1 Sterling is 36s. $7\frac{3}{10}$d. Flemiſh, and the courſe runs from 30 to 40 ſchillings.

There are here two ſorts of money, *banco* and *currency*, betwixt which there is a ſmall difference, called *agio*, which runs from 3 to 6 per cent. in favour of the bank.

When the exchange riſes above par, Britain gains by the exchange; but when it falls below par, Britain loſes.

Bank money is reduced to current and current to bank, as Sterling to Iriſh and Iriſh to Sterling.

EXAMPLES.

1. A merchant in Holland has 5741 guilders current money; how much will it reckon banco, agio 4 per cent.?

$$
\begin{array}{r}
104 : 100 :: 5741 \\
100 \\
\hline
104\,)\;574100 \\
\hline
5520 \quad 3 \quad 13
\end{array}
$$

2. 5520 guild. 3 stiv. 13 pen. banco is intended to compensate part of a debt in current money; how far will it go, agio 4 per cent.?

3. In 7159 guild. 19 stiv. 14 pen. current money, how much banco, agio $4\frac{3}{8}$ per cent.?

Answer, 6859 guild. 17 stiv. 7 pen. ban.

4. In 8974 guild. 18 stiv. 13 pen. banco, how much current money, agio $3\frac{5}{8}$ per cent.?

Answer, 9300 guild. 5 stiv. 10 pen. cur.

Sometimes bills are drawn on Holland for Flemish pounds, &c. and sometimes in guilders. The par of 20 sch. or L. 1 Flemish, is 6 guilders.

To convert either into Sterling, it will be, as the given course to L. 1 Sterling, so the given Dutch to the required Sterling.

To turn Sterling to Dutch money, reverse the last rule, or use practice.

EXAMPLES.

1. Britain draws on Amsterdam for L. 447 : 17 : 6 Sterling, how much Flemish will pay the draught, exchange at 35s. 6d. Flemish per pound Sterling?

St.		F.		St.				s.	d.
1	:	1.775	::	447.875	Or,	447.875	for	20	0
				1.775		223.9375	for	10	0
						111.96875	for	5	0
				794.978125 *Answer*.		11.196875	for	0	6
						794.978125			

2. Suppose the reckoning in Amsterdam had been guilders, how many would have been paid?

Every pound Flemish = 6 guilders; wherefore

$$\begin{array}{r} 794.978125 \\ 6 \\ \hline 4769.868750 \end{array}$$ *Answer*,

Or find how many guilders in the exchange, thus;

20 : 6
10 : 3 :: 35.5

Then 447.875 × 10.65 = 4769.86875, as before.

Or multiply by the grotes or Flemish pence in the exchange, and divide by 40, the number of grotes in a guilder, for the answer, as

$$\frac{447.875 \times 426}{40} = 4769.86875, \text{ as before.}$$

3. Britain draws on Amsterdam for L. 794 : 19 : $6\frac{1}{4}$ Flemish banco, exchange at 35s. 6d. Flemish per pound Sterling; what Sterling was received for the draught?

Flem.		St.		Flem.		Sterl.
1.775	:	1	::	794.978125	:	447.875

4. Suppose, at the same exchange, Britain had drawn for 4769 guild. 17 stiv. 6 pen.; how much Sterling was paid for the bill?

6) 4769.86875

794.978125 Flemish.

	F.		St.				
Then	1.775	:	1	::	794.978125	:	447.875

			G.		st.		G.		L.
Or,	35.5 × .3	=	10.65	:	1	::	4769.86975	:	447.875

$$\text{Or, } \frac{4769.86875 \times 40}{426} = 447.875$$

5. Britain remits to Amsterdam 5974 guild. 18 stiv. 14 pen.; how much Sterling was paid for the bill, the exchange at 34s. 9d. Flemish banco per pound Sterling?

Answer, L. 573 : 2 : $8\frac{3}{4}$.

6. Amsterdam is indebted to Britain in $8745\frac{1}{4}$ guilders, current money; for how much Sterling may Britain draw, the exchange at 36s. 4d. Flemish banco, and agio $4\frac{1}{2}$ per cent.?

Answer, L. 767 : 16 : $2\frac{1}{2}$.

7. When the exchange is at 35s. 6d. Flemish banco per pound Sterling; what is the value of the guilder banco?

Answer, $22\frac{1}{2}$d.

8. When the exchange is at 34s. 10d. Flemish banco, what is the value of the current guilder, agio $4\frac{1}{4}$ per cent.?

Answer, $21\frac{3}{4}$d.

9. Amsterdam owes to Glasgow, as the proceeds of tobacco, 10000 guilders current money, which the merchant can sell for $22\frac{1}{4}$d. per guilder; but being advised that bills on Amsterdam sell at London for 35s. 6d. Flemish banco per pound Sterling, and knowing $2\frac{1}{4}$ can be got by his bill on London, and being also advised that

the agio is at $4\frac{1}{2}$ per cent. he would know which is the most advantageous negotiation, at Glasgow or London, supposing he allows $\frac{1}{2}$ per cent. for negotiating in London?

Answer, At Glasgow he gets L. 937 : 10 : 0,

At London, including exchange here, only L. 914 : 3 : $1\frac{1}{2}$.

10. Glasgow draws on Amsterdam for 10000 guilders, and receives at the rate of $22\frac{1}{4}$d. per guilder. By way of experiment, Glasgow orders London to draw for 10000 guilders, deducing agio at 4 per cent. and remit the proceeds in bills on Glasgow, deducing $\frac{1}{2}$ per cent. commission. In compliance with this order, London draws at 35s. 9d. Flemish banco, and remits the proceeds in bills, purchased at $3\frac{1}{2}$ per cent. discount. What is the difference of these negotiations?

Answer, Better at Glasgow by L. 23 : 10 : $1\frac{1}{4}$.

11. Amsterdam ships for Leith 1000 gallons gin, at $54\frac{1}{2}$ stivers per gallon, in 100 casks at 50 stivers each; 50 hhds. flaxseed, at 32 guilders 10 stivers each; 2 boxes cinnamon, containing 24 ℔. at 121 guilders 4 stivers each: required the amount of the invoice Dutch, and the equivalent British, exchange at 35s. 4d. Flemish per pound Sterling?

Answer, 7508 guild. 16 stiv. Dutch = L. 708 : 7 : $6\frac{1}{2}$ Sterl.

II. *Exchange with Hamburgh.*

12 phennings	=	1 sol lubs.
16 sols	=	1 merk.
3 merks	=	1 rixdollar.

The par of the Flemish pound here is $7\frac{1}{2}$ merks, or $2\frac{1}{2}$ rixdollars. The par of the Hamburgh merk is 1s. 6d. Sterling. The agio here runs from 20 to 40 per cent. and all bills are paid in bank. The par of 20s. Sterling is 35s. $6\frac{3}{4}$d. Flemish banco.

EXAMPLES.

1. Britain draws on Hamburgh for L. 756 : 18 : 0 Sterling; how many merks, &c. will pay the draught, the exchange being at 33s. 6d. Flemish banco per pound Sterling?

(1.) 20 : $7\frac{1}{2}$ :: 33.5
$7\frac{1}{2}$

234.5
16.75

2) 25.125

F. St.
33 : 6, or L. 1 = 12.5625 merks.

(2.) 12.5625
756.9

1130625
753750
628125
879375

9508.55625 = 9508 m. 8.9 sch.

2. Britain draws on Hamburgh for 9508 merks 8.9 sch.; how much Sterling was paid for the bill, exchange at 33s. 6d. Flemish banco per pound Sterling?

F. Merks.
33 6 = 12.5625 | 9508.55625

L. 756.9 *Answer.*

Sterling is most expeditiously reduced to Hamburgh money, by multiplying by the grotes contained in the exchange, and dividing by 32, the grotes in a merk, and the contrary; for

$$\frac{756.9 \times 402}{4 \times 8} = 9508.55625\text{, as before.}$$

And $$\frac{9508.55625 \times 32}{402} = 756.9\text{, as before.}$$

3. Britain is indebted to Hamburgh in L. 419 : 9 : 9 Sterling; for how much is Britain debited in Hamburgh, the exchange at $34\frac{1}{2}$ sch. Flemish banco per pound Sterling?

Answer, 5400 m. 14.4 sch.

4. Hamburgh is indebted to Britain in 12745 merks 12 sch. lubs; for how much Sterling may Britain draw, when the exchange is at 32s. 5d. Flemish banco per pound Sterling?

Answer, L. 1048 : 9 : 10¼.

5. Hamburgh is indebted to Britain in net proceeds 14745 merks 15 sch.; for how much banco may Britain draw, agio 30 per cent. and how much Sterling will be paid for the draught, exchange at 33s. 4d. Flemish banco per pound Sterling?

Answer, L. 907 : 7 sch.

6. When the exchange is at 35s. 2d. Flemish banco per pound Sterling, what is the value of the merk banco?

Answer, 18⅛d.

7. When the exchange is at 33s. 3d. Flemish banco, what is the value of the current merk, the agio being 23½ per cent.?

Answer, 15½d.

8. Glasgow can draw on Hamburgh for 10000 merks banco, at 1s. 6d. per merk; bills pass at London for 33s. 9d. Flemish banco per pound Sterling, and on London at a short date for 1½ per cent. premium; in which of these ways had Glasgow better negotiate, allowing ½ per cent. at London for negotiation, and with what difference per cent.?

Answer, Better at London by L. 49 : 18 : 7¼, or 6½ per cent.

9. Leith consigns to Hamburgh a St. de Croix remittance, 235 Cwt. 3 qrs. of sugar, and 1186 gallons of rum; the sugar was sold at 27 merks 12 sch. per Cwt. and the rum at 4 merks 14 sch. per gallon; the charges of freight, impost, &c. came to 354 merks 9 sch. besides commission at 2½ per cent.: required the Hamburgh bill of sales, and Sterling to be remitted at 33s. 6d. Flemish per pound Sterling?

Answer, 11661.15469 merks = L. 928 : 5 : 0¼.

In all exchanges, when the pound Sterling is the medium, draw when the exchange is low, and remit when it is high; as it is better to receive at the rate of 35s. for your draught than a lesser number, and the contrary in remitting.

EXAMPLES.

1. When the exchange was at 36s. 6d. Flemish banco per pound Sterling, Britain remitted to Rotterdam L. 1000 Sterling; but on the fall of the exchange to 34s. 6d. Britain re-drew to be reimbursed; what was received for the draught?

Sixpences.		L.		
73	:	1000	::	69
		73		
	69)	73000		

L. 1057 19 5 *Answer.*

2. When the exchange was at 36s. Britain drew on Hamburgh for 20,000 merks, to replace which, when the draught became due, Britain was obliged to remit at 35s.; what was gained or lost by the transaction? *Answer*, L. 42 : 10 : 0 lost.

3. When the exchange was at 36s. Britain remitted to Hamburgh 20,000 merks, and when the exchange fell to 34s. Britain drew to be reimbursed; what was lost or gained by the transaction, and how much per cent.?

Answer, $1176\frac{8}{17}$ merks, which is $5\frac{22}{25}$ per cent. gained.

4. When the exchange was at 37s. Britain drew on Amsterdam for 20,000 guilders, but was obliged to remit, to keep credit there, at 34s. 6d.; what was lost or gained, and how much per cent.?

Answer, 1351.352 merks, viz. $7\frac{1}{4}$ per cent. lost.

SECT. III.

BRITISH EXCHANGES ON THE FOREIGN PIECE.

IN all cases of this nature, as the course Sterling to the foreign piece, so is the given Sterling to the required foreign; and as the piece to the course, so the given foreign to the required Sterling. Or, more expeditiously, foreign money is reduced to Sterling by practice.

I. *With France.*

12 deniers	=	1 sol.
20 sols	=	1 livre.
3 livres	=	1 crown of exchange.

Par 2s. $5\frac{1}{2}$d. per ecu, = 3 livres, course from 30 to 34.

EXAMPLES.

1. Glasgow receives a bill of sales, amounting net to L. 37456, 18s. 9d. from Bourdeaux; for how much Sterling may Glasgow draw, exchange at $32\frac{1}{2}$d. per ecu?

Liv.	d.		L.
3	: $32\frac{1}{2}$	::	37456.9375
			$32\frac{1}{2}$
			1198622
			18728.5
		3)	1217350.5
		12)	405783.5
		2,0)	3381,5.2916
			1690.764583

Or, 3)	37456.9375		
10)	12485.64583	crowns.	
4)	1248.564583	at 2s.	0d.
3)	312.141146	at 0	6
4)	104.047048	at 0	2
	26.011762	at 0	$0\frac{1}{2}$
	1690.764539		

2. Glasgow draws on France for L. 1690 : 15 : $3\frac{1}{2}$; how much will pay the bill in France, exchange at $32\frac{1}{2}$ Sterling per ecu?

Note, Foreign money is reduced to Sterling by practice, and Sterling to foreign pieces by reduction.

3. Glasgow ships for France 50 hogsheads of tobacco, containing 72800 lb. charged per invoice at $5\frac{5}{8}$d. the lb. in full of cost and charges; for how much will France credit Glasgow, the exchange at $33\frac{5}{8}$d. per ecu? *Answer*, 12178.438 crowns.

4. France ships for Britain 34597 pinees of yarn, at 15 liv. 13 sols 9 deniers per pinee; for how much will Britain credit France, exchange at $32\frac{7}{8}$d. per crown? *Answer*, L. 24781 : 7 : $6\frac{1}{4}$.

5. France ships for Britain 18 pipes claret at 720 livres per pipe, 14 casks brandy at 41 livres 8 sols per cask, 350 yards cambrics at 63s. per yard, and 547 yards lace at 37s. 6d. per yard; required Britain's debit in France and France's credit in Britain, exchange at $30\frac{7}{8}$d. per ecu?

Answer, 15667 liv. 14 sols 6 deniers = L. 671 : 17 : 3.

II. *With Portugal.*

Here they have only millreas and reas, 1000 of the latter being equal to one of the former; so that their books are kept in integers and decimals.

The par $67\frac{1}{2}$d. per millrea = 1000 reas; course from 63 to 68d.

EXAMPLES.

1. Lisbon is indebted to London in 9784 millr. 515 reas; for how much Sterling may London draw, exchange at 5s. 4d. per millrea? *Answer*, L. 2609 : 4 : $0\frac{1}{4}$.

2. London is indebted to Lisbon in L. 478 : 16 : 6: how much money stands in the Lisbon books as an equivalent, exchange at 5s. $5\frac{1}{2}$d. per millrea? *Answer*, 1754 millr. 381 reas.

3. Oporto consigns to Glasgow 14 pipes of port wine, for which the factor pays at landing at the rate of L. 21 : 1 : 6 per tun duty; he next purchases bottles at 22s. 6d. per gross, and bottles off the wine, which run at the rate of $52\frac{1}{2}$ dozen per pipe; he sold the whole at 16s. 6d. per dozen: what do the net proceeds of this consignment amount to, after deducting 8 per cent. for charges and commission, and how much must be remitted to Oporto, exchange at $65\frac{3}{4}$d. per millrea? *Answer*, 1246 millr. 235 reas.

4. Oporto sells for London a consignment of 5745 yards of broad cloth, at 3 millr. 570 reas per yard; what Sterling must be remitted to London, to even accounts with the constituent, exchange at $65\frac{2}{3}$d. per millrea? *Answer* L. 5611 : 13 : $4\frac{1}{2}$.

5. Oporto ships for Britain 15 pipes red port at 51w275 per pipe, 18 ditto white ditto at 54w420 per pipe, 12 boxes fruit at 68w725 per box, shipping charges 32w850; required the debit at Oporto and credit in Britain, exchange 5s. 3d. per millrea?

Answer, 2606w235 = L. 684 : 2 : $8\frac{1}{4}$.

III. *With Spain.*

34 mervadies = 1 rial.
8 rials = 1 piastre, or pezzo of exchange.

On the piastre; par 3s. 7d. course from 2s. 11d. to 3s. 9d. Sterling.

EXAMPLES.

1. Glaſgow imports from Cadiz goods valued per invoice at 2163 piaſtres and 4 rials; how much Sterling may Cadiz draw for, exchange at $38\frac{1}{4}$d. per piaſtre? *Anſwer*, L. 349 : 6 : $3\frac{3}{4}$.

2. Glaſgow remits to Cadiz L. 345 : 18 : $8\frac{5}{6}$ Sterling; how much Spaniſh* will compenſate the remittance at Cadiz, the exchange at $39\frac{5}{6}$d. per piaſtre? *Anſwer*, 2084 piaſt. $2\frac{1}{2}$ rials.

3. London ſends to Spain 3497 yards of broad cloth, for ſales and returns; the factor adviſes that it was ſold for 4 piaſtres and 3 rials per yard, which, deducing 6 per cent. for charges and commiſſion, the exchange being at $41\frac{1}{2}$d. per piaſtre, was 45 per cent. advance on the invoice: what did the ſales amount to in Spaniſh, what was the factor's allowance on the whole, and what was the prime coſt Sterling?

Anſwer, 15299.375 piaſt. amount of ſales in Spain,
917.962 factor's commiſſion, ditto,
L. 2486.785 Sterling net to be remitted,
1715.024 prime coſt in London.

4. Spain ſends to London 20 pipes of wine, which coſt at landing, of duty and other charges, L. 23 : 15 : 6 per tun; it was ſold at L. 57 : 13 : 4 per pipe, the factor had $5\frac{1}{2}$ per cent. for commiſſion and charges, and after all there was 37 per cent. advance on the invoice: what did the ſales amount to Sterling, what was the factor's commiſſion, what was the invoice price in Spain, exchange at $39\frac{7}{8}$d. per piaſtre, and how many piaſtres, &c. ought the factor to remit to Spain?

Anſwer, L. 1153.333 amount of ſales Sterling,
63.433 factor's commiſſion Sterling,
5128 piaſt. $7\frac{3}{5}$ rials to be remitted,
5743 6 prime coſt in Spain.

5. France ſends to Spain 756 yards of cambric at 55 ſols per yard, 374 yards of broad cloth at 178 ſols per yard, 11 pipes Burgundy at 527 livres 17 ſols 10 deniers per pipe, and 15 ditto Champaign at 539 livres 17 ſols 10 deniers per pipe; required the amount of the invoice French, and the French merchant's credit in Spain, the exchange being at $77\frac{1}{4}$ ſols per piaſtre?

Anſwer, 19312 liv. 15 ſols 8 den. = 4967 piaſt. 7 rials 11 merv.

IV. *With Genoa.*

12 denari = 1 ſoldi.
20 ſoldi = 1 pezzo.

On the pezzo; par 4s. 6d. = $5\frac{3}{4}$ lires, courſe from 47 to 58d.

EXAMPLES.

1. Glasgow owes to Genoa 3390 pez. 16 sol.; how much Sterling must be remitted to even the account, the exchange at $51\frac{7}{8}$d. per pezzo?

2. Glasgow remits to Genoa L. 732 : 18 : $1\frac{1}{4}$ Sterling; for how much will the remittance be credited at Genoa, the exchange at $51\frac{7}{8}$d. per pezzo?

3. Glasgow owes to Genoa 57459 lires 18 sol.; how much Sterling will compensate the debt, the exchange at $52\frac{3}{4}$d. per pezzo?

Answer, L. 2196 : 7 : 8.

4. Barcelona purchases 320 yards velvet from Genoa at $57\frac{1}{2}$ soldi per yard, and Genoa takes 80 dozen napkins from Barcelona at 87 rials per dozen; what is the balance, and to whom due, the exchange being at 15 soldi 6 denari per piastre?

Answer, Genoa receives a balance of 317 piastres 26 mervadies.

V. *With Leghorn.*

12 denari	=	1 soldi.
20 soldi	=	1 pezzo.

On the piastre = 6 lires = 4s. 6d. course from 47 to 58d.

EXAMPLES.

1. London owes to Leghorn L. 169 : 0 : $10\frac{1}{2}$; how much stands at London's debit in Leghorn, exchange at $55\frac{1}{2}$d. per piastre?

2. London remits to Leghorn 731 piastres; how much Sterling was paid for the bill, the exchange at $55\frac{1}{2}$d. per piastre?

3. New England consigns to Leghorn 1000 quintals of fish, with orders to remit to London; the whole cargo is sold at 13 piastres 17 sols per quintal: how much Sterling must the factor remit to London, after deduction of 3 per cent. commission, exchange at $53\frac{1}{4}$d. per piastre, and for how much currency will New England be credited at London, the exchange being at $54\frac{3}{4}$ per cent.?

Answer, L. 2980 : 15 : 7 to be remitted to London.
4612 : 15 : $1\frac{1}{2}$ New England currency.

4. Amsterdam receives from Leghorn 56 casks of oil at $37\frac{1}{2}$ piastres per cask, and 457 lb. silk at $9\frac{3}{4}$ piastres per lb.; what is Amsterdam's debit at Leghorn, and how many guilders, &c. must be remitted, at $56\frac{1}{2}$ stivers per piastre, to even the account?

Answer, $6555\frac{3}{4}$ piastres = 18519 guild. 19 stiv. 14 pen.

VI. *With Venice.*

12 deniers	=	1 sol.
20 sols	=	1 livre or ducat.

On the ducat, par $50\frac{1}{4}$d. course 45 to 55d. agio 20 per cent.

EXAMPLES.

1. London owes to Venice 1459 ducats 18 sols 1 denier banco; for how much Sterling does London credit Venice, exchange at $52\frac{1}{4}$d. per ducat?

2. London draws on Venice for L. 320 : 17 : 6; how much banco will pay the bill at Venice, exchange at $52\frac{1}{4}$d. per ducat?

3. London receives from Venice 155 pieces velvet of 25 yards each, charged per invoice at 2 duc. 18 sol. 10 den. per yard; what is the amount of the invoice, and how much Sterling must be remitted to even the account, the exchange being at $54\frac{1}{4}$d. per ducat? *Answer*, L. 2600 : 7 : 9.

4. London is indebted to Venice in 27459 duc. 15 sol. 9 den. current money; how much Sterling will compensate the debt, exchange being at par? *Answer*, L. 4791 : 3 : $2\frac{1}{4}$.

5. Venice sends to Hamburgh 16 pieces velvet at 112 ducats 15 soldi per piece, 24 pieces silver stuffs at 96 ducats 18 soldi per piece, and 25 mirrors at 24 ducats 10 soldi each; what is the amount of the bill at Venice, and how many merks, &c. will purchase a remittance, exchange at $2\frac{1}{2}$ merks per ducat?

Answer, At Venice 4742 ducats 2 soldi, and at Hamburgh 11855 merks 4 schillings.

VII. *With Denmark and Norway.*

48 stivers = 1 rixdollar.

On the rixdollar, = 4s. 6d. Sterling, course from 45 to 58d.

The rixdollar is subdivided into 6 merks, and each merk into 16 schillings.

EXAMPLES.

1. Denmark is indebted to Britain in 7456 merks 14 schillings; for how much Sterling may Britain draw, the exchange at 50d. per rixdollar? *Answer*, L. 258 : 18 : $4\frac{1}{4}$.

2. A cargo of timber was ſold at Greenock for L. 375 : 19 : 7 Sterling; what will this money reckon at Bergen, the exchange at $49\frac{1}{4}$d. per rixdollar? *Anſwer*, 1813 rixd. 4 merks $9\frac{1}{2}$ ſchil.

3. Norway ſends to Spain 5000 deals at 75 merks 14 ſchillings per 120, and 100 barrels tar at 64 merks 10 ſch. per barrel; what is the amount of the bill, and how much Spaniſh will purchaſe a remittance at 78 rixdollars per 100 piaſtres?

Anſwer, 1603 rixd. 5 merks 15 ſch. = 2056 piaſt. 3 rials 7 merv.

VIII. *With Dantzic.*

18 pence	=	1 groſhen.
30 groſhen	=	1 florin.

On the pound Sterling, at $13\frac{1}{2}$ florins par; but more commonly by Amſterdam or Hamburgh.

EXAMPLES.

1. London is indebted to Dantzic in 9750 flor. 25 groſ. 16 pence; how much Sterling ought London to remit, the exchange at $13\frac{1}{2}$ flor. per pound Sterling? *Anſwer*, L. 709 : 3 : $0\frac{1}{2}$.

2. Dantzic owes to London L. 745 : 19 : 6; how much ſtands for London in Dantzick, the exchange at $14\frac{1}{2}$ florins per pound Sterling? *Anſwer*, 10816 flor. 19 groſ. 2 pence.

3. London owes Dantzic L. 547 : 19 : 0; how much ought to be remitted to Hamburgh to clear the debt, exchange at $91\frac{1}{2}$ groſhen per rixdollar of Hamburgh, and at 34s. 6d. Flemiſh banco between London and Hamburgh?

Anſwer, 216217.6453125 groſhen of Dantzic.

4. Dantzic owes to London 5745 flor. 20 groſ. 6 pence; for how many guilders may London value on Amſterdam, exchange at $94\frac{1}{2}$ groſhen per guilder? *Anſwer*, 1828.862 guilders.

5. London imports from Dantzic 515 quarters wheat, at $12\frac{1}{2}$ florins per quarter; 100 caſks aſhes, containing 240 Cwt. at 18 florins 25 groſ. per Cwt. and 15 caſks, containing 22 Cwt. 2 qrs. at 93 flor. 20 groſ. per Cwt.; package and ſhipping charges, 47 flor. 15 groſ.: what is the amount of the invoice Dantzic, and what will a direct remittance coſt at 14 florins per 20s. Sterling?

Anſwer, 13241.25 flor. = L. 945 : 16 : $0\frac{1}{4}$.

IX. *With Stockholm.*

32 runſticks	=	1 copper dollar.
6 copper dollars	=	1 rixdollar.
$34\frac{4}{7}$ dollars	=	L. 1 Sterling.

EXAMPLES.

1. Stockholm is indebted to London in 3745 dol. 4 cop. dol. 18 runſt.; how much Sterling may be drawn for, exchange at $35\frac{1}{2}$ dollars per pound Sterling? *Anſwer*, L. 105 : 10 : 3.

2. London is indebted to Stockholm in L. 748 : 19 : 0; how much muſt be remitted to Hamburgh to even the account, exchange at 33s. 9d. Flemiſh banco per pound Sterling, and for how many dollars may Stockholm draw on Hamburgh, at $7\frac{1}{2}$ dollars per rixdollar of Hamburgh? *Anſwer*, 23697.246 dol.

3. Sweden ſhips for Spain 326 Cwt. bar iron, at 175 dollars per Cwt.; 20 coil cordage, containing 64 Cwt. 2 qrs. at $197\frac{1}{2}$ dollars per Cwt.; 70 pieces ſail cloth, at 125 dollars per piece; ſhipping charges $97\frac{1}{4}$ dollars: required the Swediſh invoice, and the piaſtres to be remitted by Spain, exchange at 15 piaſtres per 100 dollars? *Anſwer*, 78636 dol. = 11795.4 piaſt.

Note, In all exchanges on the foreign piece, the Britiſh banker muſt remit when the exchange is low, and draw when it is high, to avail himſelf of the difference.

EXAMPLES,

In drawing and remitting.

1. When bills on France ſold at 30d. per ecu, London remitted to Paris 20,000 crowns; but when the exchange got up to $32\frac{1}{2}$, London drew to be reimburſed; with what difference per cent.? *Anſwer*, $8\frac{1}{3}$ per cent.

2. When bills on Spain ſold for 46d. per piaſtre, London remitted L. 7000 to Cadiz; but when bills roſe to 50d. per piaſtre, London drew to be reimburſed: what advantage attended the negotiation per cent.? *Anſwer*, $8\frac{16}{23}$ per cent.

3. When the exchange between London and Venice was at 50d. per ducat banco, London drew for 50,000 ducats; upon the fall of exchange to 45d. London replaced the draught: how much per cent. was gained by the negotiation? *Anſwer*, $11\frac{1}{9}$ per cent.

CHAP. XVII.

ARBITRATION OF EXCHANGE.

BANKERS, who accuſtom themſelves to ſpeculations with regard to exchange, not only know how to draw and remit with advantage to any one place, as the exchange riſes or falls, but likewiſe, by comparing the courſe between two or more places, how to draw upon one, and replace that draught by remitting to ſome other place where the calculation points out the profit. This admits of two ſections.

SECT. I.

OF SIMPLE ARBITRATION.

WHEN a compariſon is made betwixt the courſe of exchange from one place to other two, in order to find the political or arbitrated par, the arbitration is called *Simple;* in which all queſtions are ſtated thus:

Let the firſt and third terms be always of the ſame kind, and the ſecond of the ſame kind with that of which the par is required.

EXAMPLES.

1. The courſe from London to Amſterdam is at 34s. 6d. and between London and Paris at 32d. per ecu; what is the arbitrational price of the French crown at Amſterdam?

Ds.St.		Sh. F.		Ds.F.		
240	:	34 6	=	414	::	32
				4		
30				165,6		4

55⅕ grotes. *Anſwer.*

2. Bills on Paris ſell at London for 32d. Sterling per ecu, and at Amſterdam for $55\frac{1}{5}$ grotes per ecu; what ought to be the courſe between London and Amſterdam, to put it on a par with the other two? *Anſwer*, 34s. 6d.

3. Bills on Paris ſell at Amſterdam for $55\frac{1}{5}$ grotes per ecu, and on London at 34s. 6d. Flemiſh banco per pound Sterling; how ſhould bills on Paris be rated at London to be on a par with the other two? *Anſwer*, 32d.

4. If bills at London on Paris be at 32d. per ecu, and on Amſterdam at 34s. 6d. what may be gained per cent. by drawing on Paris, and remitting to Amſterdam, ſhould London be adviſed that bills on Paris at Amſterdam ſell for 52 grotes per ecu?

Ds.		C.		L.St.		C.	
32	:	1	::	100	:	750	debit at Paris.
C.		Gr.		C.		G.	
1	:	52	::	750	:	39000	credit on Amſterdam.
Gr.		L.St.		Gr.		L.	
414	:	1	::	39000	:	94 4 $0\frac{5}{8}$.	

and L. 100 — L. 94 4 $0\frac{5}{8}$ = L. 5 15 $11\frac{3}{8}$ gained per cent.

5. But if the advice had been that the courſe between Amſterdam and Paris had riſen above par, ſuppoſe to 56 grotes per ecu, the courſe of the negotiation ought alſo to have been altered, ſo that the draught muſt have been made on Amſterdam, and the remittance made to Paris. Suppoſe all things as before but the exchange at Amſterdam, what profit preſents by drawing on Amſterdam? *Anſwer*, 1.43 per cent.

6. London is indebted to Riga in 5000 rubles, and can either purchaſe a bill on Riga at 50d. per ruble, or remit by Amſterdam at 90 grotes per ruble; by which of theſe ways ought London to remit, ſuppoſing bills on Amſterdam ſell at London for 36s. 4d. Flemiſh banco per pound Sterling? *Anſwer*, London ſaves by remitting via Amſterdam L. 9 : 11 : $1\frac{3}{4}$.

But had the courſe of exchange to Holland been below 36s. the political par, a direct remittance would have been moſt advantageous.

7. London exchanges with Amſterdam at 35s. Flemiſh banco per pound Sterling, and with Paris at $31\frac{1}{2}$d. per ecu; in what manner ought London to draw and remit L. 500 with advantage, when bills at Amſterdam on Paris ſell at $53\frac{1}{5}$ grotes?

Anſwer, By drawing on France, 3.25 per cent. is gained.

8. Suppoſe London exchanges with France at $30\frac{1}{2}$d. per ecu, and with Cadiz at 3s. 6d. per piaſtre; what is the arbitrated par, and how ought L. 100 to be drawn and remitted with advantage, ſuppoſing bills on Cadiz can be purchaſed at Bourdeaux for 81 ſols 9 den. per piaſtre?

Answer, Arbitrated par $82\frac{38}{61}$ sols; wherefore draw on Cadiz per 571 piast. 3 rials 14 merv. at 21s. $1\frac{1}{4}$d. Sterling profit per cent.

9. Suppose all things as before, but that the advice from Bourdeaux had been, that bills on Cadiz were up at 83 sols per piastre; how ought the draught and remittance to be made?

Answer, Draw on Bourdeaux per 786 c. 53 sols. $1\frac{1}{5}$ den. and gain 9s. $1\frac{1}{4}$d. Sterling per cent.

10. Suppose bills on Lisbon sell at Cadiz for 165 merv. per crusado of 400 reas, when Edinburgh exchanges with Cadiz at 3s. 6d. per piastre, and with Lisbon at 5s. 4d. per millrea; what is the arbitrated price of the crusado at Cadiz, and how ought Edinburgh to draw and remit L. 100 Sterling?

Answer, Arbitrated par $165\frac{83}{101}$; wherefore draw on Lisbon per 937 crusadoes 200 reas, and remit to Cadiz at 9s. $6\frac{1}{2}$d. per cent. profit.

11. But if the advice had been, that bills on Lisbon sell at 167 merv. per crusado, then draw on Cadiz and remit to Lisbon, at 14s. 6d. per cent. profit.

12. Suppose the exchange between London and Copenhagen at 4s. 6d. per rixdollar, and with Dantzic at $13\frac{1}{2}$ florins per pound Sterling; what ought to be the arbitrated par between Dantzic and Copenhagen, and how ought L. 100 to be drawn and remitted, supposing the price of the rixdollar at Dantzic 3 florins?

Answer, Arbitrated par $3\frac{3}{80}$ florins per rixdollar. Draw on Copenhagen and order a remittance from Dantzic; profit 24s. $8\frac{1}{2}$d. per cent.

13. But had the advice been, that bills on Copenhagen were at $3\frac{1}{8}$, in what manner ought the negotiation to be?

Answer, Draw on Dantzic per 1350 florins, and remit to Copenhagen 432 rixdollars, at $2\frac{4}{5}$ per cent. profit.

SECT. II.

OF COMPOUND ARBITRATION.

HERE the course of exchange between three, four, or more places is given, to find how much a remittance passing through them all will amount to at the last place, or to find the political par between the first place and the last; for which observe the following rules.

RULES.

I. Distinguish the given courses into antecedents and consequents, placing the antecedents in one column on the left, and the consequents in another on the right, by way of an equation.

II. The first antecedent and the last consequent to which an antecedent is required must always be synonimous.

III. The second antecedent must always be synonimous with the first consequent, and the third antecedent with the second consequent, &c. throughout the whole.

IV. If any of the numbers be mixed, exterminate the fraction by multiplying both sides of the equation into the denominator of the fraction.

V. Equal numbers in both columns may be exterminated, and commensurable ones abridged, till they are all in the lowest terms possible.

VI. The continual product of the consequents, divided by the continual product of the antecedents, will quote the answer.

EXAMPLES.

1. London remits to Spain L. 1000, via Holland, at 36s. Flemish banco per pound Sterling; thence, via Paris, at 54 grotes per ecu; thence, via Venice, at 100 crowns per 60 ducats; and thence to Spain, at 360 mervadies per ducat: how many piastres of 272 mervadies will be received in Spain, and how much Sterling will they get credit for, the exchange at $42\frac{1}{2}$d. per piastre?

Antecedents.			Abridged.		
L. 1 Sterling	=	432 grotes	1	=	1
54 grotes	=	1 crown	1	=	1
100 crowns	=	60 ducats	1	=	3
1 ducat	=	360 mervadies	1	=	36
272 mervadies	=	1 piastre	17	=	1
How many piastres	=	L. 1000 Sterling?			1000

$$\text{Then } 3 \times 36 \times 1000 = 108000$$

And $\frac{108000}{17} = 6352$ piast. 7 rials 18 merv.

Lastly, $6352.94117 =$ L. 1124 19 $11\frac{1}{4}$.

2. Amsterdam remits to London 4000 guilders, via Paris, at 55 grotes per crown; thence, via Venice, at 100 crowns per 60 ducats; thence, via Hamburgh, at 100 grotes per ducat; thence, via Lisbon, at 50 grotes per crusado of 400 reas; and thence to Lon-

don, at 64d. Sterling per millrea; how much Sterling will be received at London, and what is the difference in point of gain between this and a direct remittance, supposing Amsterdam could have purchased a London bill at 36s. 10d. Flemish banco per pound Sterling, and what is the political par between London and Amsterdam? *Answer*, Received at London L. 372 : 7 : 4, which is upon the whole L. 10 : 7 : 6 better than a direct remittance. The political par is 35s. 10d. nearly.

Note, For the last query, the Sterling found will be to the Flemish given as L. 1 to the political par Flemish.

3. London has 2720 piastres at Leghorn, which can be sold by drawing directly at 50d. per piastre; but to try what a circulation of them might produce, orders them to be remitted to Venice, at 94 piastres per 100 ducats; thence to Cadiz, at 320 mervadies per ducat; thence to Lisbon, at 630 reas per piastre of 272 mervadies; thence to Amsterdam, at 50 grotes per crusade of 400 reas; thence to Paris, at 56 grotes per crown, and thence to London, at $31\frac{1}{7}$d. per crown: how much was received at London, what is the arbitrated price between London and Leghorn, and what was gained by the circulation, without reckoning charges?

Answer, Arbitrated price $55\frac{5}{84}$; the circulated remittance amounts to L. 625, and the direct to L. 566 : 13 : 4; so that the difference is L. 58 : 6 : 8, or $10\frac{5}{17}$ per cent.

4. Paris remits to Amsterdam 9100 crowns, first to London at 30d. per ecu; thence to Rome, at 65d. per stamped crown; thence to Venice, at 100 stamped crowns per 140 ducats banco; thence to Leghorn, at 100 ducats banco per 100 piastres; and thence to Amsterdam, at 88 grotes per piastre: how many guilders banco will be received at Amsterdam, exclusive of charges, and how many crowns will be remitted back to Paris, at 54 grotes per crown?

Answer, 12936 guilders, or $9582\frac{2}{9}$ crowns.

Note, By this rule also are the weights and measures in different countries readily equated.

EXAMPLES.

1. If 6 yards of Hamburgh measure 5 in Holland, 7 in Holland measure 4 in France, and 5 in France measure 5 in England; how many yards English will measure 1176 yards of Hamburgh; and what will they amount to at L. 3 : 13 : 4 for 5 yards English?

Answer, 560 yards = L. $410\frac{2}{3}$.

2. If 82 bushels at London measure 27 muddles at Amsterdam, and 27 muddles at Amsterdam measure 10 setiers at Paris, 38 setiers measure 65 veertels at Antwerp, 65 veertels measure 104 hanegas at Cadiz, and 52 hanegas measure 216 alquiers at Lisbon; how many alquiers will measure 410 bushels at London?

Answer, $535\frac{5}{17}$.

By means of the following tables, a comparison between weights and measures is made by a single proportion.

TABLE I. OF WEIGHTS.

100 ℔. of England are equal to ℔.	oz.	100 ℔. of Amsterdam are equal to ℔.	oz.	At
100	0	109	8	London
91	8	100	0	Amsterdam
96	8	105	8	Antwerp
88	0	96	4	Rouen
106	0	116	0	Lyons
90	9	96	0	Rochelle
107	11	118	0	Toulouse
113	0	123	8	Marseilles
81	7	89	0	Geneva
93	5	102	0	Hamburgh
89	7	98	0	Francfort
96	1	105	0	Leipsic
137	4	150	0	Genoa
132	11	145	0	Leghorn
153	11	168	0	Milan
152	0	166	0	Venice
154	10	169	0	Naples
97	0	106	0	Seville
97	0	106	0	Cadiz
104	13	114	8	Portugal
96	5	105	4	Liege
112	$0\frac{2}{3}$	123	$0\frac{1}{12}$	Russia
107	$0\frac{1}{24}$	117	0	Sweden
89	$0\frac{1}{2}$	97	13	Denmark

As the weights of Amsterdam, Paris, Bourdeaux, Strasbourg, Besancon, and several other places, have but a very minute difference, they are all comprehended under that of Amsterdam, as that of Nuremberg is under Francfort, and others in like manner.

TABLE II. OF LONG MEASURE.

100 ells or aunes, or 125 yards of England, are equal to	100 aunes of Amsterdam are equal to	At
Aunes.	Aunes.	
100	60	London
$166\frac{2}{3}$	100	Amſterdam
$162\frac{4}{9}$	$98\frac{3}{4}$	Brabant
$97\frac{1}{12}$	$58\frac{1}{2}$	France
$200\frac{1}{24}$	120	Hamburgh
$208\frac{1}{3}$	125	Breſlau in Sileſia
$187\frac{1}{2}$	$112\frac{1}{2}$	Dantzic
$183\frac{1}{4}$	110	Bergue & Drontheim
$194\frac{1}{2}$	$116\frac{2}{3}$	Sweden
$143\frac{1}{7}$	86	St. Gall for linen
$186\frac{2}{3}$	112	St. Gall for cloth
100	60	Geneva
$164\frac{1}{2}$	$98\frac{3}{4}$	Bruſſels, Antwerp
Canes.	Canes.	
58	35	Marſeilles
$62\frac{3}{5}$	$37\frac{1}{2}$	Toulouſe
$50\frac{13}{25}$	$30\frac{1}{2}$	Genoa
$55\frac{1}{3}$	33	Rome
Varas.	Varas.	
$133\frac{1}{2}$	80	Spain
$101\frac{2}{3}$	61	Portugal
Cavidos.	Cavidos.	
$166\frac{2}{3}$	100	Portugal
Braſſes.	Braſſes.	
$170\frac{1}{5}$	102	Venice
$174\frac{1}{2}$	$105\frac{1}{4}$	Bergamo
$194\frac{1}{7}$	117	Florence, Leghorn, Lucca
$214\frac{1}{6}$	$128\frac{1}{4}$	Milan
Arſheens.	Arſheens.	
$178\frac{4}{7}$	$297\frac{5}{7}$	Peterſburgh

The aunes or ells of Haerlem, Leyden, the Hague, Rotterdam, and other places of Holland, alſo that of Nuremberg, are all equal to that of Amſterdam, and comprehended under it; the aune of Oſnaburgh is equal to that of France; and the aunes of Berne, Baſil, Francfort, and Leipſic, are equal to that of Hamburgh.

TABLE III. OF CORN MEASURE.

$10\frac{1}{4}$ quarters or 82 Wincheſter buſhels of England, or 1 laſt of Amſterdam, make, at

Aiguillon, 41 ſacks
Albi, 25 ſertiers
Alicant, 12 cahizes
Alkmaar, 35 ſacks
Amersfort, 16 muddes
Amſterdam, 27 muddles, or 1 laſt
Antwerp, $32\frac{1}{2}$ veertels
Arles, 49 ſetiers
Bayonne, 36 ſacks
Beaucaire, 28 ſetiers
Beaumont, 38 ſacks
Bergen-op-zoom, 63 ſiſters
Bois-le-Duc, $20\frac{1}{2}$ mouwers
Bommel, 18 muddes
Bourdeaux, 38 boiſſeux
Breda, 33 veertels
Bruges, $17\frac{1}{2}$ hoedts
Bruſſels, 25 ſacks
Bueren, 21 muddes
Cadillac, $33\frac{1}{3}$ ſacks
Cadiz, 52 hanegas
Cahors, 100 cartes
Campen, $24\frac{1}{2}$ muddes
Carcaſſone, 35 ſetiers
Clariac, $34\frac{1}{2}$ ſacks
Cleves, $16\frac{1}{4}$ mouwers
Condom, 41 ſacks
Coningſberg, 1 laſt
Copenhagen, 42 tuns
Dantzic, 1 laſt
Delft, 29 ſacks
Deventer, 36 muddes
Doeſbourg, 22 mouwers
Dort, 24 ſacks
Dunkirk, 18 razieres
Edam, 27 muddes
Elbing, 1 laſt
Embden, $15\frac{1}{4}$ tuns
Erfelſteyn, 21 muddes
Francfort, 27 malders
Ghent, 56 halſters
Genoa, 25 mines
Gimond, 20 ſacks
Graveline, 22 razieres
Haerlem, 38 ſacks
Hamburgh, $\frac{12}{13}$ of a laſt
Heuſden, $17\frac{1}{4}$ muddes
Hoorn, 44 ſacks
Ireland, 38 buſhels
La Brille, 40 ſacks
La Reole, 30 ſacks
Lavour, 21 ſetiers
Leyden, 44 ſacks
Libourne, 35 ſacks
Liege, 96 ſetiers
Liſle, 38 razieres
Liſbon, 216 alquiers
Leghorn, 40 ſacks
Louvain, 27 muddes
Lubeck, 95 ſchepels
Lyons, $14\frac{1}{4}$ anees
Middlebourgh, $41\frac{1}{2}$ ſacks
Montfort, 21 muddes
Muyden, 44 ſacks
Naerden, 44 ſacks
Nerac, $33\frac{1}{3}$ ſacks
Nieuport, $17\frac{1}{2}$ razieres
Oudewater, 21 muddes
Paris, 19 ſetiers
Porto Port, 180 alquiers
Purmerent, 27 muddes
Rabaſtens, 17 ſetiers
Rhenen, 20 muddes
Ruremond, 68 ſchepels
Riga, 46 loopens
Rotterdam, 29 ſacks
St. Giles, 40 charges

10$\frac{1}{4}$ quarters or 82 Winchester bushels of England, or 1 last of Amsterdam, make, at

St. Omer, 22$\frac{1}{2}$ razieres
St. Vallery, 19 setiers
Saumur, 19 setiers
Stenbergen, 35 veertels
Stockholm, 23 tuns
Terveer, 39 sacks
Thiel, 21 muddes
Toulouse, 26 setiers
Tongres, 15 muddes
Tonningen, 24 tuns
Venloo, 21$\frac{1}{4}$ mouwers
Vianen, 20 muddes
Utrecht, 25 muddes
Zuirckzee, 40 sacks

The Use of the Tables.

EXAMPLES.

1. What are 800 lb. of Lyons equal to in England?
   Say, from Table I. As 106 lb. of Lyons : 100 lb. of England :: 800 lb. of Lyons : 754$\frac{1}{2}$ of England.
2. How many aunes of Amsterdam are equal to 500 aunes of Dantzic?
   Say, from Table II. As 112$\frac{1}{2}$ aunes of Dantzic : 100 aunes of Amsterdam :: 500 aunes of Dantzic : 444$\frac{1}{4}$ aunes of Amsterdam.
3. How many setiers will 400 bushels of England make at Paris?
   Say, from Table III. As 82 bushels of England : 19 setiers of Paris :: 400 bushels of England : 92$\frac{1}{2}$ setiers of Paris.
4. How many Winchester bushels will measure 475 sacks at Leghorn?
5. How many yards of England will measure 5745 arsheens of Russia?
6. How many pounds English will equiponderate 3780 of Toulouse?

CHAP. XVIII.

INTEREST.

BY this rule is computed the allowance or gratuity made by the borrower to the lender of money; which is for the most part paid annually, if the sum lent lie in the borrower's hands; or taken up with the principal, if the money is taken up at a shorter date; or, lastly, the interest may also become a principal, if it continue in the borrower's hands beyond a year: and therefore this rule will admit of two sections, and each of the sections will also admit of some varieties.

SECT. I.

OF SIMPLE INTEREST.

When the interest at no time becomes a principal.

CASE I.

To find the interest for any given sum, at any rate per cent. and for any time.

I. If the time be any number of complete years, any aliquot part of a year, or both, compute by either of the following methods:

As 100 to the product of the rate and time, so is the principal to the interest: Or, as L. 1 to the amount of L. 1, at the rate, and for the time given, so is, &c. Or, Multiply the rate by the time, and compute by practice for the product, as in commission.

EXAMPLES.

1. What is the interest of L. 578 : 19 : 0 for $4\frac{1}{2}$ years, at 5 per cent.?

578.95	578.95	
= $4\frac{1}{2}$	4.5	
2315.80	289745	
289.475	231580	
2605.275	2605.275	
5	.05	
100) .13026.375	130.26375 = L. 130 5 $3\frac{1}{4}$	
.130.26375		

$4\frac{1}{2}$	5) 578.95	
5	8) 115.79	for 20 per cent.
$22\frac{1}{2}$	14.47375	for $2\frac{1}{2}$
	130.26375	for $22\frac{1}{2}$

2. What is the interest of L. 976 : 18 : 10 for $8\frac{1}{3}$ years, at 4 per cent. *Answer*, L. 325 : 12 : $11\frac{1}{3}$.

3. What is the interest of L. 478 : 18 : 4 for $3\frac{1}{4}$ years, at 4 per cent.? *Answer*, L. 62 : 5 : $2\frac{1}{4}$.

4. What is the interest of L. 594 : 1 : 2, at $3\frac{1}{2}$ per cent. for 5 years? *Answer*, L. 104 : 1 : 1.

5. What is the interest of L. 847 : 19 : 7, at 6 per cent. for $3\frac{1}{2}$ years? *Answer*, L. 178 : 1 : 6.

6. What is the interest of L. 974 : 19 : 11, at $4\frac{3}{4}$ per cent. for 6 years? *Answer*, L. 277 : 17 : $5\frac{3}{4}$.

II. If the time be any number of days, less than a year, multiply the given sum by the number of days, and divide the product by 7300, and the quotient will be the interest at 5 per cent. which may be increased or diminished to any other rate, by multiplying $\frac{1}{5}$ of the interest found into the given rate. Or, by multiplying the quotient into the given rate, and dividing by 5.

Note, 7300 becomes a constant divisor, because the interest of L. 100 for 73 days, or of L. 73 for 100 days, at 5 per cent. is just one pound.

EXAMPLES.

1. What is the interest of L. 378 : 14 : 0, from the 9th of November to the 16th of March, at 5 per cent.?

Nov. 21.	378.7	378.7
Dec. 31.	127	.3479 = $\frac{127}{365}$
Jan. 31.		
Feb. 28.	26509	1.3174973
Mar. 16.	45444	5
127.	73) 480.949	6.5874865 as before.
	6.588 = L. 6 11 9	

Suppose the rate had been at $4\frac{1}{2}$ per cent.

5) 6.588	10) 6.588
1.3176	6588 to be deduced.
$4\frac{1}{2}$	
5.2704	5.9292 as before.
6588	
5.9292	

2. What is the interest of L. 349 : 12 : 6 from 15th of May to the 17th of August, the interest at 4 per cent.?

Answer, L. 3 : 12 : $0\frac{1}{4}$.

3. What is the interest of L. 55 : 19 : 11, from the 17th of August to the 1st of January, at $4\frac{1}{2}$ per cent.?

Answer, 18s. 11d. nearly.

4. What is the interest of L. 111 : 11 : 10, from the 1st of January to the 11th of May in leap year, interest $4\frac{1}{4}$ per cent.?

Answer, L. 1 : 14 : 0.

5. What is the interest of L. 976 : 13 : 4, from the 15th of May 1770 to 28th of August 1771, interest 5 per cent.?

Answer, L. 62 : 17 : $7\frac{1}{2}$.

6. What is the interest of L. 597 : 10 : 0 bank stock, from the 11th of April 1770 to the 29th of September 1772, at 6 per cent. per annum? *Answer*, L. 88 : 11 : $10\frac{1}{2}$.

If several partial payments are made at different periods on the same sum, or when several sums are drawn and remitted at different dates, multiply the several balances into the number of days they are at interest, and divide the sum of the products by 7300 for the total interest at 5 per cent.

EXAMPLES.

1. Lent J. Ferguson, per bill on demand, dated 1st of June 1787, L. 1000, of which I received back the 19th of August L. 200,

on the 15th of October L. 300, on the 11th of December L. 200, on the 17th of February 1788 L. 100, and on the first of June L. 200; how much interest is due on the bill, reckoning at 5 per cent.?

1787. June 1.	Principal, per bill on demand, due by J. F.	1000	79	79000
Aug. 19.	Received in part	200		
	Balance	800	57	45600
Oct. 15.	Received in part	300		
	Balance	500	57	28500
Dec. 11.	Received in part	200		
1788.	Balance	300	68	20400
Feb. 17.	Received in part	100		
	Balance	200	105	21000
June 1.	Received in full of principal	200		
		0	73	1945.00

Answer, L. 26 12 10¼

2. Borrowed from J. Crosse, the 11th of November 1787, L. 800, which I returned in the following partial payments, at their respective dates,

15th of January 1788	L. 250	0	0
17th of April	150	0	0
18th of June	100	0	0
19th of August	125	0	0
25th of September	175	0	0

How stands it between us with respect to interest, reckoning at 5 per cent.?

Answer, L. 20 : 19 : 3.

3. Upon my cash account with the Royal bank, my year's negotiations were as follow;

Jan.	10.	drawn L.	200	Jan.	25.	remitted L.	150
	27.		300	Feb.	1.		300
Feb.	15.		150	Mar.	6.		200
	25.		160		20.		300
Mar.	11.		370	April	15.		500
April	10.		500	May	2.		150
	29.		200	June	3.		700
May	18.		300		15.		300
	27.		500	July	18.		750
June	12.		300	Aug.	20.		300
	24.		450	Sept.	30.		500
July	7.		350	Oct.	14.		550
	28.		250		15.		1100
Aug.	19.		150		20.		100
	29.		320	Nov.	15.		500
Sept.	17.		200	Dec.	20.		450
Oct.	12.		500		30.		400
	24.		500				
Nov.	11.		500				
	27.		200				
Dec.	12.		300				
	24.		200				
	28.		200				

Jan. 1. to be settled.

What balance is due on the account of principal and interest, reckoning at 5 per cent. per annum?

Answer, Balance due me L. 150 : 0 : 0 principal.
Interest due the bank 18 : 5 : 0.

4. Upon my cash account with the bank of Scotland, I negotiated as under:

Jan.	8.	drawn L.	500	Jan.	20.	remitted L.	380
	24.		500		27.		820
	30.		350	Feb.	4.		200
Feb.	8.		420		12.		500
	18.		300		20.		80
	23.		400		27.		500
Mar.	3.		500	Mar.	8.		300
	12.		400		18.		500
	20.		180		24.		220
	28.		420		31.		300
April	4.		370	April	9.		500
	13.		480		16.		370
	20.		510		24.		600
	28.		380		30.		420

May	8.	drawn	L. 800	May	12.	remitted	L. 200
	14.		400		18.		300
	16.		300		20.		300
	24.		500		30.		600
	28.		500		31.		700
June	11.		300	June	14.		500
	20.		400		24.		300
	28.		500		30.		500
July	2.		350	July	10.		300
	4.		470		15.		200
	6.		100		20.		400
	24.		700		30.		800
Aug.	5.		200	Aug.	9.		300
	15.		350		20.		400
	23.		480		25.		500
	28.		500		30.		480
Sept.	8.		200	Sept.	18.		500
	12.		370		24.		600
	15.		430		30.		300
Oct.	4.		800	Oct.	30.		1000
	13.		700	Nov.	8.		200
	19.		400		12.		300
	24.		200		16.		500
Nov.	18.		500		26.		400
	24.		500		30.		500
Dec.	3.		820	Dec.	5.		400
	8.		720		13.		480
	17.		600		24.		590
	28.		800		30.		700
	29.		500		31.		750

What balance is due of principal and interest, at Jan. 8. 1789, reckoning 5 per cent. in favour of the bank, and 4 per cent. for the balances in my favour?

Answer, Principal due the bank	L.	410	0	0
Interest ditto		25	7	$2\frac{1}{4}$
Principal and interest	L.	435	7	$2\frac{1}{4}$

III. If the interval between the partial payments be greater than a year, the usual method is to add the interest, at the date of each payment, to the principal, and deduct the payment from that amount, as in the following specimen.

EXAMPLES.

1. Borrowed on bond, dated the 15th of May 1784, the sum of L. 1000, of which paid at different dates as under, viz.

25th of June 1785	L. 250	0	0
24th of October 1786	150	0	0
11th of November 1787	300	0	0
20th of December 1788	150	0	0
15th of May 1789	250	0	0

How stands it between the lender and me at the 15th of May 1789?

1784.	May 15.	Principal per bond	L. 1000
		1 year and 41 days interest	55.616
		Amount	1055.616
1785.	June 25.	Paid in part	250
		Balance	805.616
		1 year and 121 days interest	53.634
		Amount	859.250
1786.	Oct. 24.	Paid in part	150
		Balance	709.250
		1 year and 18 days interest	37.211
		Amount	746.461
1787.	Nov. 11.	Paid in part	300
		Balance	446.461
		1 leap year and 39 days interest	24.769
		Amount	471.230
1788.	Dec. 20.	Paid in part	150
		Balance	321.230
		146 days interest	6.424
		Amount	327.654
1789.	May 15.	Paid in part	250
		Balance	77.654

Equal to L. 77 : 13 : 1.

2. Lent on bond, the 1st of January 1778, the sum of L. 1500, to bear legal interest, and received thereof the following payments, viz.

15th of May 1780	L.	300	0	0
11th of November 1781		200	0	0
15th of January 1782		150	0	0
15th of May 1783		150	0	0
20th of November 1784		200	0	0
29th of May 1785		100	0	0
11th of November 1786		250	0	0
1st of December 1787		100	0	0

I am to be paid up the 20th of December; required a true state of the case at that date?

Answer, Balance L. 625 : 3 : 8½.

3. Purchased an estate the 15th of May 1783, with the burden of L. 1000, borrowed upon it the 11th of November 1777, of which no part had been paid of principal or interest. I made the following payments, viz.

Paid 1st of June 1783	L.	500	0	0
11th of November 1784		300	0	0
12th of May 1786		300	0	0

What had I to pay 15th of June 1787 to retire the bond?

Answer, L. 289 : 3 : 8½.

CASE II.

To find the present value of any sum, due any time hence, at any rate of interest.

Find the amount of L. 100 at the rate and for the time given, or find the amount of L. 1 at the rate and for the time given; then will the given sum be to its present value as the first amount to L. 100, or as the second amount to L. 1.

EXAMPLES.

1. What ready money will pay a debt, due 15 years hence, of L. 1198 : 9 : 2, interest being reckoned at 5 per cent.?

$$15 \times 5 + 100 = \underline{175 \ : 100} \ ::$$

$$7 \ : \ 4 \ :: \ 1198 \ \ 9 \ \ 2$$

$$4$$

$$7\,)\,4793 \ \ 16 \ \ 8$$

Answer, L. 684 16 8

$$1.75 \ : \ 1 \ :: \ 1198.458\dot{3}$$

$$1$$

$$5\,)\,1198458\dot{3}$$

$$5\,)\,239691\dot{6}$$

$$7\,)\,47938\dot{3}$$

$$684.8\dot{3}$$

2. What ready money will take up a bond for L. 1500, payable 18 months hence, bearing interest at $4\frac{1}{2}$ per cent. per annum?
Answer, L. 1405 : 3 : $0\frac{1}{2}$.

3. What ready money will take up a bill, due at 395 days distance, contents L. 375? *Answer*, L. 355 : 15 : $0\frac{1}{4}$.

4. What ready money will discount a bill due in 97 days, for L. 157 : 10 : 0? *Answer*, L. 155 : 8 : $8\frac{1}{2}$.

Note, When the rate of interest is not mentioned, 5 per cent. the legal interest, is always understood.

PRACTICAL DISCOUNT.

BANKERS, and others, who keep money for the purpose of discounting bills, do not follow the above plan of operation, as being no better than lending money upon interest; but they consider the sum to be paid in the bill, bond, &c. as a principal, find the interest thereof, for the time to run, adding three days of grace; they deduce the interest so found from the content of the bill, and pay the balance to him for whom the discount is made. Some likewise, charge $\frac{1}{4}$, $\frac{1}{2}$, nay, perhaps, 1 per cent. for trouble, which, before deduction, is added to the interest.

EXAMPLES.

1. A bill is presented the 5th of May for discount, of L. 176, 18s. payable 27th of July; how much money does the porteur receive, after deduction of interest, and ½ per cent. commission?

```
200 ) 176.9
        86   days.
   ─────────
 73 ) 152.134
   ─────────
      2.084   interest.
       8845   commission.
    ─────────
     2.9685   sum to be deducted.
    ─────────
   173.9315   paid the porteur of the bill.
```

2. A bill for L. 547 : 18 : 6, payable the 24th of August, is discounted 1st of June, at ¾ per cent. commission; how much money did the porteur receive? *Answer*, L. 537 : 5 : 8¼.

3. A bill for L. 345 : 15 : 8, payable 19th of September, is discounted the 15th of June at ¼ per cent. commission; how much money did the porteur receive? *Answer*, L. 340 : 4 : 7.

4. On the 15th of May, discounted A. B.'s bill, due the 19th of July, for L. 417 : 17 : 8, for which received 1 per cent commission; how much money did I pay? *Answer*, L. 409 : 16 : 3¼.

CASE III.

To find the rate of interest, at which any amount flowed from any principal, in any time.

As the product of the time and principal to the whole interest, so is 100 to the rate.

EXAMPLES.

1. At what rate of interest will L. 200 become a stock of L. 300 in 5 years time?

$5 \times 200 = 1000 : 100 :: 100 : 10$ per cent. *Answer*.

2. Discounted A. B.'s bill the 15 of July, for L. 500, due the 13th of August, for which received $\frac{1}{2}$ per cent. commission; how much money did I pay, and at what rate per cent. did I lend my money?

Answer, Paid L. 495 : 6 : $2\frac{1}{4}$ lent at 10.8 per cent.

3. Discounted A. B.'s bill the 14th of July, for L. 600, due the 11th of August, at $\frac{1}{2}$ per cent commission; how much money did I pay, and at what rate per cent. did I lend my money?

Answer, Paid L. 594 : 9 : $0\frac{1}{4}$ lent at 10.885 per cent.

CASE IV.

To find the time any sum was at interest, when the principal rate of interest, and amount are given.

As 1 year's interest of the given principal is to 1 year, so is the whole interest to the time required.

EXAMPLES.

1. At 10 per cent. interest, L. 200, in a certain time became L. 300; required the time? 20 : 1 :: 100 : 5 years, *Answer*.

2. In what time will L. 584 : 6 : 8 amount to L. 682 : 18 : $9\frac{1}{2}$ at $4\frac{1}{2}$ per cent.? *Answer*, 3 years 273 days.

3. In what time will L. 275 gain L. 8 : 5 interest, at 5 per cent. per annum? *Answer*, 219 days.

SECT. II.

Of Compound Interest.

If a man lend L. 100 for one year, and exact the payment of principal and interest when due, he will receive L. 105; if he lends out this money to the first holder, or any other, for another year precisely, he will receive back L. 110 : 5, &c. Hence arises compound interest, which, though it is prohibited by the laws, every banking company exact in effect, as they take particular care, that no cash account shall remain unsettled beyond a year. This section also admits of four varieties, which we shall not illustrate with many examples, as all questions which occur in this rule are briefly answered by the Tables subjoined to Annuities, which are constructed by the rules here laid down; as also by logarithms, of which examples are given.

CASE I.

To find the amount of intereſt, of any principal ſum, at any rate of intereſt, and for any time.

Multiply the amount of L. 1 for a year ſo often into itſelf as are years propoſed, abating one, and the laſt product, multiplied by the principal, gives the amount; from which deduce the principal, for the intereſt.

If there are alſo days beyond complete years, add to the amount, or principal and compound intereſt formerly found, the ſimple intereſt of that amount for them.

EXAMPLES.

1. What is the intereſt and amount of L. 700 for $3\frac{1}{2}$ years, at 5 per cent. per annum?

	Or practically,
1.05	20) 700
1.05	35
1.1025	20) 735 1ſt year.
1.05	36.75
1.157625	20) 771.75 2d year.
700	38.5875
40) 810.3375 amount at three years.	40) 810.3375
20.2584375	20.2584375
830.5959375 amount at $3\frac{1}{2}$ years.	830.5959375

2. What is the amount of L. 780, forborn 6 years, at 5 per cent. per annum, compound intereſt? *Anſwer*, L. 1045 : 5 : 6.

3. What is the amount of L. 960 at 5 per cent. per annum, compound intereſt, from the 11th of November 1773 to the 25th of February 1779? *Anſwer*, L. 1243 : 0 : $5\frac{1}{4}$.

By this caſe is computed Table I. which gives the amount of L. 1, at 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, and 5 per cent. for any ſeries of years, from 1 to 50; by the help of which the amount of any ſum, at any of thoſe rates, for any number of years in the Table is already found, by multiplying the amount of L. 1 found oppoſite the number of years in the column under the given rate by the given principal.

EXAMPLES.

1. What is the amount of L. 500, forborn 20 years at 5 per cent. compound interest?

Opposite 20, and under 5 per cent. we have

$$\begin{array}{lr} & 2.6532977 \\ \text{Multiply by} & 500 \\ \hline & 1326.64885 \end{array}$$ *Answer.*

2. What is the amount of L. 800, forborn 30 years, at $4\frac{1}{2}$ per cent. compound interest? *Answer*, L. 2996 : 5 1.

3. What is the amount of L. 1000, forborn $40\frac{1}{2}$ years, at 4 per cent. compound interest? *Answer*, L. 4897 : 0 : 9.

CASE II.

When the amount, rate, and time are given, to find the principal, or present worth.

Find the amount of L. 1 for the given time and rate, by which divide the given amount, and the quotient is the answer.

EXAMPLES.

1. What is the present worth of L. 810 : 6 : 9, due 3 years hence, discounting at 5 per cent. compound interest?

$$\begin{array}{r} 1.05 \\ 1.05 \\ \hline 1.1025 \\ 1.05 \\ \hline 1.157625 \end{array} \Big) \; \begin{array}{l} 810.3375 \\ \hline \text{L. } 700 \end{array}$$ *Answer.*

2. What ready money will pay a debt of L. 562.432, due 3 years hence, discounting at the rate of 4 per cent. compound interest? *Answer*, L. 500.

By this Case is Table II. constructed, to find the value of L. 1, discounting at 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, and 5 per cent. for any number of years from 1 to 50, and is used by multiplying the tabular number into the given amount for the present value. The same answer may also be found by dividing the given amount by the tabular number found in Table I. as in the above example.

EXAMPLES.

1. What ready money will pay a debt of L. 1000, due 20 years hence, discounting interest at 5 per cent. per anuum, compound interest?

Opposite 20 years, and under 5 per cent. we find in Table II.

	.3768895
Multiply by	1000
	376.8895 present value.

Or thus:

The amount of L. 1 per Table I. at 20 years	= 2.6532977)	1000.000000
		376.8895 as before.

2. What ready money will take up a debt of L. 2000, to be paid 30 years hence, at 4 per cent.? *Answer*, L. 616 : 12 : 9.

3. What ready money will take up a debt of L. 3000, to be paid 25 years hence, at $4\frac{1}{2}$ per cent.? *Answer*, L. 998 : 3 : 10.

CASE III.

When principal, amount, and time are given, to find the rate.

Divide the amount by the principal, and the quotient will be the amount of L. 1, which will be found on Table I. even with the given time.

EXAMPLES.

1, At what rate of interest will L. 376.8895 amount to L. 1000, forborn 20 years?

376.8895) 1000.000

2.6532977 found under 5 per cent.

2. At what rate of interest will L. 500 amount to L. 1039.4641 in 15 years?

CASE IV.

When we have the principal, amount, and rate given, to find the time.

Divide the amount by the principal, and the quotient will be the amount of L. 1 which find under the given rate, and on the side, in a line with it, you have the time in Table I.

EXAMPEES.

1. In what time will L. 376.8895 become L. 1000, at 5 per cent. compound interest?

376.8895) 1000.0000

2.6532977 found opposite 20 years.

2. In what time will L. 500 become L. 1039.4641 at 5 per cent. compound interest?

More examples for practice in all the foregoing cases.

1. What will L. 40 amount to, forborn 400 years, at 5 per cent. compound interest? *Answer*, L. 14882516363 : 7 : $10\frac{4}{5}$.

2. A. owes several mortgages at distant periods, viz. L. 500 at 5 years, L. 600 at 10 years, L. 1000 at 7 years, and L. 900 at 4 years; but having got a considerable legacy, is resolved to pay them off, on discount at 5 per cent. compound interest; how much will do it? *Answer*, L. 2211 : 4 : 6.

Of Compound Interest by Logarithms.

CASE I.

The principal rate and time given to find the amount.

Multiply the logarithm of the rate + 1 into the time, and add the logarithm of the product to the logarithm of the principal for answer.

EXAMPLES.

1. What is the amount of L. 700 for three years at 5 per cent. compound interest?

$$\begin{array}{rr} 1.05 = & 0.02119 \\ & 3 \\ \hline & 0.06357 \\ \text{Principal } 700 = & 2.84510 \\ \hline \text{Amount L. } 810.3375 = & 2.90867 \end{array}$$

2. What is the amount of L. 800 forborn 30 years at $4\frac{1}{2}$ per cent. compound interest?

$$\begin{array}{rr} 1.045 = & 0.01912 \\ & 30 \\ \hline & 0.57360 \\ 800 = & 2.90309 \\ \hline \text{Amount L. } 2997 = & 3.47669 \end{array}$$

3. Suppose in the way of a tontine L. 500 were subscribed for, to lie at 5 per cent. compound interest, till there should be only one survivor of the subscribers, who should be entitled to call up the money. Suppose also by this time 80 years had elapsed; queritur how much would the survivor be entitled to receive?

$$
\begin{array}{rcl}
1.05 & = & 0.02119 \\
 & & 80 \\
\hline
 & & 1.69520 \\
L.\ 500 & = & 2.69897 \\
\hline
L.\ 14830 \text{ nearly} & = & 4.39417
\end{array}
$$

CASE II.

The amount, rate, and time given, to find the principal or present worth.

From the logarithm of the amount, subtract the logarithm of the amount of L. 1, at the rate and for the time given, and the remainder will be the present value.

EXAMPLES.

1. Required the present worth of L. 810 : 6 : 9 due 3 years hence, discounting at the rate of 5 per cent. compound interest?

$$
\begin{array}{rcl}
810.3375 & = & 2.90867 \\
\text{Amount of L. 1 at 3 years} & = & 0.06357 \\
\hline
\text{Present value L. } 700 & = & 2.84510
\end{array}
$$

2. Suppose a bond for L. 2997 due 30 years hence, were to be taken up, discounting compound interest at $4\frac{1}{2}$ per cent.; how much ready money will it require?

$$
\begin{array}{rcl}
2997 & = & 3.47669 \\
\text{Amount of L. 1 at 30 years} & & 0.57360 \\
\hline
\text{Principal L. } 800 & = & 2.90309
\end{array}
$$

3. Suppose a bond for L. 14.830 to be payable 80 years from this time, discounting compound interest at 5 per cent.; how much ready money will it require?

$$
\begin{array}{rcl}
14.830 & = & 4.39417 \\
\text{Amount of L. 1 at 80 years} & = & 1.69897 \\
\hline
\text{Present worth L. } 500 & = & 2.69520
\end{array}
$$

CASE III.

Principal, amount, and time given to find the rate.

From the logarithm of the amount, ſubtract the logarithm of the principal, and the remainder will be the logarithm of the amount of L. 1, which divided by the time will quote the logarithm of the rate + 1.

EXAMPLES.

1. L. 700 was lent out for three years, when the amount was L. 810 : 6 : 9, at what rate of intereſt was it lent out?

$$
\begin{array}{rcl}
810.3375 & = & 2.90867 \\
700 & = & 2.84510 \\
\hline
 & 3\,) & 0.06357 \\
\hline
1.05 & = & 0.02119
\end{array}
$$

2. A bond of L. 2997 due 30 years hence, was paid with L. 800, at what rate of intereſt was it diſcounted?

$$
\begin{array}{rcl}
2997 & = & 3.47669 \\
800 & = & 2.90309 \\
\hline
 & 30\,) & 0.57360 \\
\hline
1.045 & = & 0.01912
\end{array}
$$

3. L. 14830 at the end of 80 years was called up by the ſurvivor of a tontine ſubſcription, the original ſubſcription was only L. 500; at what rate of compound intereſt was it lent?

$$
\begin{array}{rcl}
14830 & = & 4.39417 \\
500 & = & 2.69897 \\
\hline
 & 80\,) & 1.69520 \\
\hline
1.05 & = & .02119
\end{array}
$$

CASE IV.

Principal, amount, and rate given, to find the time.

From the logarithm of the amount, ſubtract the logarithm of the principal, and the remainder will be the amount of L. 1 at the rate given, which divided by the logarithm of the rate + 1, will quote the time.

EXAMPLES.

1. L. 700 lay at 5 per cent. compound intereſt, till it became L. 810.3375; how long did it continue before it was called up?

$$\begin{array}{lcl} 810.3375 & = & 2.90867 \\ 700 & = & 2.84510 \end{array}$$

Log. of the rate .02119) 0.06357 (3 years. *Anſwer.*
.06357

0

2. L. 800 was put out at $4\frac{1}{2}$ compound intereſt, and lay till it amounted to L. 2297; what was the time?

$$\begin{array}{rcl} 2297 & = & 3.47669 \\ 800 & = & 2.90309 \end{array}$$

.01912) .57360 (30 years. *Anſwer.*
5736

0

CHAP. XIX.

ANNUITIES.

AN annuity is a ſum of money, payable at equal intervals, as a year, half-year, or quarter, to continue for ſome number of years, for life, or for ever.

When an annuity continues unpaid, after it falls due, it is then ſaid to be in arrear; and when the purchaſer, upon paying the price, does not immediately enter upon poſſeſſion, the annuity not commencing till ſome time after, it is, in that caſe, ſaid to be in reverſion.

SECT. I.

OF ANNUITIES FOR A CERTAIN TIME.

PROBLEM I.

Annuity, rate, and time given, to find the amount or ſum of yearly payments and intereſt.

Multiply the given annuity by the amount of L. 1 for a year, to that product add the given annuity, and the ſum is the amount in two years; multiply that ſum by the amount of L. 1 for a year, and to the product add the given annuity, and the ſum is the amount in three years, &c. for any number of years.

EXAMPLES.

1. An annuity of L. 100 payable yearly, is forborn and unſettled till the end of three years, what will then be due, reckoning compound intereſt at 5 per cent. per annum on all the payments then in arrear?

100 1st year.	Or, 100 Annuity 1st year.
1.05	5 Int.
105	105
100	100 An. 2d year,
205 2d year,	205
1.05	10 : 5 Int.
215.25	100 An. 3d year.
100	
315.25 3d year.	315 : 5

By this problem is the amount of L. 1 annuity calculated in Table III. at 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, and 5 per cent. for 50 years; and all questions demanding the amount of an annuity, for any number of years, and at any rate per cent. in the table are answered by multiplying the tabular number under the rate and opposite the time by the annuity.

EXAMPLES.

1. What will L. 20 per annum annuity amount to in 40 years, compound interest being allowed at 5 per cent?

Under 5 per cent. and opposite 40, we find
120.7997742
Multiply by 20 the annuity.

L. 2415.9954840 *Answer.*

2. What will L. 100 amount to, lent out yearly, with the accumulated interest on each L. 100, for 20 years at 5 per cent.?
Answer, L. 3306 : 11 : 11.

PROBLEM II.

Annuity, rate, and time given, to find the present worth, or the sum of money that will purchase the annuity.

Find the amount of the given annuity, by the last problem, at the rate and time given; and then, by Case II. in compound interest, find the present value of that amount.

EXAMPLES.

1. What sum of money will purchase L. 40 annuity, to continue 15 years, at 5 per cent. compound interest?

By Table III. the amount of L. 40 annuity for 15 years is

$$\begin{array}{r} 21.5785636 \\ 40 \\ \hline 2.0789282\ )\ 863.1425440 \text{ amount.} \\ \hline \text{L. } 415 \quad 3 \quad 8\tfrac{1}{2} \end{array}$$

2. What is the present worth of L. 12 annuity, to continue 37 years, at $4\frac{1}{2}$ per cent.? *Answer*, L. 214 : 6 : 11.

By this problem we have the construction of Table IV. which, by multiplying the tabular number into the annuity, gives the present worth in the product: For instance, in the last example, under and even with 15, we have

$$\begin{array}{lr} & 10.3796580 \\ \text{Multiply by} & 40 \\ \hline \text{The present worth, L.} & 415.18632 \end{array}$$

3. What is the present worth of L. 200 annuity, to continue 45 years, at 3 per cent. compound interest? *Answer*, L. 4903 14 10.

4. What is the present value of an estate of L. 300 per annum, to be sold at thirty years purchase, compound interest at 5 per cent.? *Answer*, L. 4611 : 14 : $8\frac{1}{2}$.

PROBLEM III.

The present worth, rate of interest, and time of continuance given, to find the annuity.

Find the present worth of L. 1 annuity, for the rate and time given, by the last problem, by which divide the present worth given.

EXAMPLES.

1. What annuity to continue 5 years, will L. 173.179 purchase, compound interest at 5 per cent. being allowed?

$$5 : 100 :: 100 : 20$$

But $1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 = 1.2762815625$ amount of L. 1 in 5 years.

$$\text{And } \frac{20.0000000000}{1.2762815625} = \frac{\overset{20}{15.6705}}{4.3295}\ ) \ 173.179$$

Answer. L. 40 annuity.

By this problem we have the method of constructing Table V. in which the tabular number answering to the rate and time, multiplied by the sum intended for the purchase, will produce the annuity at once.

2. A gentleman has L. 1000, which he proposes to lay out upon an annuity to continue 20 years; how much will it purchase, allowing compound interest at 5 per cent.?

By Table V. corresponding to the rate and time we find

$$\begin{array}{r} .0802426 \\ 1000 \\ \hline 80.2426 \end{array}$$ *Answer*.

3. A farmer is asked L. 500 down for the benefit of a 19 years lease, which he is willing to convert into an additional rent; how much ought the farmer to be charged for it per annum, compound interest being allowed at 5 per cent*? *Answer*, L. 41 : 7 : $5\frac{1}{2}$.

More examples for exercise in the two last chapters.

1. A pays L. 2000 for an annuity of L. 100 to continue 50 years; B puts L. 2000 out at interest, which he proposes to manage so as to have compound interest for his money: which of them will amount to the greatest sum, at the end of the 50 years, at the rate of 5 per cent. each? *Answer*, B gains L. 2000.

2. A owes B L. 1000, which he agrees to pay by an annuity to continue 10 years; what must he pay per annum as an equivalent for L. 1000 due now, compound interest being allowed at $4\frac{1}{2}$ per cent? *Answer*, L. 126 : 7 : $6\frac{1}{4}$.

* The Tables here alluded to may be seen at the end of the work.

3. Which is most advantageous, a term of 20 years in an estate of L. 309 per annum, or the reversion of the same estate ever after, reckoning compound interest at 5 per cent.?

Answer, The term of 20 years by L. 3919 : 15 : 9.

4. What annuity would be sufficient to pay off a debt of 140 millions in the space of 40 years, at 4 per cent. compound interest?

Answer, L. 7073.276.

SECT. II.

Annuities for ever, or Freehold Estates.

In the annuities for ever, or in fee simple, are to be considered, 1. The annuity, or yearly rent; 2. The price, or present worth; 3. The rate of interest: all which fall under one or other of the following problems.

PROBLEM I.

Annuity or rent and rate of interest given, to find the price.

As the rate of L. 1 to L. 1, so is the rate to the price.

EXAMPLES.

1. The yearly rent of an estate is L. 400; what is it worth in ready money, computing interest at 5 per cent.?

.05 : 1 :: 400 : L. 8000. *Answer*.

2. The yearly rent of an estate is L. 500; what is the value paid down, at $4\frac{1}{2}$ per cent. interest? *Answer*, L. 11111 : 2 : $2\frac{1}{4}$.

PROBLEM II.

Price and rate of interest given, to find the rent or annuity.

As L. 1 to its rate, so the price to its rent.

EXAMPLES.

1. A gentleman purchases an estate for L. 8000, and has 5 per cent. for his money; what is the yearly rent?

1 : .05 :: 8000 ; L. 400 per annum.

2. A gentleman purchases an estate for L. 2000, and has 5 per cent. for his money: what is the yearly rent? *Answer*, L. 100.

PROBLEM III.

Price and rent given, to find the rate of interest.

As the price to the rent, so is L. 100 to the rate.

EXAMPLES.

1. An estate of L. 400 a year is purchased for L. 8000; what rate of interest has the purchaser for his money? *Answer*, 5 per cent.
2. An estate of L. 700 per annum was purchased for L. 19000; what rate of interest has the purchaser for his money?

Answer, $3\frac{2}{8}$ nearly.

PROBLEM IV.

The rate of interest given, to find at how many years purchase an estate may be bought.

Divide L. 100 by the rate, and the quotient gives the years.

EXAMPLE.

A gentleman would purchase an estate to have 4 per cent. for his money; how many years purchase should he offer?

4) 100

25 *Answer*.

PROBLEM V.

The number of years purchase at which an eſtate is bought or ſold being given, to find the rate of intereſt.

Divide 100 by the number of years.

EXAMPLE.

A gentleman gives 25 years purchaſe for an eſtate; what intereſt has he for his money?

25) 100

4 per cent.

SECT. III.

Annuities in Reversion.

PROBLEM I.

The rate of intereſt and the rent of a freehold eſtate in reverſion given, to find the preſent worth of reverſion.

1. Find the value of the eſtate as if poſſeſſion were to be immediate.
2. Find the preſent value of the rent or annuity for all the years prior to the commencement.
3. Subtract the laſt found value from the firſt, and the remainder is the value of the reverſion,

EXAMPLES.

1. A. has the poſſeſſion of an eſtate of L. 500 per annum, to continue 10 years; B. has the reverſion of the ſame eſtate from that time for ever; what is the value of the eſtate, what is the value of 10 years poſſeſſion, and what is the value of the reverſion, reckoning compound intereſt at 5 per cent.

05) 500.00

L. 10000 value of the estate.

$7.7217349 \times 500 = 3860.86745$ value of the possession.

6139.13255 value of the reversion.

2. A. has the possession of an estate of L. 400 per annum to continue seven years, after which B. has the reversion for ever; what is the value of the estate, what is the value of the 7 years possession, and what is the value of the reversion, reckoning compound interest at $3\frac{1}{2}$ per cent.?

Answer,	Value of the estate,	L. $11428 : 11 : 5\frac{1}{2}$
	Value of 7 years possession,	$2445 : 16 : 4\frac{1}{4}$
	Value of the reversion,	$8982 : 15 : 1\frac{1}{4}$

PROBLEM II.

The price or value of a reversion, the time prior to the commencement, and rate of interest given, to find the rent or annuity.

1. Find the amount of the price of the reversion for the year prior to the commencement, by Table I.
2. Find the annuity which that amount will purchase for the answer.

EXAMPLES.

1. The reversion of a freehold estate, to commence 10 years hence, is bought for L. 6139.13255, compound interest being reckoned at 5 per cent.; required the annuity or rent?

Answer, L. 400.

2. The reversion of a freehold estate, upon which there is an annuitant for 8 years, is purchased for L. 5000, compound interest being allowed at $4\frac{1}{2}$ per cent.; what is the yearly rent?

Answer, L. $451 : 0 : 7\frac{1}{4}$.

SECT. IV.

ANNUITIES FOR LIVES.

THE value of annuities on lives, as far as probability goes, is determined from obſervations made on the bills of mortality by various writers on political arithmetic. In all calculations of this kind, the age of 86 is conſidered as the utmoſt limit of human life, and the difference between any given age and 86 is called the complement.

PROBLEM I.

To find the value of an annuity of L. 1, during the life of a ſingle perſon of any given age.

1. Subtract the given age from 86 for the complement.
2. Find the value of L. 1 annuity to continue during ſaid complement.
3. Multiply this value into the amount of L. 1 for 1 year, and divide the product by the complement.
4. Subtract the quotient from 1, and divide the remainder by the intereſt of L. 1 for a year, and the quotient will be the value of the annuity required, or number of years purchaſe it is worth.

EXAMPLE.

What is the value of an annuity of L. 40, for an age of 40 years, intereſt at 5 per cent.?

86 — 40 = 46 complement.

Value of 46	17.8800663	per Table IV.
	1.05	
46)	18.774069615	
Subtract	.40820447	
from	1.........	
Intereſt of L. 1 = .05)	.59179553	
	11.83591	years purchaſe.
	40	
	473.43640	

By this Problem is constructed the following Table, shewing the value of L. 1 annuity for a single life, at 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, and 5 per cent.

Age.	3 per cent.	$3\frac{1}{2}$ percent.	4 per cent.	$4\frac{1}{2}$ percent.	5 per cent.
9 or 10	19.87	18.27	16.88	15.67	14.6
8 or 11	19.74	18.16	16.79	15.59	14.53
7 or 12	19.6	18.05	16.64	15.51	14.47
13	19.47	17.94	16.6	15.53	14.41
6 or 14	19.33	17.82	16.5	15.35	14.34
15	19.19	17.71	16.41	15.27	14.27
16	19.05	17.59	16.31	15.19	14.2
5 or 17	18.9	17.46	16.21	15.1	14.12
18	18.76	17.33	16.1	15.01	14.05
19	18.61	17.21	15.99	14.92	13.97
4 or 20	18.46	17.09	15.89	14.83	13.89
21	18.3	16.96	15.78	14.73	13.81
22	18.15	16.83	15.67	14.64	13.72
23	17.99	16.69	15.55	14.54	13.64
3 or 24	17.83	16.56	15.43	14.44	13.55
25	17.66	16.42	15.31	14.34	13.46
26	17.5	16.28	15.19	14.23	13.37
27	17.33	16.13	15.04	14.12	13.28
28	17.16	15.98	14.94	14.02	13.18
29	16.98	15.83	14.81	13.90	13.09
30	16.8	15.68	14.68	13.79	12.99
31	16.62	15.53	14.54	13.67	12.88
2 or 32	16.44	15.37	14.41	13.55	12.78
33	16.25	15.21	14.27	13.43	12.67
34	16.06	15.05	14.12	13.3	12.56
35	15.86	14.89	13.98	13.17	12.45
36	15.67	14.71	13.82	13.04	12.33
37	15.46	14.52	13.67	12.9	12.21
38	15.29	14.34	13.52	12.77	12.09
1 or 39	15.05	14.16	13.36	12.63	11.96
40	14.84	13.98	13.20	12.48	11.83
41	14.63	13.79	13.02	12.33	11.7
42	14.41	13.59	12.85	12.18	11.57
43	14.19	13.4	12.68	12.02	11.43

Age.	3 per cent	$3\frac{1}{2}$ per cent.	4 per cent.	$4\frac{1}{2}$ percent	5 per cent.
44	13.96	13.2	12.5	11.87	11.29
45	13.73	12.99	12.32	11.7	11.14
46	13.49	12.78	12.13	11.54	10.99
47	13.25	12.56	11.94	11.37	10.84
48	13.01	12.36	11.74	11.19	10.68
49	12.76	12.14	11.54	11.0	10.51
50	12.51	11.92	11.34	10.82	10.35
51	12.26	11.69	11.13	10.64	10.17
52	12.0	11.45	10.92	10.44	9.99
53	11.73	11.2	10.7	10.24	9.82
54	11.46	10.95	10.47	10.04	9.63
55	11.18	10.69	10.24	9.82	9.44
56	10.9	10.44	10.01	9.61	9.24
57	10.61	10.18	9.77	9.39	9.04
58	10.32	9.91	9.52	9.16	8.83
59	10.03	9.64	9.27	8.93	8.61
60	9.73	9.36	9.01	8.69	8.39
61	9.42	9.08	8.75	8.44	8.16
62	9.11	8.79	8.48	8.19	7.93
63	8.79	8.49	8.2	7.94	7.68
64	8.46	8.19	7.92	7.67	7.43
65	8.13	7.88	7.63	7.39	7.18
66	7.79	7.56	7.33	7.12	6.91
67	7.45	7.24	7.02	6.83	6.64
68	7.1	6.91	6.75	6.54	6.36
69	6.75	6.57	6.39	6.23	6.07
70	6.38	6.22	6.06	5.92	5.77
71	6.01	5.87	5.72	5.59	5.47
72	5.63	5.51	5.38	5.26	5.15
73	5.25	5.14	5.02	4.92	4.82
74	4.85	4.77	4.66	4.57	4.49
75	4.45	4.38	4.29	4.22	4.14
76	4.05	3.98	3.91	3.84	3.78
77	3.63	3.57	3.52	3.47	3.41
78	3.21	3.16	3.11	3.07	3.03
79	2.78	2.74	2.7	2.67	2.64
80	2.34	2.31	2.28	2.26	2.23

An example will be the best illustration of the Table.

1. A gentleman, 47 years of age, wants to purchase an annuity of L. 100, what ready money ought he to pay, reckoning interest at 5 per cent.?

By the Table he must pay	10.84	years purchase.
Multiply by the annuity	100	
	L. 1084	*Answer.*

2. A young trader marries a widow of 30 years of age, with a jointure of L. 200 per annum, and proposes to dispose of the jointure, to enable him to carry on his trade to better purpose; what sum ought he to receive, reckoning interest at $4\frac{1}{2}$, in lieu of the jointure? *Answer*, L. 2758.

3. A young lady has an exchequer bounty for life of L. 20 per annum, but wanting to raise a sum on it to enable her to marry with greater advantage, she offers it to sale; what sum should she receive at 18 years of age, reckoning interest at 4 per cent.?
Answer, L. 322.

4. A gentleman obtains a pension for his son of 10 years of age, during life, for L. 150 per annum; what is it worth in ready money, supposing the interest of money $3\frac{1}{2}$ per cent.?
Answer, L. 2740 : 10 : 0.

Note, If an annuity for life, together with a reversion for a term of years, or the reversion by itself, only comes in question, reduce the year's purchase found in the table to years certain per Table IV. and to the years certain add the years in reversion, and the present worth corresponding to the sum of the years in Table IV. will be the value of the life and reversion; from which, if the value of the life be taken, the value of the reversion will remain.

5. What is the present value of an estate of L. 80 per annum, together with a reversion of 20 years after the demise of the present possessor, aged 40 years; and what is the separate value of the reversion, interest at 5 per cent.?

By the Table, the value of the life = 11.38, which, per Table IV. is opposite 18 years nearest; 18 + 20 = 38, and the value of L. 1 annuity, for 38 years, at 5 per cent. = 16.8678926; which multiplied into 80, = 1349.4314 for the value of the life in reversion.

2dly, From	16.8678926
Take	11.6895869
Value of L. 1 reversion	5.1783057
	80
Value of L. 80 reversion	414.2644560

Note, If the value of a reversion in fee simple, or for ever, after the life of a given age, be required, from the value of the fee simple subtract the value of the life in possession, and the remainder will be the value of the reversion.

6. An annuitant of 60 years of age liferents an estate of L. 500 per annum; what is the reversion in fee simple worth to the heir, interest being reckoned at 5 per cent.?

Value of L. 1 in fee	20
Value of the life per Table	8.39
Value of L. 1 in reversion	11.61
Multiply by	500
	L. 5805 *Answer.*

PROBLEM II.

To find the value of an annuity depending on the joint continuance of two lives.

CASE I.

If the two persons be of one age, take the value of any one of the lives from the Table, and multiply by one year's interest of L. 1; subtract the product from 2, and divide the value formerly found by the remainder; the quotient will give the number of years purchase sought.

EXAMPLES.

1. An annuity of L. 400 depends on the joint lives of two persons, aged 30 years each, either of which failing, the annuity to cease; what is the value, reckoning interest at 4 per cent.?

One life of 30 years per Table =	14.68
Rate of interest	.04
Deduce	.5872
From	2.
	1.4128) 14.68
	10.39
	400
Answer, L.	4156

2. An annuity of L. 500 depends on the joint continuance of two lives, each going 40; what is the value of the annuity, interest being reckoned at $4\frac{1}{2}$ per cent.? *Answer*, L. 4338.

CASE II.

If the ages are different, multiply the value of the one life into the value of the other, as they appear in the Table; multiply the product by the interest of L. 1 for a year; add the value of the two lives, and from their sum deduce the last product; divide the first product by the remainder for the year's purchase required.

EXAMPLES.

1. What is the value of L. 500 annuity for the joint lives of two persons, whereof one is 40 and the other 50 years of age, reckoning interest at 5 per cent.?

By the Table 40 years is worth	11.83
And 50 ditto is worth	10.35
	122.4405
Rate of intereſt	.05
Take this product	6.122025
From ſum of the two lives, viz.	22.18
	16.057975) 122.4405
Year's purchaſe	7.62
	500
Anſwer,	L. 3810

2. What is the value of L. 300 annuity for the joint lives of two perſons, the one going 20, and the other 30, reckoning intereſt at $4\frac{1}{2}$ per cent.? *Anſwer*, L. 3159 : 12 : 0.

PROBLEM III.

To find the value of an annuity upon the longeſt of two lives.

From the ſum of the values of the ſingle lives, ſubtract the value of the joint lives, and the remainder will be the value ſought.

EXAMPLES.

1. What is the value of an annuity of L. 100 upon the longeſt of two lives, the one 30 and the other 40 years of age, intereſt at 4 per cent.?

30 years =	14.68
40 years =	13.2
Sum	27 88
By Prob. II. Caſe II. value joint	9 62
Value of L. 1	18 26
	100
Value required,	L. 1826

2. What is the value of an annuity of L. 150, upon the longest of two lives, the one 20 and the other 50 years of age, interest at $4\frac{1}{2}$ per cent.? *Answer*, L. 2529 : 12 : 0.

PROBLEM IV.

To find the value of the next presentation to a living.

From the value of the successor's life, deduce the joint value of his and the incumbent's life, and the remainder will be the year's purchase, which, multiplied into the annuity, gives the answer.

EXAMPLES.

1. A. enjoys a living of L. 300 a year, and B. would purchase it for his life after A.'s death; what should he pay for it, reckoning interest at 5 per cent. and allowing B. to be 25 years of age, and A. 60?

B.'s life =	13.46	
A.'s life =	6.97	
Difference	6.49	year's purchase.
	300	
L.	1947	value of next presentation.

2. A. enjoys a living of L. 230 per annum, and is aged 70; B. would purchase the survivancy, who is only aged 20; how much should he pay for it, interest at $4\frac{1}{2}$ per cent.?

Answer, L. 2049 : 6 : 0.

PROBLEM V.

To find the value of a reversion for ever after two successive lives.

Find the value of the longest of two lives, which being subtracted from the value of the perpetuity, the remainder will be the number of years purchase.

EXAMPLES.

1. A. aged 50 years, has an estate of L. 200 a year, upon which B. aged 30, has a liferent after A.'s death; upon account of incum-

brances the estate must be sold; what may it bring, reckoning the burden of these annuities at 4 per cent. interest?

A.'s life	50	=	11.34	
B.'s life	30	=	14.68	
Sum			26.02	
Value of their joint lives			8.60	reckoned at L. 1.
Value of the longest life			17.42	
Value of the subject			25.00	
			7.58	year's purchase of the reversion.
			200	
			L. 1516	*Answer.*

2. A. aged 60, has an estate of L. 300 a year, and marries a young wife of 18, whom he also infefts in liferent. Two years ago, by bad œconomy, he was obliged to sell the subject, with the burden of his wife's liferent and his own; how much should it have been valued at, allowing interest at 5 per cent.? *Answer*, L. 2940 : 18 : 0.

PROBLEM VI.

To find the value of an annuity during the joint continuance of three lives, one of which failing, the annuity to cease.

1. Multiply their single values continually, and the last product by the interest of L. 1 for a year.
2. Double this product, and subtract it from the sum of the several products of the lives taken two and two.
3. By the last remainder divide the continued product of the three lives, and the quotient will be the number of years purchase.

EXAMPLES.

1. A. is 18 years of age, B. 34, and C. 56; what is the value of their joint lives, reckoning interest at 4 per cent.?

A.	=	16.1			
B.	=	14.12	The continued product	=	2275.6
C.	=	10.01			.04
					91.024
					2
			Product first		182.048

Product of A. and B.	=	227.33
A. and C.	=	161.16
B. and C.	=	141.34

Sum 529.83
Deduce 182.048

347.782 Product second.

Lastly, $\frac{2275.6}{347.782} = 6.54$ Product third.

2. A. is 20 years of age, B. 40, and C. 60; they enjoy an annuity, to cease with the first that dies, of L. 500 per annum; what is the value of that annuity, reckoning interest at 5 per cent.?

Answer, L. 2850 nearly.

PROBLEM VII.

To find the value of an annuity upon the longest of three lives.

From the sum of the values of three single lives separately from the Table, subtract also the sum of all the joint lives found per Problem VI. and that sum will be the number of years purchase sought.

EXAMPLES.

1. A. is 18 years of age, B. 34, and C. 56; what is the value of the longest of these three lives, interest at 4 per cent.?

A. = 16.1
B. = 14.12
C. = 10.01

Sum of the single lives	40.23	
Sum of the joint lives	26.6	
Remainder	13.63	
Value of their joint lives	6.54	per Problem VI.
Value of the longest of three lives	20.17	in years purchase.

A. and B. = 10.76
A. and C. = 8.19 } per Problem II.
B. and C. = 7.65

26.6

2. A. is 20 years of age, B. 30, and C. 60; and they enjoy an annuity, to ceaſe with the longeſt liver of the three; what is the value of L. 500 annuity, at 5 per cent. intereſt, upon theſe conditions? *Anſwer*, L. 8805.

The foregoing problems are ſufficient for all the uſeful purpoſes that occur in the buſineſs of buying and ſelling annuities; wherefore, to conclude the ſubject, for the ſatisfaction of dealers in this way, we have ſubjoined a Table, ſhewing the number of years poſſeſſion that will reimburſe an annuitant of the purchaſe-money at the ſeveral rates per cent. at which it may have been purchaſed.

TABLE, exhibiting the number of years poſſeſſion that will reimburſe a purchaſer.

Years purch.	3 p. cent. Yrs.days.	$3\frac{1}{2}$ p. cent. Yrs.days.	4 p. cent. Yrs.days.	$4\frac{1}{2}$ p. cent. Yrs.days.	5 p. cent. Yrs.days.
5	5 182	5 216	5 252	5 289	5 327
$5\frac{1}{2}$	6 37	6 79	6 122	6 168	6 216
6	6 261	6 311	6 364	7 55	7 113
$6\frac{1}{2}$	7 124	7 184	7 247	7 374	8 20
7	7 356	8 62	8 137	8 217	8 303
$7\frac{1}{2}$	8 227	8 311	9 34	9 129	9 231
8	9 104	9 200	9 304	10 51	10 172
$8\frac{1}{2}$	9 359	10 97	10 217	10 348	11 125
9	10 236	11	11 138	11 290	12 92
$9\frac{1}{2}$	11 128	11 274	12 69	12 245	13 75
10	12 24	12 191	13 9	13 212	14 75
$10\frac{1}{2}$	12 292	13 115	13 324	14 194	15 94
11	13 200	14 48	14 286	15 190	16 134
$11\frac{1}{2}$	14 115	14 354	15 259	16 203	17 195
12	15 36	15 305	16 246	17 234	18 285
$12\frac{1}{2}$	15 329	16 265	17 246	18 285	20 38
13	16 264	17 235	18 261	19 358	21 189
$13\frac{1}{2}$	17 206	18 216	19 292	21 90	23 13
14	18 156	16 209	20 340	22 215	24 247
$14\frac{1}{2}$	19 115	20 245	22 43	24 5	26 168
15	20 82	21 234	23 132	25 195	28 151
$15\frac{1}{2}$	21 59	22 267	24 245	27 60	30 209
16	22 45	23 316	26 18	28 336	32 360
$16\frac{1}{2}$	23 41	25 16	27 185	30 300	35 264

Explanation and use of the foregoing Table.

The left hand column contains the number of years purchase paid for the annuity, and the other columns shew the yearly possession that will reimburse the annuitant at all the usual rates per cent.

EXAMPLES.

1. A gentleman purchases an annuity for 11 years purchase; how long must he retain it to reimburse him, interest $4\frac{1}{2}$ per cent.?
   In a line with 11, and under $4\frac{1}{2}$, we have 15 years 190 days for answer.

2. An annuity of L. 200 is bought at $12\frac{1}{2}$ years purchase; how long must the possession be enjoyed to reimburse the annuitant?

COMPUTATION OF ANNUITIES BY LOGARITHMS.

PROBLEM I.

Annuity, time, and rate given, to find the amount, at compound interest.

Multiply the logarithm of the rate + 1 into the time, and to the product add the logarithm of the annuity. From the natural number represented by the sum subtract the annuity, and from the logarithm of the remainder subtract the logarithm of the rate, and the remainder will be the logarithm of the amount required.

EXAMPLES.

1. What will an annuity of L. 20 per annum amount to in 40 years, compound interest being allowed at 5 per cent. per annum?

$$
\begin{array}{rcl}
1.05 & = & 0.021189 \\
 & & 40 \\
\hline
 & & 0.847560 \\
20 & = & 1.301030 \\
\hline
140.8 & = & 2.148590 \\
20 & & \\
\hline
120.8 & = & 2.082067 \\
\hline
0.05 & = & 8.698970 \\
\hline
\textit{Anſwer}, \ 2416 & = & 3.383097
\end{array}
$$

2. What will L. 100 annuity amount to in 20 years, compound intereſt being allowed at 5 per cent.? *Anſwer*, L. 3308 nearly.

3. A gentleman purchaſes an eſtate of L. 1000 a year, and draws $7\frac{1}{2}$ per cent. by diſcounting bills with the rents; what will be the neat produce of the L. 1000 at the end of 20 years?

Anſwer, L. 43302.

PROBLEM II.

Annuity, rate, and time given, to find the preſent worth.

1. Multiply the logarithm of the rate + 1 into the given time, and from the number repreſented by the product ſubtract 1, and multiply the remainder into the annuity for a dividend, to which the rate, multiplied into the number anſwering to the firſt product, will be a diviſor.

EXAMPLES.

1. What ſum of money will purchaſe L. 40 annuity, to continue 15 years, at 5 per cent. compound intereſt?

1.05 =	0.021189	
	15	
	.317835	
40 =	1.602060	
83.155 =	1.919895	
40		
43.155	1.634988	From
	.317835	
Rate .05 =	8.678970	
	8.996805	Take
L. 414.14 =	2.638183	

2. An annuity of L. 12 a year is to continue 37 years; what ready money will purchase it, discount being allowed at the rate of $4\frac{1}{2}$ per cent. compound interest? *Answer*, L. 214 : 7 : 0.

3. What is the present worth of L. 200 annuity, to continue 45 years, discounting compound interest at 3 per cent.?

Answer, L. 4903 : 15 : 0 nearly.

PROBLEM III.

Present worth, time of continuance, and rate of interest given, to find the annuity.

1. Multiply the logarithm of the rate + 1 into the time, and the number answering to the product — 1 will be a divisor.

2. Multiply the logarithm of the rate + 1 into the time + 1, and from the number corresponding to the product. Subtract the number corresponding to the first, and multiply the difference by the given value for a dividend; the quotient will give the annuity required.

EXAMPLES.

1. A gentleman has L. 1000, which he proposes to lay out upon an annuity, to continue 20 years; how much will it purchase, compound interest being allowed at 5 per cent.?

1. $1.05 = 0.021189$
20

$0.423780 = 2.6534$

2. 0.021189
21

$0.444969 = 2.7855$

$.1330$

3. $.1320 \times 1000 = \dfrac{133.0000}{1.6535} = 80.4$

2. A. owes B. L. 1000, which he agrees to pay by an annuity, to continue 10 years; what ought he to pay per annum, as an equivalent for L. 1000 due now, computing interest at $4\frac{1}{2}$ per cent.?

Answer, L. 126 : 8 : 0 nearly.

Note, The operation by logarithms is abundantly short and easy; but, to obtain an answer with sufficient exactness, the logarithmic numbers must be carried much higher than in the common tables.

CHAP. XX.

THE POWERS OF NUMBERS, AND THEIR ROOTS.

Definitions.

I. The power of any number is a product arising by multiplying that number by itself, the product by the same number, this product by the same number, &c. to any number of multiplications.

II. The given number is called the *first power*, or *root*.

The product of the first power by itself is the second power, or square.

The product of the second power by the first is the third power, or cube.

The product of the third power by the first is the fourth power, or biquadrate, &c.

III. The natural numbers 1, 2, 3, &c. are sometimes placed over these powers, denoting the number of multiplications used in producing them, or shewing what powers they are, and are called *indices* or *exponents*, as in the following scheme:

Indices,	0,	1,	2,	3,	4,	5,	6,	7,	&c.
Powers,	1,	2,	4,	8,	16,	32,	64,	128,	&c.

IV. The raising any root or number given to any power required is called *involution*, and is performed by multiplying the given root into unity continually, as taught above. If the root of any power not exceeding the seventh power be a single digit, it may be obtained by inspection, from the following Table.

TABLE OF POWERS.

Root, or firſt power,	1	2	3	4	5	6	7	8	9
Square, or ſecond power,	1	4	9	16	25	36	49	64	81
Cube, or third power,	1	8	27	64	125	216	343	512	729
Biquadrate, or fourth power,	1	16	81	256	625	1296	2401	4096	6561
Surſolid, or fifth power,	1	32	243	1024	3125	7776	16807	32768	59049
Cube ſquared, or ſixth power,	1	64	729	4096	15625	46656	117649	262144	531441
Seventh power,	1	128	2187	16384	78125	279936	823543	2097152	4782969

V. The finding the root of a given power is called *evolution*, or *extraction of roots*.

VI. The root of the given number, considered as a power, is a number which, being raised to the index of that power, will either be equal to the given number, or approach very near to it.

To extract the Square Root of any Number.

RULE.

1. Begin at the unit's place, put a point over it, and also over every next figure but one, reckoning to the left for integers and to the right for decimals; and there will be as many integer places in the root as there are points over the integers in the given number: the points over decimal fractions likewise determine the number of decimals in the root. The figure under a point, with its left hand place, is called a *period*.

2. Under the left hand period write the greatest square contained in it, and set the root thereof in the quotient; subtract the square, and to the remainder bring down the next period, as in division.

3. On the left of this remainder write the double of the root, or quotient, for a divisor; seek how often this may be had in the remainder, except the right hand place; write what ariseth both in the root, and on the right of the divisor.

4. Multiply this increased divisor by the last quotient figure; subtract, and to the remainder bring down the next period; double the root for a divisor, and proceed as before.

Note, Fractional places will often arise in the root, by annexing to the remainders periods of two ciphers each, till there be no remainders, or till the decimal part of the quotient repeat or circulate, or till you think proper to limit it.

EXAMPLES.

1. Required the square of 38.

$38 \times 38 = 1444 =$ square of 38, by involution.

Required the ſquare root of 1444.

Illuſtration.

Put a point over the unit's place 4, and alſo over the place of hundreds. Now the number conſiſts of two periods, and will have two integer places in the root: then the greateſt ſquare in 14, the left hand period, is 9, and its root 3; write 9 under the period, and 3 in the root: now 9 from 14 leaves 5, to which annex the next period 44; the root 3 doubled makes 6, which in 54 is contained 8 times, annex 8 to the 3 in the quotient, and to the diviſor 6, makes the root 38, and the diviſor 68; then 8 times 68 is 544; and there remaining 0 on ſubtraction, it may be concluded that 38 is the true root.

```
     1444 ( 38 = root
     9      by evolution.
     ----
68 ) 544
×8   544
     ----
     0
```

2. Required the ſquare of $234\frac{1}{2}$?

234.5 × 234.5 = 54990.25 = ſquare number.

Required the ſquare root of 54990.25?

```
         54990.25 ) 234.5 = root.
         4
         ----
   43 )  149
   ×3    129
         ----
  464 )  2090
   ×4    1856
         -----
468.5 )  23425
   ×.5   23425
         -----
         0
```

Note I. It is plain that number muſt be a ſquare whoſe root may be extracted without a remainder; and ſuch number may be as well decimal as integral, as in the following example.

3. Required the ſquare root of 50.2681?

```
        50.2681 ) 7.09 = root required.
        49
        -----
1409 )  12681
  ×9    12681
        -----
        0
```

4. Required the ſquare root of 0.00015625?

$$
\begin{array}{rl}
 & 0.000\dot{1}\dot{5}\dot{6}\dot{2}5\ )\ 0.0125 = \text{root.} \\
 & \quad 1 \\
 & \overline{} \\
22\ ) & 56 \\
\times 2 & 44 \\
 & \overline{} \\
245\ ) & 1225 \\
\times 5 & 1225 \\
 & \overline{} \\
 & 0
\end{array}
$$

II. The number that is not a ſquare, or whoſe root cannot be extracted, is ſaid to be *ſurd* or *irrational*; but the roots of ſuch ſurd numbers may be approximated in decimals as near the truth as required, by annexing to the given ſurd ſo many pairs of ciphers as you would have decimal places in the root.

5. Required the ſquare root of 2?

$$
\begin{array}{rl}
 & \dot{2}.\dot{0}0\dot{0}0\dot{0}0\dot{0}0\dot{0}0\ (\ 1.41421 = \text{root.} \\
 & 1 \\
 & \overline{} \\
24\ ) & 100 \\
\times 4 & 96 \\
 & \overline{} \\
281\ ) & 400 \\
\times 1 & 281 \\
 & \overline{} \\
2824\ ) & 11900 \\
\times 4 & 11296 \\
 & \overline{} \\
28282\ ) & 60400 \\
\times 2 & 56564 \\
 & \overline{} \\
282841\ ) & 383600 \\
\times 1 & 282841 \\
 & \overline{} \\
 & 100759
\end{array}
$$

Hence it appears the ſquare root of 2 is 1.41421 true to five places of decimals; if a greater degree of accuracy be required, more places may be obtained by dividing the remainder 100759 in the common way by the diviſor 28284, omitting the firſt figure 1, thus:

```
28284 ) 100759 ( 3562
        84852
        ------
        159070
        142420
        ------
         176500
         169704
         ------
          67960
          56568
          -----
          11292
```

Therefore the ſquare root of 2 ſtill approximates nearer 1.414213562.

III. To find the ſquare root of any number made up of integers and decimals, or of decimals alone.

Make the decimal places complete periods of two figures each, if not ſo given, by annexing a cipher, or ciphers, to the right hand. Then proceed in the operation as above directed, thus:

6. Required the ſquare root of 234.5?

```
To be placed thus:  234.50,00,00 ( 15.313 = root,
                    1
                    ---
          25 )      134
          ×5        125
                    ---
          303 )     950
          ×3        909
                    ---
          3061 )    4100
          ×1        3061
                    ----
          30623 )   103900
          ×3         91869
                    ------
                     12031
```

7. Required the ſquare root of .001?

To be placed thus: .0010,00,00 (.0316, &c. = root.

IV. The proof of all these operations is, by multiplying the root into itself, and taking in the remainder, if any, the product will be ever equal to the square, or number given, whose root is required.

V. When the square root of a repetend is required.

Rule, Instead of annexing periods of ciphers, annex periods of the repetends, thus:

8. Required the square root of the repetend $.\dot{1}$.

Placed thus: .11,11,11, &c. (.333

```
        9
      -----
   63 ) 211
   ×3   189
      ------
  663 ) 2211
   ×3   1989
      -------
 6663 ) 22211
   ×3   19989
      --------
66663 ) 222211
   ×3   199989
        ------
         22222
```

9. Required the square root of the repetend $.\dot{4}$.

Answer, $.\dot{6}666$, &c. = root.

VI. When the square root of a circulate is required.

Rule, Annex to the remainders periods of the circulating figures, and proceed as in the above operations.

10. Required the square root of the circulate $13\dot{8}.5\dot{1}8$; or, which is the same thing, of 13,8.51?

Placed thus: 1.3,8.51,85,18,51, &c.

Answer, 11.769389 root.

VII. To extract the ſquare root of any vulgar fraction.

Rule, Extract the root of the numerator and denominator, for the fractional root.

Thus the ſquare root of $\frac{25}{36}$ is $\frac{5}{6}$; for 5 is the root for 25, and 6 the root of 36: the proof is, that $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$. So that the ſquare root of $\frac{144}{256}$ is $\frac{12}{16}$, or $\frac{3}{4}$; and ſo you proceed for any other.

Note, As vulgar fractions are ſeldom commenſurable, the beſt and moſt elegant way is to convert the vulgar into a decimal fraction, and extract the root in decimals, as above taught.

We have, by a variety of examples, explained the method of extracting the ſquare root of numbers, vulgar and decimal; to which we ſhall ſubjoin ſome examples illuſtrating the uſe of this operation, in the various affairs of life.

11. A gentleman has three fields, one of which is 7,75 links by 6,47 links, another of 12,87 links by by 9,85 links, and another 14,83 links by 11,87 links, which he wants to exchange for one park or field that lies convenient for him; required the ſide of that field of equal content? *Anſwer*, 18.786 links.

12. A maltſter has a kiln, whoſe diameter is 14,7 feet, which is too little by three fourths for his buſineſs; required the diameter of a kiln to anſwer his purpoſe? *Anſwer*, 19.44 feet.

Note, The areas of circles are to one another as the ſquares of their diameters; therefore ſquare the diameter, add to it the parts deficient, extract the ſquare root of the laſt ſum, which will be the anſwer. If leſs be required, ſubtract the overplus, and the ſquare root of the remainder is the anſwer.

13. Suppoſe the length of a horſe's tether is $22\frac{1}{4}$ feet, which allows him to eat 172.88 ſquare yards of graſs; what additional ſquare yards of graſs would he have liberty to eat by lengthening his tether $12\frac{3}{4}$ feet? *Anſwer*, 254.9 ſquare yards.

14. If a pipe, whoſe diameter is 1.6 inches, fill a ciſtern in four hours, in what time will a pipe, whoſe diameter is 3.2 inches, fill the ſame? *Anſwer*, 1 hour.

15. There is a wall of a fortification 245.5 feet high, at the bottom of which there is a moat or ditch 187.91 feet broad: required the length of a ſcaling ladder that will reach from the fartheſt ſide of the moat to the embattlements? *Anſwer*, 309.16 feet.

N. B. The ſquare of the hypothenuſe, or longeſt ſide of a right angled triangle, is equal to the ſum of the ſquares of the other two ſides.

16. Three towns, A., B., and C., are ſo ſituated, that B. lies 80 miles ſouth of A., and C. 60 miles weſt of A.; what is the diſtance between B. and C.? *Anſwer*, 100 miles.

17. To find the radius or ſemidiameter of the ſenſible or viſible horizon; that is, how far a man can ſee by looking around him on the ſurface of the earth or ſea, or how far any height can be ſeen.

Suppoſe a man on the top of a mountain that is 4 miles high, including the height of the ſpectator's eye; required the limits of his viſible horizon? *Anſwer*, 178.45792 miles.

N. B. The earth's diameter, from the lateſt experiments, is 42078016 Engliſh feet; conſequently its ſemidiameter is 21039008 Engliſh feet.

18. A ladder 40 feet long may be ſo placed, that, upon one ſide of the ſtreet, it will reach a window 33 feet from the ground; and, without moving it at the bottom, it will reach another window on the other ſide, 21 feet high: what is the breadth of the ſtreet?
Anſwer, 56.6 feet.

To extract the Cube Root of any Number.

RULES.

I. Over the unit's place of the given number put a point, and alſo over every third figure from the unit's place, to the left for integers, and to the right for decimal fractions; and the root will have as many integer places as there are points or periods of the integral part of the given number.

II. Under the left hand period write the greateſt cube it contains, and ſet the root in the quotient; ſubtract the cube from the period, and to the remainder annex the remaining periods; which you may call the reſolvend.

III. To the quotient annex as many ciphers as there were periods remaining: call this the root.

IV. Divide the reſolvend by the root, add the quotient to thrice the ſquare of the root, let the ſum be a diviſor to the reſolvend, and the quotient figures annexed to the right of the firſt root, without the cipher, will be the cube root ſought.

V. If the second figure of the root be 1, or 0, then generally 3 or 4 figures of the root will be obtained at the first operation: but if the second figure exceed 2, it will be best to find only two places at first.

VI To renew the operation: Subtract the cube of the figure found in the root from the given number; then form a division, and divide as directed in the 4th precept; and this will give the root true to 5 or 6 places; for each operation commonly triples the figures found in the last root.

EXAMPLES.

1. Required the cube of 214?

$$214 \times 214 \times 214 = 9800344 = \text{cube or solid content.}$$

Note, The solid content of any body, which is called its cube, is the product arising from the multiplying its length by its breadth, and that by its thickness. When the solid content is given, to find the length of its side every way, is called the *extraction of the root*, and is to be performed by the above precepts. Thus:

Required the cube root of 9800344?

$$\dot{9}80\dot{0}34\dot{4} \;)\; 2$$

$$2 \times 2 \times 2 = 8$$

$$2.00 \;)\; 18003{,}44 \text{ Resolvend.}$$

$$9001 = \text{Quotient.}$$
$$120000 = \text{Thrice the square of the root.}$$

$$129001 \;)\; 1800344 \;(\; 14$$
$$129001$$

$$510334$$
$$516004$$

The root is 214

Illustration.

Put a point over the unit's place 4, another over the place of thousands, and another over that of millions; or, in general, over every third figure, denoting the three dimensions of the cube; and

because there are three points, there will be three places in the root. Under the left hand period 9, write 8, the greatest cube therein, and its root 2 write in the quotient; then subtracting, the resolvend is 1800344: Now, because there are two periods remaining, therefore two ciphers annexed to the root 2 make it 200; by which dividing the resolvend, the quotient is 9001; also the square of 200 is 40000, the triple thereof 120000 being added to 9001, makes 129001 for a divisor, by which dividing 1800344, the quotient is 14 nearly, and is taken as 14, because it is much nearer than 13; now 14 being annexed to the former root 2, makes 214, the root sought.

2. Required the cube root of 114604290.028?

```
           114604290.028 ( 4           48
            64                         48
           ----                       ----
    4,00 ) 50604290                   384
           --------                  192
           126510                    -----
           480000                    2304
           ------                      48
           606510 ) 50604290 ( 8      -----
           ----------------------    18432
                                     9216
                                     ------
                                     110592
```

Here 480 is taken for the root at the first operation:

```
Then 114604290.028 ( 48
     110592
     -----------
480 ) 4012290
     ---------
Quotient = 8358.9
          691200 = triple the square of 580, viz.
          ------          230400 × 3.
Divisor 699558.9 ) 4012290.028 ( 5736
                   34977945
                   --------
To 480 the first root  5144955
add     5.736          4896912
       -------         -------
Sum 485.736             248043
                        209867
                        ------
                         38176
```

The work of the division is supposed to be done on a spare piece of paper.

Here, instead of bringing down the figures of the dividend to the remainder, the divisor is lessened each time, by pointing off a place on the right; but regard is to be had to the carriage which will arise from the places thus omitted.

3. Required the cube root of $28\frac{3}{4}$?

To be placed and pointed thus:

$$
\begin{array}{rll}
270000 & 28.750000 & (\ 3.06 = \text{root.} \\
540 & 27 & \\
36 & \overline{} & \\
\overline{} & 1750000 & = \text{resolvend.} \\
\text{Divisor } 275436 & \times\ 61652616 & = \text{product.} \\
 & \overline{} & \\
 & 97384 & = \text{remainder.}
\end{array}
$$

To prove the operations in this rule: Cube the root found, and to the product add the remainder, if any, the sum will be equal to the cube whose root is required. Thus,

$$3.06 \times 3.06 \times 3.06 + 97384 = 28.750000.$$

4. Required the cube root of .000485613?

$$
\begin{array}{rll}
 & .000485613 & (\ .078 = \text{root.} \\
14700 & 343 & \\
1680 & \overline{} & \\
64 & 142613 & = \text{resolvend.} \\
\overline{} & & \\
\text{Divisor } 16444 \times 8 = & 131552 & = \text{product.} \\
 & \overline{} & \\
 & 11061 & = \text{remainder.}
\end{array}
$$

OBSERVATION.

If the cube root of a vulgar fraction is required, find the cube root of the given numerator for a new numerator, and the cube root of the given denominator for a new denominator. Thus the cube root of $\frac{64}{343}$ is $\frac{4}{7}$, and the cube root of $\frac{27}{216}$ is $\frac{3}{6}$ or $\frac{1}{2}$; and the cube root of $\frac{125}{8} = 15\frac{5}{8}$ is $\frac{5}{2} = 2\frac{1}{2}$.

But if the root of either the numerator or denominator cannot be extracted without a remainder, reduce the vulgar fraction to a decimal, and then extract the root, as in Example 4.

The extraction of the cube root is of great use in solving questions and problems in the practical parts of the mathematics. We shall subjoin a few examples, and leave the solution of them to the genius of the reader.

EXAMPLES.

1. Suppose a bullet of 3 inches diameter weigh 72 lb.; what will a bullet weigh whose diameter is 4 inches?

Answer, $170\frac{2}{3}$ lb.

N. B. Like solids are in the triplicate proportion of their like sides; that is, the solid contents of similar figures are to one another as the cubes of their similar sides, or diameters.

2. There is a cubical vessel whose side is 12 inches; required the side of a vessel that will hold three times as much?

Answer, 17.306 inches.

3. There is a cistern whose length is 56 inches, breadth 45 inches, and depth 24; required the dimensions of another cistern that will contain five times as much?

		Inches.
Answer,	Length	95.6.
	Breadth	71.7.
	Depth	41.

4. There is a granary whose dimensions are as follow, viz. length 5, breadth 4, and depth 3 feet, which suppose holds 10 bolls barley: required the dimensions of a similar granary to hold 125 bolls of the same grain?

$\frac{125}{10} = 12.5$, the ratio of the one granary to the other.

		Feet.
Answer,	Length	11.1.
	Breadth	9.4.
	Depth	6.9.

5. Admit the length of a ship's keel is 76.95 feet, the breadth of the midship beam 29.58 feet, and the depth of her hold 14.877 feet, and the burden of this ship to be 360 tons: required the dimensions of another ship of the same mould that shall carry only 72 tons? *Answer*, Keel 45.02, breadth 17.3, depth 8.7.

6. There are three boxes, the content of one is 10000 ſolid or cubical inches, of another 16656, and of the third 20000: required the ſide of a cubical box that ſhall contain as much as all the three? *Anſwer*, 36.25.

7. Required the ſide of a cubical veſſel that will contain 80 wine gallons? *Anſwer*, 21.885 inches.

N. B. 231 inches in a wine gallon.

To extract the Biquadrate Root.

RULE.

Extract the ſquare root of the given number; the ſquare root of which extract again, and the laſt of theſe roots is the root ſought.

EXAMPLES.

1. Required the biquadrate root of 5308416?

```
 .  . . .     . .
 5308416 ) 2304 ( 48 root.
 4          16
 ---       ----
34 ) 130   88 ) 704
     129        704
 --------  ----
4604 ) 18416    0
       18146
       -----
         0
```

2. Required the biquadrate root of 64013554081? *Anſwer*, 503.

It will not be improper here to give the rationale of extracting the ſquare and cube root, or more properly the extraction of roots in general.

THE RATIONALE OF EXTRACTING ROOTS.

I. *Of the Square Root.*

WHEN a number is given, whose square root is to be extracted, the first thing to be done is to point it; that is, to place a point over the first figure, and over every other figure afterwards; which points resolve the number into periods of two figures each, unless the number of figures be odd, for then the last period can have but one figure; the reason of which is, that the square of each of the nine digits will produce but two places of figures; and 10 is the first number whose square will produce three places of figures; and 100 is the first that will produce five places, when squared; and so on. Therefore the points denote the number of figures in the root. Thus in $6\dot{4}$ there is but one figure in the root; in $\dot{1}4\dot{4}$, there are two; in $\dot{9}9\dot{8}5\dot{6}$, there are three; in $\dot{1}0\dot{0}2\dot{0}0\dot{1}$, there are four figures in the root; and so for any other number.

In order to shew the reason of the rules, we must have recourse to algebra, in which a letter, as a, is put to represent any number at pleasure; and aa denotes the square of a, and aaa its cube; ay denotes a product, and $2ay$ denotes a double product; and the factor into which any letter is multiplied is called the *co-efficient*.

Now, since there are always so many places of figures in the root as there are points, or parcels, in the given number, the figures of each place may be represented by letters, a, b, c, &c. Thus if there be but one period, as 64, there will be but one figure, viz. $8=a$, and so $64=a^2$ only; but in a number of two periods, as 144, there will be two figures in the root, viz. $12=10+2=a+b$. A number of three places, as $\dot{9}9\dot{8}5\dot{6}$, will have a root of three places of figures, viz. $316=300+10+6=a+b+c$; that is, $a=300$, $b=10$, and $c=6$; and thus you may proceed for any larger number of places.

Again, since $\dot{1}4\dot{4}=a+b=a^2+2ab+b^2=a^2+\overline{2a+b}\times b=100+44$; therefore $a^2=100$, and $2a+b=44$, consequently $a=10$, and $2a=20$; and since $\overline{20a+b}\times b=44$, it plainly shews that 20)44(2$=b$, because $\overline{20+2}\times 2=44$; therefore the work in symbols and in numbers will stand as below.

	$\dot{1}4\dot{4}$)	10		$a^2+2ab+b^2$)	$a+b$
	100	$\times 2$		a^2	
$20+2$)	44	12 = root.	$2a+$)	$2ab+b^2$	
$\times 2$	44		$2ab\times$	$2ab+b^2$	

OBSERVATION.

When the proposed number has three periods, as $\dot{9}9\dot{8}5\dot{6}$, then there will be three places in the root, $a+b+c$; and therefore $=\overline{a+b+c}^2=a^2+2ab+b^2+2ac+2bc+c^2=99856$; here then $a^2=90000$, and $a=300$, and $2a=600$; consequently $9856=2ab+b^2+2ac+2bc+c^2$; therefore $600)9856(10=b$; and now $\overline{2a+b}\times b=610\times 10=6100$; therefore $9856-6100=3756=2ac+2bc+c^2=\overline{2a+2b+c}\times c=620+c=c$: therefore $620)3756(6=c$; and so $626\times 6=3756$. See the operation below.

$$
\begin{array}{rrl}
 & 99856\ (& 300=a \\
 & 90000 & \\
\hline
 & & 10=b \\
 & 9855 & \\
600+10 & 6100 & \\
\times 10 & \overline{} & 310=a+b \\
520+6\ ) & 3756 & 6=c \\
\times 6 & 3756 & \\
 & \overline{} & 316=a+b+c \\
 & 0 &
\end{array}
$$

In the above operation, the ciphers being every where omitted, the work will be contracted, and appear in the common form, since then we take down but one period at each division; for we must always make as many divisions as there are places of figures in the root, and we can get no more than one at a time.

$$
\begin{array}{rl}
 & \dot{9}0\dot{8}5\dot{6}\ (\ 316 \\
 & 9 \\
 & \overline{} \\
61\ ) & 98 \\
=1 & 61 \\
 & \overline{} \\
626\ ) & 3756 \\
\times 6 & 3756 \\
 & \overline{} \\
 & 0
\end{array}
$$

OBSERVATION.

By this analytic process, the reason of extracting the root of any number, we presume, is very evident.

II. *Of the Cube Root.*

In extracting the cube root, the given number is resolved into periods of three figures, by the points placed over the first, fourth, &c. because the cube of the greatest digit produces but three places of figures, or 10 is the first number whose cube makes four places; and 100 cubed makes seven places, and so on; therefore over the first, fourth, seventh, tenth, &c. figures, we place a point, to see how many places or figures the root will consist of.

The rationale for extracting the cube is similar to that for extracting the square root. Thus, let the cube number $\dot{1}72\dot{8}$ be proposed; there are two points; the root, therefore, will have two places or points, viz. $a+b$, whose cube is $a^3+3a^2b+3ab^2+b^3=1728=1000+728$. Here it is evident $a^3=1000$, and so $a=10$. Also $3a^2b+3ab^2+b^3=728$, and $3a^2=300$; therefore $300)728(2=b$, whence we shall have $3a^2b=600$, $3ab^2=120$, and $b^3=8$; consequently $3a^2b+3ab^2+b^3=600+120+8=728$, the next remainder: therefore the cube root is $a+b=10+2=12$.

2. Let the number $1\dot{3}824$ be the cube whose root is required. $a^3+3a^2+3ab^2+b^3=13000+824$; and it appears evident, from the table of powers, that the next cube number less than 13000 is 8000, the root of which is $20=a$; therefore $a^3=8000$, which, subtracted from the given number, leaves $5824=3a^2b+3ab^2+b^3$. Now, $3a^2=1200)5824(4=b$; therefore we have

$$\begin{aligned} 3a^2b &= 4800 \\ 3ab^2 &= 960 \\ b^3 &= 64 \\ \hline & 5824 = 3a^2b + 3ab^2 + b^3. \end{aligned}$$

And consequently the cube root is $a+b=20+4=24$. From all which it is evident, the common rule, given for extracting the cube root, is nothing more than the above analytical process in words. By the same process may the rationale for extracting the roots of higher powers be demonstrated.

To extract the Root of the Fifth Power or Sursolid.

PRECEPTS.

1. Let the given number be divided into periods of five figures, then find the nearest root of the left hand period, place the figure

so found in the quotient, subtract its fifth power, and to the remainder annex the next period for a resolvend.

2. Let $a+b$ represent the root, and then the sursolid, or fifth power, will be $aaaaa+5aaaab+10aaabb+10aabbb+5abbbb+bbbbb$. Now, $aaaaa$ being already subtracted, there remains the other five parts to find b, divide by its quotient, viz. by $5aaaa+10aaab+10aabb+bbbb$: that is, try how often $5aaaa$ is contained in the resolvend; and, by the help of the quotient figure, make up the other four parts of the divisor.

EXAMPLE.

Required the fifth power, or the sursolid root of 33554432?

$$
\begin{array}{rcl}
 & & \dot{3}355\dot{4}4432 \;(\; 32 = \text{root.} \\
 & & 243 \\
\hline
 &) & 9254432 \\
405000 & = & 5aaaa \\
54200 & = & 10aaab \\
3600 & = & 10aabb \\
1200 & = & 5abbb \\
16 & = & bbbb \\
\hline
\text{Divisor } 4627216 \times 2 & = & 9254432 \text{ product.} \\
\hline
 & & 0
\end{array}
$$

To extract the Root of the Sixth Power, or Cube squared.

PRECEPT.

Extract the square root of the given number, and then extract the cube of that root, the last is the root sought. The same answer may be obtained, by first extracting the cube root, and then the square root of that root.

EXAMPLE.

Required the sixth power of 191102976?

191102976 (3824 = square root.

3824 (24 = cube root.

So that 24 is the sixth power or root of 191102976.

To extract the Root of the Seventh Power.

PRECEPT.

Let $a+b$ express the root, and the seventh power will be $aaaaaaa + 7aaaaaab + 21aaaaabb + 35aaaabbb + 35aaabbbb + 21aabbbbb + 7abbbbbb + bbbbbbb$, by the aid of which proceed as in extracting the root of the fifth power.

EXAMPLE.

Required the 7th power or root of 3404825447?

3404825447 (23 = root.
128

2124825447

448000000	=	$7aaaaaa$
201000000	=	$21aaaaab$
50400000	=	$35aaaabb$
7560000	=	$35aaabbb$
680400	=	$21aabbbb$
34020	=	$7abbbbb$
729	=	$bbbbbb$

Divisor 708275149 × 3 = 2124825447 = product.

0

To extract the Root of the Eighth Power.

PRECEPT.

Extract the square root of the given number continually, till you have three roots; and the last of these is the root sought.

EXAMPLE.

Let 17857939904896 be the eighth power: *1st*, By extracting the square root you get the biquadrate, or fourth power, viz. 1336336: *2dly*, And by extracting the square root of the biquadrate, you get the square, or second power, viz. 1156: *3dly*, The square root of this, viz. 34, is the root sought.

To extract the Root of the Ninth Power.

PRECEPT.

Extract the cube root of the given number, which will be the cube or third power; the cube root of this last is the ninth power, or root sought.

EXAMPLE.

Let 5159780352 be the ninth power; the cube root of which is 1728, whose cube root 12 is the root sought.

OBSERVATIONS.

1. In general, whatever the given power be, let $a+b$ represent the root, and by involution raise $a+b$ to the power of the given number; then with this, as a condition in your operation, extract

the root, as has been directed and exemplified in the extractions of the roots of the fifth and seventh powers.

2. If the index of the given power be a multiple of 2, the operation is easy; for, by extracting the square root of the given number, you obtain a power whose index is one half of the index of the given power. Thus the square root of the tenth power is the fifth power, and the square root of the twelfth power is the sixth power, &c.

3. If the index of the given power be a multiple of 3, the cube root of it gives a power whose index is one third of the index of the power given. Thus the cube root of the ninth power is the cube or third power, &c.

4. As involution is directly contrary to extraction or evolution, therefore the squaring a square number will produce the biquadrate, or fourth power; and if this last be squared, it will give the eighth power. Likewise, if a cube number be cubed, it will produce the ninth power; and if the biquadrate be cubed, it will give the twelfth power, &c.

CHAP. XXI.

NUMERAL SERIES, OR PROGRESSION.

DEFINITIONS.

1. A RANK of three or four numbers, that increaſe or decreaſe by a conſtant and uniform progreſſion, is called a *Numeral Series.*

2. If the progreſſion be made by equal differences, that is, by the conſtant addition or ſubtraction of the ſame number, the ſeries is called an *Arithmetical Progreſſion.*

Thus	1	2	3	4	5	6	7	8	9,	&c. incr. by adding 1.
	4	8	12	16	20	24	28	32	36,	&c. incr. by adding 4.
	49	43	37	31	25	19	13	7	1,	&c. decr. by ſubtr. 6.

are ranks of numbers in arithmetical progreſſion; and of ſuch ranks there may be an infinite number.

3. If the progreſſion be made by a conſtant multiplication or diviſion with the ſame number, the ſeries is called a *Geometric Progreſſion.*

Thus	1	2	4	8	16	32	64,	incr. by 2.
	1	6	36	216	1296	7776	46656,	incr. by 6.
	6561	2187	729	243	81	27	9, 3	decr. by 3.

are ranks of numbers in geometric progreſſion; and of ſuch ranks there may be an infinite variety.

4. The common multiplier or diviſor is called the *ratio;* thus 2 is the ratio in the firſt rank, 6 in the ſecond, and 3 in the third.

OF ARITHMETIC PROGRESSION.

THEOREM I.

IN any ſeries of terms in arithmetic progreſſion, the ſum of any two terms, conſidered as extremes, is equal to the ſum of any two terms taken as means equally diſtant from the extremes.

Thus in 3 terms, where the first and third are extremes, and the other mean, viz. 6. 9. 12. then $6+12=9+9=18$.

And in 4 terms, viz. 13. 19. 25. 31.
Then $13+31=19+25=44$.

Also in the terms 49. 43. 37. 31. 25. 19. 13. 7. 1.
Then $49+1=43+7=37+13=31+19=25+25=50$.

THEOREM II.

In any arithmetic progression, the sum of any two terms lessened by the first term; or their difference increased by the first term, will be a term also in that progression.

Thus in the progression 1. 3. 5. 7. 9. 11. 13. 15. 17. 19. 21, &c.

For $7+11=18$, and $18-1=17$ is a term of the progression.

Also $11-7=4$, and $4+1=5$ is a term of the progression.

PROBLEM I.

In an arithmetic progression, given the first term, the common difference, and the number of terms, to find the last term.

RULE.

Multiply the number of terms less one by the common difference, to the product add the first term, and the sum will be the last term.

EXAMPLES.

1. A person agrees to discharge a certain debt in a year, by weekly payments, viz. the first week 4s. the second week 8s. constantly increasing by 3 each week; required the last period?

$5 =$ first term	Now	$52 - 1 =$	51
$3 =$ com. difference	And	$51 \times 3 =$	153
$52 =$ No. of terms.	Then	$153 + 5 =$	158s.

Answer, L. 7 : 18 : 0, the last payment.

2. Suppose 1 and 9 to be the first and second terms of an arithmetic progression of 1074 terms; required the last term?

Here $9-1=8$ is the common difference,
And $1073\times8=8584$.
Now $1074-1=1073$;
Then $8584+1=8585=$ last term.

3. How many strokes does a clock strike in one revolution of the index, viz. in 12 hours? *Answer*, 78 strokes.

4. A man had 12 children; the youngest was three years old, and the common difference of their ages was four years; what was the age of the eldest? *Answer*, 47 years.

PROBLEM II.

Given the first term, the last term, and the number of terms, in an arithmetic progression, to find the sum of all the terms.

RULE.

Add the first and last terms together, the sum multiplied by half the number of terms gives the sum of all the terms.

EXAMPLES.

1. Required the sum of the first thousand numbers in their natural order.

Here $1=$ first term, $1=$ com. diff.
$1000=$ No. of terms, its $\frac{1}{2}$ is 500.
Now $1000+1=1001$.
Then $1001\times500=500500$ is the sum required.

2. Thirteen persons gave their charity to a poor man in arithmetical progression; the first gave 2d. the last 26d.; how much did the poor man get in all? *Answer*, $2+26=28$, and $28\times13=364$, and 364 divided by $2=182$d.

3. Suppose a basket and 500 apples were placed in a streight line, a yard distant from each other; required in what time a man could bring them one by one to the basket, allowing him to walk at the rate of three miles an hour?

Between the basket and apples are 500 spaces, which is the number of terms. Now $500+1=501$. Then $501\times 250=125250=$ sum of the terms. But as he goes backwards and forwards, he walks 250500 yards; which divided by 1760, the yards in one mile, gives 142.329 miles; which, at 3 miles an hour, will take 47 hours 26 minutes 34 seconds.

4. A person finishes a journey in 13 days, the number of miles travelled each day being in arithmetical progression; the seventh day he travelled 22 miles: required the length of his journey?

Answer, 286 miles.

OF GEOMETRIC PROGRESSION.

THEOREM I.

In a series of terms, in geometric proportion, the product of any two terms, considered as extremes, is equal to the product of any two intermediate equidistant terms, considered as means.

Thus, in three terms, viz. 5 25 125. Or 3 9 27.

Then $5\times 125=25\times 25=625$. Also $3\times 27=9\times 9=81$.

And in four terms, 4 8 16 32.

Then $32\times 4=16\times 8=128$.

Also in the terms 1 4 16 64 256 1024 4096 16384.

Then $16384\times 1=4096\times 4=1024\times 16=256\times 64=16384$.

THEOREM II.

In any geometric progression, the product of any two terms divided by the first term; or the quotient of any two terms multiplied by the first term, will give a term also in that series.

Thus, in the progression 3 6 12 24 48 96 192 384 768, &c.

Then $\dfrac{12\times 96}{3}=384$; and $\dfrac{192}{12}\times 3=48$, are terms in the progression.

THEOREM III.

If over a ſeries of terms in geometric progreſſion be wrote a ſeries of terms in arithmetic progreſſion, whoſe firſt term is 0, and common difference is 1, term for term; then any term in the arithmetic ſeries will ſhew how far its correſponding term in the geometric ſeries is diſtant from the firſt term.

Thus $\begin{cases} 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6, \text{\&c. arithmetic ſeries.} \\ 1 \quad 3 \quad 9 \quad 27 \quad 81 \quad 243 \quad 729, \text{\&c. geometric ſeries.} \end{cases}$

Here 729 is diſtant from the firſt term 6 terms, 243 is diſtant 5 terms, 81 is diſtant 4 terms.

Note, The terms of the arithmetical ſeries are called indices to the terms of the geometric ſeries.

Thus 5 is the index to 243, 3 to 27, and 1 to 3, &c.

PROBLEM I.

Given the firſt term, the ratio, and the laſt term in the geometric progreſſion; required the ſum of all the terms?

RULE.

Multiply the laſt term by the common ratio; from the product ſubtract the firſt term for a dividend.

Subtract 1 from the ratio for a diviſor; then divide, and the quotient will be the ſum of all the terms.

EXAMPLES.

1. Suppoſe the firſt term of a ſeries to be 3, the ratio 3, and the laſt term 6561; required the ſum of all the terms?

	6561 = laſt term.		3 = ratio.
Mult. by	3 = ratio.	Subt.	1
	19683 = product.	Rem.	2 = diviſor.
Subt.	3 = firſt term.		
	19680 = dividend.		

Then 2) 19680 (9840 = ſum of all the terms.

2. Let the first term be 2, the second term 10, and the last term 156250; required the sum of the series?

Here 5 = ratio.

$156250 \times 5 = 781250$, and $781250 - 2 = 781248$ = dividend.

$5 - 1 = 4$ = divisor. Then 4)781248(195312 sum of the series.

3. A gentleman, who has a daughter married on a new-year's day, gave the husband towards her portion 4s. promising to triple that sum the first day of every month, for nine months after the marriage; the sum paid on the first day of the ninth month was 26244s.: required the lady's portion?

Answer, 39364s. = L. 1968 : 4 : 0.

PROBLEM II.

The first term, the ratio, and the number of terms, of a series of numbers in geometric progression, being given, to find the last.

RULES.

1. Write down 6 or 7 of the leading terms in the geometric series, and over them their indices.
2. Add together the most convenient indices, to make an index less by unity than the number expressing the place of the term sought.
3. Multiply together the terms of the geometric series belonging to those indices which were added; make the product a dividend.
4. Raise the first term to a power whose index is equal to the number, less one, of the terms multiplied; make the result a divisor to the former dividend, and the quotient will be the term sought.

EXAMPLES.

1. Required the 12th term of a geometric series whose first term is 3, and second term 6?

Here $\frac{6}{3} = 2$ the common ratio.

$$\text{And} \begin{cases} 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6, \text{ \&c. indices.} \\ 3 \quad 6 \quad 12 \quad 24 \quad 48 \quad 96 \quad 192, \text{ \&c. geometric terms.} \end{cases}$$

Then $6 + 5 = 11$, is the index to the 12th term.

And $192 \times 96 = 18432$ is the dividend.

The number of terms multiplied is 2, and $2-1=1$, the power to which the first term 3 is to be raised; but the first power of 3 is 3.

Then $\frac{18432}{3} = 6144$ is the 12th term of the given series.

2. A person being asked to dispose of a fine horse, said he would sell him on condition of having $\frac{1}{4}$d. for the first nail in his shoes; $\frac{1}{2}$d. for the second nail; 1d. for the third; 2d. for the fourth; 4d. for the fifth; 8d. for the sixth, &c. doubling the price of every nail to thirty-two, the number of nails in the four shoes: what would the horse be rated at in this way?

Here the first term is 1, the ratio 2, and the number of terms 32.

First, to find the last term.

$$\text{Now}\begin{cases} 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8, \text{ \&c. indices.} \\ 1 \quad 2 \quad 4 \quad 8 \quad 16 \quad 32 \quad 64 \quad 128 \quad 256, \text{ \&c. geometrical terms.} \end{cases}$$

And 31 is the index to the 32 term.

Then $8+8=16$, $16+8=24$, $24+7=31$.

The first term being 1, any power thereof is 1; so the 4th article of the rule is useless in this question.

Now $246 \times 256 = 65536$ is the 17th term,
$65536 \times 256 = 16777216$ is the 25th term,
$16777216 \times 128 = 2147483648$ is the 32d term.

$$\begin{array}{rl} \text{Then } 2147483648 & \\ 2 & \\ \hline 4294967296 & \\ -\ 1 & \text{the 1st term.} \\ \hline 4294967295 & \end{array}$$

$2-1=1$) 4294967295 the sum of the terms, or the price of the horse in farthings; or L. 4473924 : 5 : $3\frac{1}{4}$, the *Answer*.

3. A corn merchant buys 12 stacks of wheat, and was to pay 2d. for the first stack, 6d. for the second, &c. tripling the price for every following stack; what sum had he to pay?

Answer, L. 6603 : 0 : 2.

CHAP. XXII.

SUPPOSITION.

THIS rule, by the help of ſuppoſed numbers, finds an anſwer to many queſtions which have not terms enough given to be reſolved by Proportion; and therefore it is ſometimes called the *Rule of Falſe*, and ſometimes the *Rule of Poſition*. In this rule there are two varieties, comprehended under the titles *Simple* and *Compound*.

SECT. I.

SIMPLE SUPPOSITION.

WHEN there is only occaſion for one ſuppoſition, which the nature of the queſtion will eaſily evince, the anſwer is found by this analogy:

As the reſult ariſing from the number ſuppoſed
To the total given,
So is the ſuppoſed number itſelf
To the number ſought.

EXAMPLES.

1. A gentleman bought a Phaeton, horſe, and harneſs, for L. 100: The horſe was valued at twice as much as the harneſs, and the Phaeton at double both horſe and harneſs; required the particulars?

	L.
Suppoſe the harneſs coſt	5
Then the horſe	10
And Phaeton	30
In all	45

	L.		L.		L.		L.	s.	d.	
Then	45	:	5	::	100	:	11	2	$2\frac{2}{3}$	Harneſs.
							22	4	$5\frac{1}{3}$	Horſe.
							66	13	4	Phaeton.
							100	0	0	Proof.

2. Three men, A., B., and C., were disputing about their ages. Says A., I am certainly older than B. by $\frac{1}{2}$ of C.'s age. This I know, says B., that my age is just $\frac{3}{4}$ of C.'s; and I know, says C., that our ages together will reckon just 130: Required the particulars?

Suppose	C. = 16	Then	48 : 16 :: 130 : $43\frac{1}{3}$	C.
Then	B. = 12		48 : 12 :: 130 : $32\frac{1}{2}$	B.
And	A. = 20		48 : 20 :: 130 : $54\frac{1}{6}$	A.
	48		130	Proof.

3. Divide 60 into two such parts that the greater may be triple the lesser.

12 will admit of 3 for the lesser, and 9 for the greater number.

Wherefore 12 : 3 :: 60 : 15 the lesser,
and 12 : 9 :: 60 : 45 the greater number.

4. Divide L. 1200 among three men, A., B., and C., so as A. may have $\frac{1}{2}$, B. $\frac{1}{3}$, and C. $\frac{1}{4}$, in proportion.

$$\left.\begin{array}{l}\frac{1200}{2} = 600\\ \frac{1200}{3} = 400\\ \frac{1200}{4} = 300\end{array}\right\} 1300 : \begin{array}{c}600\\400\\300\end{array} :: 1200 : \left.\begin{array}{r}553.846\\369.230\\276.923\end{array}\right\} 1200$$

5. Divide 80 into two such parts that $\frac{2}{3}$ of the one will just be equal to $\frac{3}{4}$ of the other. *Answer*, $42\frac{6}{17}$ and $37\frac{11}{17}$.

6. A., B., and C. bought a ship, towards payment of which A. contributed a certain sum, B. the double of A., and C. as much as both the other two. The price of the ship was L. 2000. Required the sum paid by each?

Answer,	A. paid L.	333	6	8
	B.	666	13	4
	C.	1000	0	0
	L.	2000	0	0

7. One being asked to lend his neighbour 100 guineas, made answer that he had not so many; but if $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{1}{12}$ of his guineas were taken together, they would just make 100: Required his number? *Answer*, 60.

8. Divide 64 into two ſuch parts that $\frac{1}{6}$ of the one may be equal to $\frac{1}{4}$ of the other? *Anſwer*, $38\frac{2}{5}$ and $25\frac{3}{5}$.

9. A gentleman who had ſeveral children founds a queſtion on that circumſtance to try a ſtudent's ingenuity, and tells him his youngeſt child's years were juſt equal to the number of children he had; moreover, there were juſt 3 years between each of them, and the oldeſt had only completed his 49th year: how many children had he? *Anſwer*, 13.

10. Divide L. 200 between two men, and give the one L. 73 more than the other?

Anſwer, L. 136 : 10 : 0, and L. 63 : 10 : 0.

SECT. II.

Compound or Double Supposition.

When a queſtion is involved in ſo many circumſtances, as a ſimple diviſion of any numbers cannot be readily aſſimilated to theſe circumſtances, the queſtion, in that caſe, may be reſolved by the following

RULES.

1. Make choice of any number at pleaſure, and arrange it according to the conditions propoſed; and this number may perhaps, from a compariſon, turn out to be the number ſought.
2. If it does not, and the reſult of your ſuppoſition be more or leſs than the truth, the exceſs or defect is called the *firſt error.*
3. Make choice of a ſecond number, and proceed with it as before, calling the exceſs or defect the *ſecond error.* Then will the number required be found by the following

RULES.

1. If both errors are of the ſame kind, *i. e.* both exceſſes, or both defects, multiply the firſt ſuppoſition into the ſecond error, and the ſecond ſuppoſition into the firſt error, the difference of theſe products, divided by the difference of the errors, will quote the anſwer.
2. But if the errors are diſſimilar, *i. e.* the one exceſs, and the other defect, the ſum of the products obtained as before, divided by the ſum of the errors, will quote the anſwer.

EXAMPLES.

1. Three beggars, upon a festival, had 324 sixpences thrown among them, of which each seized as many as he could; upon comparing circumstances, it was found that B. had scrambled 15 more than A., and that C. had $\frac{1}{5}$ of the sum seized by the other two: Required the particulars.

1. Suppose A. got	60	2. Suppose A. got	70
Then B.	75	Then B.	85
	135		155
And C. $\frac{1}{5}$	27	And C.	31
	162		186
	324		324
First error	162	Second error	138

$$138 \times 60 = 8280$$
$$162 \times 70 = 11340$$
$$162 - 138 = 24\,)\,3060$$

127.5 to A
142.5 to B.

270
54 to C.

324 Proof.

2. Three men bought a ship for L. 760, for which B. paid L. 10 more than A., and C. as much as both together; required the particulars?

1. A. =	200	2. A. =	120
B. =	210	B. =	130
C. =	410	C. =	250
	820		500
	760		760
	60 +		260 −

$$\begin{array}{lrcrcr} \text{Then} & 260 & \times & 200 & = & 52000 \\ \text{And} & 120 & \times & 60 & = & 7200 \end{array}$$

$$260 + 60 = 320\ )\ 59200$$

185 paid by A.
195 paid by B.
380 paid by C.

760 Proof.

3. Three men, A., B., and C., bought a ship, and paid down L. 2760; of this B. paid as much again as A., and L. 12 more, and C. thrice as much as B., and L. 12 more; required the particulars?

1st sup. 500 paid by A. 2d sup. 400
1st error 1800 + 2d error 900 +

Answer, L. 300 paid by A.
612 by B.
1848 by C.

4. One begins the world with a certain sum of money, which he improved so well in the way of trade, that he increased it by $\frac{1}{7}$ yearly, allowing out of his profits L. 100 for the annual expence of his family, and at the end of three years found he had doubled his capital; what was his original stock? *Answer*, L. 1110.

5. A man travelling with a certain number of guineas met a robber, who took half of what he had and $\frac{1}{2}$ guinea over; the same treatment he meets with from a second, a third, and a fourth robber, till at last he had only 7 guineas left; how many had he at first? *Answer*, 127.

6. A general disposing his men in square battalia, found he had 60 more than would stand in the figure; but when he thought of enlarging the side of his square, though but by one more, he found he should want 41 for that purpose; what number of men had he, and how many did they stand of a side?

Answer, 2560 men, and 50 of a side.

7. A., B., and C. owe me each a sum of money; A.'s and B.'s together amount to L. 47, A.'s and C.'s to L. 71, and B.'s and C.'s to L. 88; required the particulars?

Answer, A. owes L. 15
B. 32
C. 56

103

Questions of this kind may be more easily solved than by the common rule, thus: Add the given sums, and $\frac{1}{2}$ of the aggregate sum will be the amount of the debts; from that amount subtract the greatest given sum, which will give the least debt, the next greatest for the next debt, and the least sum for the greatest; thus:

$$47 + 71 + 88 = \frac{206}{2} = 103 - \begin{cases} 88 = 15 & \text{A.} \\ 71 = 32 & \text{B.} \\ 47 = 56 & \text{C.} \end{cases}$$

8. A man bought two horses, A. and B., and a saddle worth L. 5: Now, when the saddle is put upon A., he is then worth twice B.; but if put upon B., he is then worth three times A.: Required the value of the horses?

Answer, A. = L. 3, and B. = L. 4.

9. Two persons, A. and B., travelling together, A. with L. 100, and B. with L. 48, met with an acquaintance, who borrowed twice as much from A. as from B., and yet left A. thrice as much as he left B.: How much got he from each? *Answer*, He borrowed L. 44 from A., and L. 88 from B.

10. A., stealing apples, was catched by B., and, to make friends with him, gave him half of all he had, and gets back 10; A., going farther, meets with C., who extorted from him half of all he had left, but afterwards returns him 4; a little thereafter D. comes up with him, who plundered him of the half of the apples he had remaining, and afterwards returns him 1; after which he had just 13: How many had he at first? *Answer*, 60.

11. A clock has two hands turning upon the same center, whereof the swifter makes a revolution every 12 hours, and the slower every 16 hours; what is the synodical period of these two hands?

Answer, 48 hours.

12. Divide 90 into four such parts, that the first increased by 2, the second diminished by 2, the third multiplied by 2, and the fourth divided by 2, will all be equal? *Answer*, 18, 22, 10, and 40.

13. A., B., and C. bought a ship for 200 guineas. A. said, that with $\frac{1}{2}$ of B.'s money he would pay the ship; B. replied, that with $\frac{1}{3}$ of C.'s money he could pay the ship; and C. rejoined, that with $\frac{1}{4}$ of A.'s money he could pay the ship: Required the particulars?
Answer, A. had L. 128; B., L. 144; and C., L. 168.

14. There is a certain fishing rod, consisting of two parts, whereof the upper part is to the lower as 5 to 7, and moreover, 9 times the upper part, together with 13 times the lower, is 11 times the whole rod, and 36 inches over; required the length of each part?
Answer, The upper part 45 inches, and the lower 63 inches.

15. A man, his wife, and son's years make 96, of which the father and son's equal the wife's, and 15 years over, and the wife's and son's equal the man's, and 2 years over; what was the age of each? *Answer*, The man 47, the wife $40\frac{1}{2}$, and son $8\frac{1}{2}$.

16. One farmer asked another, how many lambs he had: The ewes, replied he, brought me 2000, but by paying the tithe, and other losses, they are much reduced; for at one time I lost half the number I now have, at another time $\frac{1}{3}$ the same number, and at another time $\frac{1}{4}$ the same number: How many has he remaining?
Answer, 864.

17. There is a certain floor, in form of a rectangle, whose dimensions are such, that, had it been 2 feet broader and 3 feet longer, it would have measured 64 square feet more than it did; but, on the contrary, had it been 3 feet broader, and only 2 feet longer, it would have measured 68 feet more than it did: Required the dimensions of the floor? *Answer*, 14 feet by 10 feet.

18. A certain company at a tavern found, when they came to pay their reckoning, that if they had been 3 more in company to the same reckoning, they would have paid 1s. each less than they did; and, on the contrary, had they been 2 fewer, they must have paid 1s each more than they did: Required the number in company, and their quota? *Answer*, 12 in company, at 5s. each.

19. Three persons, A., B., and C., were talking of their money: Says A. to B. and C., give me $\frac{1}{2}$ of your money, and I will have 17 guineas; says B. to A. and C., give me $\frac{1}{3}$ of your money, and I shall have 17 guineas; nay, says C. to A. and B., give me but $\frac{1}{4}$ of your money, and I shall have 17 guineas: Required the particulars?
Answer, A. had 5, B. 11, and C. 13.

APPENDIX.

PROMISCUOUS EXAMPLES

IN ALL THE PRECEDING RULES.

1. A Gentleman went to the races, where by betting he loſt and won as follows, viz. to A. he paid L. 217 : 11 : 10; to B. he paid L. 514 : 10 : 8; from C. he received L. 614 : 4 : 9; from D. he received L. 209 : 19 : 9; to E. he paid L. 411 : 11 : 11; and from F. he received L. 509 : 18 : 11; his expences came upon the whole to L. 12 : 4 : 11; and, on reckoning his caſh when he came home, he found L. 1597 : 18 : 0 ſtill in his pocket: *quæritur*, How much money did he ſet out with?

Anſwer, L. 1419 : 13 : 11.

2. There are two neighbours, whoſe ages when added reckon 140 years 6 months, and when the one is ſubtracted from the other, the remainder is 47 years 9 months; how old is each?

Anſwer, The elder 94 years $1\frac{1}{2}$ month, the younger 46 years $4\frac{1}{2}$ months.

3. An undertaker on a canal had finiſhed a mile in length when he gave in his account to the proprietors; the canal was 30 feet broad at top, 20 at bottom, and 8 feet deep; how many ſolid yards had he finiſhed, and what was his charge, at 2s. 6d. per ſolid yard?

Anſwer, $39111.\dot{1}$ ſolid yards, which at 2s. 6d. amounts to L. 4888 : 17 : $9\frac{1}{2}$.

4. What is the value of 179 hogſheads of tobacco, weighing net 1725 Cwt. 3 qrs. 21 ℔. at 3s. $7\frac{1}{2}$d. per ℔.?

Anſwer, L. 35036 : 10 : $7\frac{1}{2}$.

5. Bought 1000 ells Holland by the Flemiſh ell, and paid down L. 90; how muſt I charge it per Engliſh ell, to gain L. 10 on the whole?

Anſwer, 3s. 4d.

6. A privateer, conſiſting of a captain, firſt and ſecond mates, a ſurgeon, a carpenter, fifty ſailors, and forty marines, took a

prize, which, after deduction of the necessary charges, netted L. 30,000. The conditions of the adventure were these, viz. the proprietors had $\frac{3}{4}$; and of the remainder the captain had 8 shares, his first mate 6, the second 4; the surgeon and carpenter had each 6, each sailor 1 share, and each marine $\frac{3}{4}$ of a share. Required a division of the prize.

Answer, The proprietors draw	L.	22500	0	0
The captain - -		545	9	$1\frac{1}{11}$
First mate, carpenter, and surgeon, each		409	1	$9\frac{9}{11}$
Second mate - -		272	14	$6\frac{6}{11}$
Each sailor - -		68	3	$7\frac{7}{11}$
Each marine - -		51	2	$8\frac{31}{44}$

7. One bought 327 gallons of rum at 9s. per gallon, and sold it again at 4s. 6d. per the Scots pint; what profit had he?

Answer, L. 16 : 15 : 11.

8. In 7453 double joes, 72s. each, how many guineas, crowns, shillings, and sixpences, of each an equal number?

Answer, $19513\frac{17}{55}$.

9. A grocer bought 87 Cwt. 2 qrs. 14 ℔. of sugar at the rate of 88s. per Cwt. and sold it out at $9\frac{3}{7}$d. per ℔. Troy, declaring at the same time that it was at the rate he bought it; what was his gain upon the whole, admitting 17 ℔. Troy to be just equal to 14 ℔. Avoirdupois?

Answer, L. 82 : 12 : $4\frac{1}{2}$.

10. A gentleman purchased 275 acres of land, and paid at the rate of L. 24 : 17 : 0 per acre, surveyed with a Scots chain; he sold it again at the same price per acre, but the survey was made with an English chain; what was his profit?

Answer, L. 1849 : 10 : $9\frac{1}{2}$.

11. One side of a roof 30 feet by 12 is to be covered with slates 8 inches by 5; how many will it require, allowing two inches in length of each to be covered?

Answer, 1728.

12. A. can do a piece of work in 30 days, which with the help of B. he can execute in $17\frac{1}{7}$ days; in what time will B. do it by himself?

Answer, 40 days.

13. If the breadth of a board be 8 inches, how many feet in length will it require to make a door 4 feet in breadth and $6\frac{1}{2}$ in length?

Answer, 39 feet.

14. There is a piece of ground 200 falls by 80 to be planted with firs, each at 5 feet distance from the other every way throughout; how many will it require?

Answer, 220077.

15. A ſquare piece of ground, containing $5\frac{1}{2}$ acres, is to be ſown with flaxſeed, of which every $\frac{1}{4}$ peck is to poſſeſs 12 ells by 10; how many pecks will be ſufficient for the field?

Anſwer, 66 pecks.

16. After the conqueſt of Canada from the French, a gentleman made a purchaſe of 976 acres French meaſure; how many Engliſh acres may he reckon upon, ſuppoſing 16 of the former equivalent to 19 of the latter?

Anſwer, 1159.

17. When flour ſells at 35s. 9d. per bag, the ſixpence loaf weighs $5\frac{1}{2}$ lb.; what ought to be the price of flour when the ſixpence loaf weighs $4\frac{3}{4}$ lb.?

Anſwer, 41s. $4\frac{3}{4}$d. nearly.

18. A carpenter with his two ſervants undertook to finiſh a gentleman's houſe in 52 days; the maſter's wages were to be 2s. 11d. the one ſervant 7d. and the other 5d. a-day; when the work was completed, each had the ſame ſum to receive: how many days were they reſpectively at work?

Anſwer, The maſter 4 days, firſt ſervant 20 days, and ſecond 28 days.

19. A ciſtern is ſupplied with water by two pipes A. and B., which together fill the ciſtern in 12 hours, and A. alone in 20 hours; in what time would it be filled by B. alone?

Anſwer, In 30 hours.

20. A baker bought 570 bolls of wheat at 18s. per boll, freight and incidental charges came to L. 12 : 17 : 6; by the way it was ſo damaged, that he was obliged to ſell it at 20 per cent. loſs; what did he charge per boll at that rate?

Anſwer, 14s. 9d.

21. A room 70 yards in perimeter and 4 yards high is to be lined with paper $\frac{5}{8}$ yard wide: how much will it require?

Anſwer, 448 yards.

22. Bought $\frac{1}{7}$ of $\frac{2}{9}$ of a ſhip for L. 370 : 15 : 0; what will $\frac{5}{6}$ of $\frac{3}{8}$ coſt at that rate?

Anſwer, L. 3649 : 11 : 5.

23. Conſigned to Jamaica ſundry goods, which netted per account of ſales L. 343 : 1 : 8 Sterling; in return I had 46 Cwt. 3 qrs. ſugar at 24s. 6d. per Cwt. and 1570 lb. of indigo at 2s. 4d. per lb.; what does my factor owe me ſtill?

Anſwer, L. 102 : 12 : $11\frac{1}{2}$.

24. Sold my India ſtock at 220 per cent. and had 25 per cent. profit between buying and ſelling; had I kept it one day longer, I could have got 235 per cent.; what would my profit per cent. have been then?

Anſwer, $33\frac{1}{2}$.

25. A room 60 feet in perimeter by 8 in height is to be hung with yard-broad stuff valued at 14s. 6d. per yard; how many yards will it require, and what will the charge be, allowing for a window of 5 feet by 4?

Answer, $51\frac{1}{9}$ yards = L. 37 : 1 : $1\frac{1}{9}$.

26. Consigned to my factor in Amsterdam 15 hogsheads tobacco, containing net 157 Cwt. 3 qrs. 18 lb. which I am advised he has sold at 22 st. 9 pen. per lb.; for how many guilders may I draw, deducing 8 per cent. for commission and charges, and how much Sterling will they bring at 35s. $7\frac{1}{2}$d. Flem. banco per pound Sterling?

Answer, L. 1717 : 10 : $1\frac{3}{7}$.

27. Consigned to my factor in Hamburgh 375 Cwt. 3 qrs. 14 lb. of sugar, the sales of which he advises at 36 marks $10\frac{2}{3}$ sch. per Cwt. for which, deducing impost and petty charges 218 m. 8 sch. with 6 per cent. commission, and standing to bad debts, he desires me to draw at usance: for how many rixdollars may I draw, and what Sterling will they bring me, exchange 33s. 6d. Flem. banco per pound Sterling?

Answer, 4245 rixd. $26\frac{1}{4}$ sch. = L. 1013 : 17 : $3\frac{1}{4}$.

28. A. had L. 570 interest on board the Betsy, B. L. 750, C. L. 1590, D. L. 960, E. L. 2210, and the Betsy was valued at L. 1500; in a storm, of C.'s goods to the value of L. 900 was thrown overboard; what part of that average falls upon each?

Answer, A. pays	L. 67	13	7
B.	89	1	$0\frac{1}{4}$
C.	188	15	$8\frac{3}{4}$
D.	113	19	6
E.	262	8	$0\frac{1}{4}$
Ship	178	2	0
	L. 899	19	$10\frac{1}{4}$
			$1\frac{3}{4}$ lost with the remainders.

29. Agreeable to an order from Bourdeaux, Glasgow ships 75 hogsheads of tobacco, containing 785 Cwt. 2 qrs. 19 lb. net, charged at 1s. $5\frac{3}{8}$d. per lb.; in return for which Bourdeaux sends by the same ship, for account of the Glasgow shipper, 15 pipes of brandy, at 57 sols 3 deniers per gallon, and 15 pipes claret, at 84 sols 8 den. per gallon; how stands the account between the correspondents, and for how much may the creditor draw, exchange at $31\frac{1}{2}$ per ecu?

Answer, The Glasgow correspondent is creditor for L. 5783 : 14 ; $8\frac{1}{2}$, and can draw for 44066 crowns 33 sols $3\frac{1}{2}$ deniers.

30. London receives from Oporto 48 pipes Port wine, charged at 458 reas per gallon; in return London sends 55 pieces broad

cloth, $18\frac{1}{2}$ yards each, at 12s. 6d. per yard, and 95 pieces linen, 25 yards each, at 2s. 9d. per yard; what is the difference of theſe tranſactions in the money of that country to which it belongs, exchange at $62\frac{1}{2}$d. per millrea?

Anſwer, Due to the London merchant L. 241 : 3 : 0.

31. Glaſgow orders from Dantzick ſundry goods, amounting per invoice to 79854 florins 25 groſ. 16 d.; how many guilders muſt Glaſgow remit to Amſterdam, at 91 groſ. per guilder, and how much Sterling muſt be paid for the bill, exchange between London and Amſterdam being at 35s. 6d. Flem. banco per pound Sterling?

Anſwer, 26325 guild. 15 ſtiv. 9 pennings = L. 2471 : 18 : 1.

32. A butcher agrees with a grazier to feed 20 oxen for 12 months; but at 2 months end the butcher adds 5 more, and at the end of $6\frac{1}{3}$ months he adds 10 more; and they agreed that the 35 oxen ſhall be kept only ſo long as will be equivalent to the original contract; required how long the whole may continue after the laſt 10 were put in?

Anſwer, 25 may be fed 8 months, after adding the 5, in lieu of 20 12 months, 35 may continue 1 month after putting in the 10, and then the butcher has made good his bargain.

33. Paid L. 35 to A. for 9 weeks ſalary, and to B. L. 64 for 15 weeks ditto; I have laid by L. 120 to pay them at a certain period together; at what time may they make their demand?

Anſwer, 14 weeks 5 days.

34. Bought by the Engliſh yard, and paid 11s. 2d.; how may I ſell by the Scots ell to gain 40 per cent.?

Anſwer, At 16s. $0\frac{3}{4}\frac{22}{90}$d.

35. London can draw on Amſterdam at 35s. 6d. and on Paris at $30\frac{1}{4}$d. the exchange between Amſterdam and Paris is at $52\frac{1}{2}$d. per ecu; in what manner ſhould London draw and remit L. 500 to take the advantage of the courſe, and with what profit?

Anſwer, Draw on Paris for 3967 crowns, and remit to Amſterdam L. 488 : 18 : $8\frac{1}{2}$, which will juſt compenſate the remittance from Amſterdam to retire your draught on Paris; profit L. 11 : 1 : $3\frac{1}{2}$.

36. Bills at London on Paris ſell at 30d. per ecu, and on Cadiz at 42d. per piaſtre; bills at Paris on Cadiz alſo ſell at 4 livres per piaſtre; in what manner and with what advantage ought London to draw and remit L. 500?

Anſwer, Draw on Cadiz for $2857\frac{1}{7}$ piaſtres, and remit $3809\frac{1}{2}$ crowns to Paris, which will replace the piaſtres, and leave L. 23 : 16 : $2\frac{1}{4}$ in the negotiator's pocket.

37. When wine sold at L. 30 per pipe, the value of L. 20 served a ship's crew, consisting of 336 men, for 4 days, at an allowance of 1 pint per day to each; how long will the value of L. 500 serve 250 men, at an allowance of $1\frac{1}{2}$ pint a-day, when wine is at L. 24?

Answer, 112 days.

38. If the beam of a balance be 58 inches long, and 20 ℔. on the one end will equiponderate 30 on the other, at what point of the beam is it divided?

Answer, The shorter end is 23.2 inches, and the longer 34.8.

39. If the shorter end of the beam of a balance be 23.2 inches, and the longer from the point of suspension 34.8 inches, how much on the longer end will equiponderate 30 on the shorter?

Answer, 20 ℔.

40. A merchant hath a balance, the shorter of whose ends is 23.2 inches, and the longer 34.8; he buys upon the shorter, and sells upon the longer, and moreover charges 5 per cent. on his invoice; how much per cent. does he make in whole?

Answer, 55 per cent.

41. The insured, upon a total loss, produced a policy for L. 575, at the rate of 25 guineas per cent. and 2 per cent. discount allowed by custom; how much money will he draw?

Answer, L. 412 : 11 : 3.

42. A. B. has a property of L. 412 : 11 : 3 aboard the Swan, bound for Jamaica; how much must he insure to cover that property, at 25 guineas per cent. 2 per cent. discount in case of loss?

Answer, L. 575.

43. A. sells his cloth at $19\frac{1}{7}$ per cent. profit; B. buys at 9s. and sells at 10s. 6d.; C. buys at 9d. and sells at 11d.; which of these charges highest?

Answer, B. has $16\frac{2}{3}$, and C. $22\frac{2}{9}$ per cent.

44. A. lends out L. 100 at compound interest 5 per cent. and B. double that sum, at the same time and rate, simple interest; neither touches principal nor interest, by agreement, till the amounts shall be equal: at what period may they make their demand?

Answer, At the end of 34 years 328 days nearly.

45. A certain banker discounts bills at 2 months date, or, at an average, including days of grace, at 63 days, for which he retains interest at 5 per cent. and commission at $\frac{1}{2}$ per cent.: at what rate per cent. per annum does he lend his money?

Answer, L. 8 : 11 : $5\frac{3}{4}$ per cent.

46. A gentleman wants to sink L. 2000 for an annuity to continue 20 years; how much ought he to receive per annum as an

equivalent, allowing compound intereſt to both parties at 5 per cent.?

Anſwer, L. 160 : 9 : $8\frac{1}{2}$.

47. A gentleman ſet out with L. 152 in his pocket, but by ſome accident loſt a conſiderable part of it, which was luckily ſupplied by a purſe which he afterwards found; upon comparing circumſtances at the end of his journey, he found that he had then about him juſt double the money he loſt, and moreover, that, if he had loſt what he found, and found what he loſt, he would have had then L. 53 ſhort of what he ſet out with: Required the ſum he loſt, and the ſum he found?

Anſwer, He loſt L. 102 : 10 : 0, and found L. 155 : 10 : 0.

48. Three companions met at an inn, and, comparing circumſtances, relative to their money, found they had altogether L. 780. Now, if from the ſum of A.'s and B.'s money C.'s be taken, there will remain A.'s and L. 82 over; but if A.'s be taken from B.'s and C.'s, the remainder will be C.'s, excepting L. 43. Required the particulars?

Anſwer, A. had L. 316, B. L. 273, and C. L. 191.

49. Suppoſe a ſhopkeeper ſhould cheat his cuſtomers out of 40 per cent. by weighing in his goods on the ſhorter end of an unequal beam of 6 inches, and weighing out on the longer, what are the different ends of the beam?

Anſwer, The ſhorter 2.5 inches, and the longer 3.5 inches.

50. A gentleman wants a garden 22 falls by 10, and two walks croſſing each other at right angles, whoſe contents ſhall be juſt $\frac{1}{2}$ of the whole garden; required the breadth?

Anſwer, 3.425 falls.

51. There is a bullet whoſe diameter is 4 inches, and weight 9 ℔.; required the weight of another whoſe diameter is $6\frac{1}{4}$ inches?

Anſwer, 34.3322 ℔.

52. There is a gun whoſe concave diameter is $1\frac{1}{2}$ inch, for charging which $\frac{1}{4}$ ℔. of powder is ſufficient; how much will be ſufficient to charge a gun whoſe concave diameter is 7 inches?

Anſwer, 25.4 ℔.

53. A privateer, running at the rate of 5 knots an hour, diſcovers a ſhip 18 miles off, making way at the rate of 4 knots or 8 miles an hour; required the number of miles the ſhip can run before the privateer come up with her?

Anſwer, 72 miles.

54. A reckoning of L. 7 currency in Jamaica was paid with L. 5 Sterling; what was the exchange at that rate?

Anſwer, 40 per cent.

55. A debt in Ireland of L. 529 : 3 : 4 Iriſh was paid with 464 guineas 10s. $2\frac{3}{4}$d. Britiſh; what was the exchange?

Anſwer, $8\frac{1}{2}$ per cent.

56. The toll of a bridge was 4d. for a carriage, 2d. for a horſe, and $\frac{1}{4}$d. for a foot traveller. At the year's end the collector paid in to his conſtituent L. 94 : 15 : 10, and, for a proof of his fidelity, added, that, as often as 5 carriages paſſed, 9 paſſed on horſeback, and 15 on foot; how many paſſed of each denomination?

Anſwer, 2500 carriages, 4500 horſemen, and 7500 foot paſſengers.

57. Imported 785 lb. of Spaniſh ſilk, duty 7s. 8$\frac{2}{5}$d. and 418 lb. thrown ſilk, duty 3s 11$\frac{17}{20}$d. and $\frac{1}{2}$ of $\frac{1}{20}$ duty; what is the collector's charge?

Anſwer, L. 385 : 12 : 1$\frac{1}{2}$.

58. Imported 718 lb. ſilk nubs, duty 4$\frac{12}{20}$d. and $\frac{2}{5}$ of $\frac{1}{20}$, and 784 lb. Italian wrought ſilk, duty 12s. 1$\frac{11}{20}$d. per lb.; required the total duty?

Anſwer, L. 489 : 5 : 8$\frac{1}{4}$.

59. One being aſked what o'clock it was, made anſwer, that $\frac{1}{4}$ of the time from noon was equal to $\frac{3}{5}$ of the time from midnight; required the hour from noon?

Anſwer, 20 min. paſt five.

60. Imported 375 hogſheads of tobacco, which, being weighed on the quay, netted 411548 lb. for the duties on which I am to pay down $\frac{3}{4}$d. old ſubſidy, and give bond at the rate of 6$\frac{4}{20}$ and $\frac{2}{7}$ of $\frac{1}{20}$ for each lb.; required the old ſubſidy and bonded duty on the whole?

Anſwer, L. 1286 : 1 : 9 paid down, L. 10688 : 16 : 3$\frac{1}{4}$ bonded.

61. I have a bill on Dublin for L. 356 : 10 : 0 Sterling, which I propoſe to ſell here; what ſhould I receive for it, the exchange being at 10 per cent.?

Anſwer, L. 351 : 1 : 11$\frac{1}{2}$.

62. A bond for L. 14000 due 11 years hence, is allowed to be taken up upon diſcount at 12$\frac{1}{2}$ per cent. compound intereſt; how much money inſtantly paid down, will diſcharge the bond, upon theſe conditions?

Anſwer, L. 3927 : 2 : 0$\frac{1}{2}$.

OF MENSURATION.

I. OF SURFACES.

IN the menſuration of ſurfaces, the area, or ſuperficial content, is only conſidered, without regard to the thickneſs, and eſtimated in ſquares of yards, feet, or ſuch other denomination as beſt ſuits the figure.

PROBLEM I.

Given the ſide of a ſquare in lineal meaſure, to find the ſuperficial content.

RULE.

Square the given ſide for the anſwer.

EXAMPLES.

1. There is a ſquare garden, each of whoſe ſides is $384\frac{2}{3}$ yards; how many ſquare yards doth it contain?

$$\begin{array}{r} 384.\dot{6}66 \\ 384\frac{2}{3} \\ \hline 147968.\dot{4} \end{array} \quad \textit{Anſwer.}$$

2. There is a ſquare court, each of whoſe ſides is 126 feet 6 inches; how many ſtones will pave it, each 12 inches ſquare?

Anſwer, 1600225.

3. There is a ſquare plot, each of whoſe ſides is 220 yards, and within it a ſquare area is to be left, each of whoſe ſides to be 180 yards; what is the ſuperficial content of each, and what will the exterior ſquare or difference amount to, if valued at 8s. $9\frac{1}{2}$d. per ſquare yard?

Answer,	Whole square	48400 yards
	Inclosed ditto	32400 ditto
	Difference	16000 at 8s. $9\frac{1}{2}$d. = L. 7033 : 6 : 8.

PROBLEM II.

To find the superficial content of a parallelogram or oblong square.

RULE.

Multiply the length by the breadth, and the product is the answer.

EXAMPLES.

1. A floor 76 feet 9 inches by 32 feet 4 inches is to be laid with stones uniformly 2 feet 3 inches by 1 foot 4 inches; what is the superficial content of the room, and how many stones of the above dimensions will be required?

Answer, 2481 feet 7 inches, superficial content;
And 827.194 stones.

2. A footpath on the side of a street 396 feet 8 inches by 9 feet 6 inches is to be paved with stones uniformly 2 feet 8 inches by 1 foot 10 inches; required the number?

Answer, 770.8 nearly.

PROBLEM III.

To find the superficial content of a rhombus, whose sides are all equal, but two of its angles obtuse, and two acute.

RULE.

Multiply one of the sides by the height, measured on a perpendicular dropped from an obtuse angle to its opposite side.

EXAMPLES.

1. There is a quarry of glass, in form of a rhombus, each of

whose sides is 15 feet 6 inches, and the length of the perpendicular 13 feet 5 inches; required the superficial content?

Answer, 207 square feet $11\frac{1}{2}$ inches.

2. There is a rhombus, each of whose sides is 19 feet, and the perpendicular 18 feet; required the superficial content?

Answer, $18 \times 19 = 342$.

PROBLEM IV.

To measure a rhomboides.

RULE.

This figure has four sides, whereof the opposite are parallel and equal, and also four angles, of which the opposite are equal: wherefore multiply one of the longer sides into the perpendicular dropped from one of the obtuse angles for the superficial content.

EXAMPLE.

Suppose one of the longer sides 19 feet 6 inches, and the perpendicular 10 feet 2 inches; required the area?

Answer 198 feet 3 inches.

PROBLEM V.

To measure a triangle.

RULE.

Every triangle is the half of some quadrilateral figure, and therefore may be measured by multiplying the longest side by $\frac{1}{2}$ the perpendicular dropped from the opposite angle.

EXAMPLES.

1. In the right angled triangle A. B. C. whose hypothenuse A. C. is 18 feet, and perpendicular C. D. 8.9; required the area?

Answer, $8.9 \times 9 = 80.1$.

Or because A. B. C. is right angled, B. C. will be perpendicular to A. B. then let A. B. $=13.35$, and B. C. $=12$.

$13.35\times6=80.1$, as before.

2. In the oblique angled triangle A. B. C. suppose the base A. B. 20, and the perpendicular C. D. $=12$; required the content?

Answer, $20\times6=120$.

The area of a triangle may also be found by measuring the sides; thus, from the half of their sum, subtract the sides severally, then will the square root of the continued product of the half sum and differences be the area required; which is very easily performed by logarithms.

EXAMPLES.

1. Suppose an oblique angled triangle whose sides measure severally 20, 30, and 40; required the area?

Answer, $20+30+40=90$ and $\frac{90}{2}=45$

$45-20=25$ and $45-30=15$ and $45-40=5$

Then $45\times25\times15\times5=84375$, whose square root $=290.47$.

$$\text{Logarithmically,}\quad \left.\begin{array}{r} 45=1.65321 \\ 25=1.39794 \\ 15=1.17609 \\ 5=0.69897 \end{array}\right\} 2 \;\left|\; \begin{array}{l} 4.9262 \\ \hline 2.46310=290.47. \end{array}\right.$$

2. A slater agreed to slate a pavilion roof at 21s. *per* square yard. It consisted of 4 triangles, each of which measured 40 feet by the base, and 45 feet by each of the oblique sides; required the slatter's charge?

$$\begin{array}{r} 40 \\ 45 \\ 45 \\ \hline 130 \\ \hline \end{array}$$

$$\left.\begin{array}{r} 65=1.81291 \\ 25=1.39794 \\ 20=1.30103 \\ 20=1.30103 \end{array}\right\} \begin{array}{l} 2) \\ =5.81291 \\ \hline \end{array} 9)$$

$2.90645=806.2.$

89.57 Guineas.

PROBLEM VI.

To meaſure a trapezium.

RULE.

Divide it into two triangles by a diagonal drawn between the two oppoſite angles that are fartheſt diſtant from each other: from the angles ſubtending the diagonal, let fall perpendiculars upon it, then will the product of the diagonal into the half ſum of the perpendiculars be the area required.

EXAMPLES.

1. There is a quadrangular park, the oppoſite ſides of which are neither parallel nor equal, and the greateſt diſtance between the oppoſite angles being meaſured with a Scots chain, was found to meaſure 25 chains 50 links, the one perpendicular meaſured 8 chains 60 links, and the other 11 chains 80 links; what is its content in land meaſure?

Anſwer, 26 acres, 1 fall, $21\frac{1}{5}$ ells.

2. There is an irregular quadrangular park, the diagonal of which meaſures 13 chains 60 links, one perpendicular 6 chains 25 links, and the other 3 chains 42 links; required the content in Engliſh meaſure?

Anſwer, 6 acres, 2 roods, 11 poles.

Note, Irregular figures of more ſides than 4, muſt be divided into trapezia and triangles, as ſeems moſt convenient, and the content computed accordingly.

PROBLEM VII.

To meaſure a regular polygon, the ſides and angles of which are all equal.

RULE.

Multiply the ſum of the ſides by half the perpendicular let fall from the centre to any of the ſides, and the product will give the area.

EXAMPLE.

The side of a hexagon is 14.6 feet, and the perpendicular 12.64 feet; required the area?

Answer, $14.6 \times 6 \times 6.32 = 553.632$.

But the area of any regular polygon may be found more easily by the help of the following

TABLE.

Sides.		Names.		
5	———	Pentagon,	———	1.720477
6	———	Hexagon,	———	2.598076
7	———	Heptagon,	———	3.633959
8	———	Octagon,	———	4.828427
9	———	Enneagon,	———	6.181827
10	———	Decagon,	———	7.694209
11	———	Endecagon,	———	8.514250
12	———	Dodecagon,	———	9.330125

RULE.

Multiply the tabular number by the square of the side for the area of the polygon.

The last Example resumed.

$$14.6 \times 14.6 \times 2.598076 = 553.80588016.$$

PROBLEM VIII.

Having the diameter of a circle to find the circumference.

RULE.

The circumference of a circle, whose diameter is 1, hath been found to be 3.141593, therefore multiply 3.141593 by the given diameter for the circumference required.

EXAMPLES.

1. The diameter of a circle is 22 feet; what is the circumference?

Answer, $3.141593 \times 22 = 69,115046$.

2. The diameter of a circular plot is 20 chains; what is the circumference?

Answer, 62.83186.

PROBLEM IX.

Having the circumference to find the diameter.

RULE.

Reverse the former rule.

EXAMPLES.

1. The circumference of a circle is 69.115046 feet; required the diameter?

Answer, 22 feet.

2. The circumference of a circle is 62.83186 chains; what is the diameter?

Answer, 20 chains.

PROBLEM X.

The diameter of a circle given to find the area.

RULE.

The area of a circle whose diameter is 1, hath been found to be .7854 nearly, and all circles are to one another, as are the squares of their diameters: therefore multiply the square of the diameter into the decimal .7854, and the product will give the area.

EXAMPLE.

The diameter of a circular plot is 20 chains; what is the area in Scots land measure?

A. R. F. Ells.

Answer, $.7854 \times 20 \times 20 = 31 : 1 : 26 : 20$.

PROBLEM XI.

The circumference of a circle given to find the area.

RULE.

Find the diameter by Problem IX., and the area will be found by Problem X.

EXAMPLE.

The circumference of a circle 69.115046 feet; required the area?

Ans. $\dfrac{69.115046}{3.141593} = 22$ diameter, and $484 \times .7854 = 380.1536$. Area.

PROBLEM XII.

The diameter of a circle given to find the side of a square equal in area thereto.

RULE.

The side of a square equal in area to a circle whose diameter is 1, hath been found to be .8862 nearly: therefore multiply that number into the given number for the side of the square required; or find the area of the circle as before, and the square root of that area will be the answer.

EXAMPLE.

The diameter of a circle measures 20 chains; what is the side of a square that will be equal in area?

Answer, $.8862 \times 20 = 17.7240$.

The area of a circle whose diameter is 20, was found to be 314.16 chains, whose square root is 17.724, as before.

PROBLEM XIII.

Having the circumference given, to find the side of a square equal in area.

RULE.

The side of a square equal in area to a circle whose circumference is 1, hath been found to be .2821: therefore, if this number be multiplied into the given circumference, the product will be the answer: or find the area of the circle, and the square root of that area will be the answer.

EXAMPLE.

The circumference of a circle is 62.83186; required the side of a square equal in area?

Answer, $62.83186 \times 2821 = 17.724$, &c.

PROBLEM XIV.

The diameter given to find the side of the inscribed square.

RULE.

The side of a square inscribed in a circle whose diameter is 1, hath been found $= .7071$: therefore the product of the diameter multiplied into 7071 will give the answer; or the square root of twice the square of the same diameter will give the answer.

EXAMPLE.

Required the side of a square that can be inscribed in a circle, whose diameter is 20?

$$\begin{array}{l} .7071 \\ \quad 20 \\ \hline 14.1420 \end{array} \qquad 10 \times 10 + 10 \times 10 = 200 \text{ whose square root } \sqrt{2} = 14.142$$

PROBLEM XV.

The circumference given to find the side of the inscribed square.

RULE.

It hath been found that if the circumference be 1, the side of the inscribed square will be .2251, wherefore the product of .2251 into the circumference will give the answer; or find the diameter, and work as before.

EXAMPLE.

The circumference of a circle is 62.83186; what is the side of an inscribed square?

Answer, $62.83186 \times .2251 = 14.143451686$.

PROBLEM XVI.

The area given to find the diameter.

RULE.

Divide the given area by .7854, and the square root of the quotient will give the diameter.

EXAMPLE.

The area of a circle is 314.16; what is the diameter?

$$\overline{\frac{314.16}{.7854}}\,\sqrt{^2} = 20$$

Note, If the circumference be required, find the diameter by the last problem, and then the circumference.

PROBLEM XVII.

The area given to find the ſide of a ſquare inſcribed.

RULE.

The area of a ſquare inſcribed in a circle, the area of which is 1 hath been found to be .6366; and therefore, if the area be multiplied by that number, the ſquare of the ſide will be produced.

EXAMPLE.

The area of a circle is 314.16; required the ſide of the inſcribed ſquare.

Anſwer, $\sqrt[2]{314.16 \times .6366} = 14.142$.

PROBLEM XVIII.

Given the ſide of a ſquare, to find the diameter of the circumſcribing circle.

RULE.

If the ſide of the ſquare be 1, the diameter of the circumſcribing circle will be 1.4142; conſequently multiply the given ſide into that number for the diameter ſought.

EXAMPLE.

What is the diameter of that circle, the ſide of whoſe inſcribed ſquare is 14.142?

Anſwer, $14.142 \times 1.4142 = 19.9996164 = 20$

PROBLEM XIX.

Having the ſide of a ſquare, to find the diameter of a circle, the content whereof ſhall be equal.

RULE.

When the side of a square is 1, it hath been found that the diameter of the equivalent circle is 1.128.

EXAMPLE.

The side of a square is 17.752; required the diameter of the equivalent circle?

Answer, $17.752 \times 1.128 = 20$, *&c.*

PROBLEM XX.

Having the side of a square, to find the circumference of a circumscribing circle.

RULE.

If the side of the square be 1, the circumscribing circle is 4.443: wherefore multiply the given side by that number for the circumference required.

EXAMPLE.

Suppose the side of the square 20 what is the circumference of the circumscribing circle

Answer, 88.86.

Note, The area of a emicircle is found by taking $\frac{1}{2}$ of the area of the circle; the area of a quadrant, by taking $\frac{1}{4}$, *&c.*

PROBLEM XXI.

To find the length of the arch line of a sector, or segment of a circle.

RULE.

Multiply the chord of half the arch by 8, and from that product ſubtract the chord of the whole arch, $\frac{1}{3}$ of the remainder will be the anſwer.

EXAMPLES.

1. Of a ſegment of a circle, the chord line of which meaſures $22\frac{1}{2}$ inches, and the chord of half the arch 13; required the length of the arch?

Anſwer, $$\frac{13 \times 8 - 22.5}{3} = 27.1\dot{6}$$

2. Of a ſegment, the chord line meaſures 34.4, and the chord of half the arch 19.8; required the length of the arch?

Anſwer, $41\frac{1}{3}$.

Or mathematically.

Find the center, complete the circle, and with a ſector, ſcale, or protractor, find the number of degrees contained in the arch; find alſo the circumference in parts: then will the arch in degrees be to the arch in parts, as 360 to the circumference in parts.

PROBLEM XXII.

To find the area of a ſector.

RULE.

A ſector is a part of a circle comprehended between two ſemidiameters that are not in the ſame ſtraight line, and may be either greater or leſs than a ſemicircle. The area of a ſector leſs than a ſemicircle is found by multiplying the arch line, found by the laſt Problem, into the ſemidiameter, then will $\frac{1}{2}$ the product be the area: and if the ſector be greater than a ſemicircle, from the area of the whole circle ſubtract the area of the leſſer ſector, and the remainder will be the area of the greater.

EXAMPLES.

1. There is a ſector whoſe ſemidiameter is 15 feet, the chord of the whole arch 22.3, and the chord of half the arch 12.5; requir-ed the area of that ſector, and alſo the area of the remaining part of the circle, which is a ſector greater than a ſemicircle?

Anſwer, 1.

$$\begin{array}{r} 12.5 \\ 8 \\ \hline 100.0 \\ 22.3 \\ \hline 3)\ 77.7 \\ \hline \text{Arch line, } 25.9 \end{array} \qquad \text{Then } \begin{array}{r} 25.9 \\ 15 \\ \hline 2)\ 388.5 \\ \hline \text{Sect. area } 194.25 \end{array}$$

2. $15 + 15 = 30$ Diameter.
and $30 \times 30 \times .7854 = 706.86$ Circle's area.
Leſſer Sector's area $= 194.25$

Greater ſector's area, 512.61

2. There is a ſector whoſe ſemidiameter is 24.5, the chord of the whole arch = 3.92, and the chord of half the arch 2.2; required the areas of the leſs and greater ſectors reſpectively?

Anſwer, the leſs, 558.6; and the greater 1327.1454.

PROBLEM XXIII.

The meaſure of the chord line, and height of a ſegment of a circle given to find the diameter of the whole circle.

RULE.

Square half the chord line, and divide the ſquare by the height, and the quotient added to the given height will be the diameter required.

EXAMPLES.

1. The chord of a ſegment is 22.3, and the height 5.4; requir-ed the diameter of the circle?

Anſ. $\dfrac{11.15 \times 11.15}{5.4} + 5.4 = 28.422$, diameter required.

2. The chord of a ſegment is 36, and height 6; required the diameter?

Anſwer, 60.

Note, The diameter may be found mathematically, by completing the circle.

PROBLEM XXIV.

To find the area of the ſegment of a circle.

RULE.

Find the diameter by the laſt Problem, then find the area of the ſector by Problem XXII. From the ſemidiameter ſubtract the height of the ſegment, and multiply the remainder into $\frac{1}{2}$ the chord line, the product ſubtracted from the area of the ſector will leave the area of the ſegment.

EXAMPLES.

1. Of a ſegment of a circle, the chord is 19, and height 4; required the area?

Anſ. 13.833 × 13.14 = 182, area of the ſector.

13.833 × 9.14 = 126.436, area of the triangle.

55.563, area of the ſegment.

2. Required the area of a ſegment, the chord line of which is 35, and height 9.6?

Anſwer, 236.06.

Note, If the ſegment be greater than a ſemicircle, find the area of the compliment as before, and ſubtract it from the area of the whole circle for the anſwer.

PROBLEM XXV.

To find the area of an ellipſis.

RULE.

An ellipsis, like the circle, is bounded by a regular curve line, which returns into itself, but its transverse diameter is longer than the conjugate. The area is found by multiplying the one diameter into the other, and that product by .7854.

EXAMP ES.

1. The transverse diameter of an ellipsis is 20, and conjugate 13. ; required the area ?

Answer, $.7854 \times 20 \times 13 = 204.204$.

2. The transverse diameter of an ellipsis is $61\frac{3}{5}$, and conjugate $44\frac{2}{5}$; required the area ?

Answer, 1948.100416.

PROBLEM XXVI.

To measure a parabola.

RULE.

A parabola is a curvilineal figure, formed from the section of a cone, cut by a plane parallel to one of its sides. It is $\frac{2}{3}$ of its circumscribing parallelogram, and therefore the area is found by taking $\frac{2}{3}$ of the product of the base and perpendicular height.

EXAMPLES.

1. There is a parabola, whose base or greatest ordinate is 30, and perpendicular height 22 ; required the area ?

Answer, $\dfrac{30 \times 22 \times 2}{3} = 440$

2. There is a parabola, whose base is 53.75, and perpendicular height 39.25 ; required the area?

Answer, 1406.4583.

MENSURATION.

II. OF SOLIDS.

SOLIDS consist of length, breadth, and thickness.

PROBLEM I.

To find the solidity of a cube.

RULE.

A cube is a solid body, comprehended under six geometrical squares as a die: therefore involve the given side to the 3d power for the solidity.

EXAMPLLE.

There is a cubic piece of mahogany which measures 3 feet 8 inches every way; required its solidity?

F. I.

Answer, 3 : 8 × 3 : 8 × 3 : 8 = 3 : 8 × 3.8 = 49 : 3⅔.

PROBLEM II.

To measure a parallelopepidon.

RULE.

Multiply length, breadth, and thickness continually.

EXAMPLE.

There is a piece of wood 25 feet 6 inches in length, 2 feet 3 inches deep, and 4 feet 8 inches broad; required the solid content?

Answer, $25.6 \times 2.3 \times 4.8 = 267$ feet 9 inches.

PROBLEM III.

To find the solidity of a triangular prism.

RULE.

Multiply the area of the base into the length of the solid.

EXAMPLES.

1. There is a triangular prism, each side of the base of which is 18 inches, and the length of the solid 20 feet 6 inches; required the solidity?

Ans. $\dfrac{1.5 \times 3}{2} = 2.25$ and $2.25 - 1.5 = 1.75$.

Then $2.25 \times 1.75 \times 1.75 \times 1.75 = 11.90724375$, the sqare root, of which is 3.465, the area of the base.

Lastly, $3.465 \times 20.5 = 71.0325$ solidity.

2. Suppose each side of the base had been 15.6 inches, and the length 19.5; required the solidity?

Ans. In this case, as in every other, the perpendicular may be found. Therefore perpendicular $= 13.51$.

$$\text{Then } \frac{13.51 \times \frac{15.6}{2} \times 19.5}{144} = 14.27 \text{ feet.}$$

PROBLEM IV.

To measure a Pyramid.

RULE.

A pyramid is a solid, whose base is a polygon, and sides plane triangles, terminating in a point at the vertex: wherefore multiply the area of the base by one third of the altitude for the solidity.

EXAMPLES.

1. There is a pyramid each side of whose square base is 8 feet, and perpendicular height 24 feet 6 inches; required the solidity?

Answer, $8 \times 8 \times 24.5 = 1568$.

2. In the triangular pyramid let each side of the base be $21\frac{1}{2}$ inches, and perpendicular height 16 feet; required the solidity?

Answer, $7.41\dot{3}$ feet.

3. Suppose the base a heptagon, of which each side is 15 inches, and perpendicular let fall from the center 15.58 inches; suppose also the perpendicular height 13 feet 6 inches; required the solidity?

Answer, 25.56 solid feet.

PROBLEM V.

To find the solid content of a cylinder.

RULE.

A cylinder is a round solid body, whose bases are circular, parallel, and equal: wherefore multiply the area of the base into the length for the solidity.

EXAMPLES.

1. The diameter of a cylinder is 4 feet 6 inches, and length 12 feet 9 inches; required its solidity

Answer, $4.5 \times 4.5 \times .7854 \times 12.75 = 202.7797$.

2. Suppose the diameter of the base only $21\frac{1}{2}$ inches, but the length 16 feet; required the solidity?

Answer, 40.34 feet.

PROBLEM VI.

To find the solidity of a cone.

RULE.

Peculiar to a cone is a circular base, which tapers gradually, and terminates in a point at the vertex. It is $\frac{1}{3}$ of its circumscribed cylinder: wherefore multiply the area of the base by $\frac{1}{3}$ of the height for the solidity.

EXAMPLES.

1. There is a cone, the diameter of whose base is $4\frac{1}{3}$ feet, and perpendicular height $15\frac{1}{3}$ feet; required the solidity.

Answer, $4\frac{1}{3} \times 4\frac{1}{3} \times .7854 \times 5\frac{1}{9} = 75.379$.

2. Suppose the base's diameter 26.5 inches, and height $16\frac{1}{2}$ feet; required the solidity?

Answer, 21.06.

PROBLEM VII.

To find the solidity of the frustum of a pyramid.

RULE.

When the top of a pyramid is so cut off, that the bases of the remainder are parallel, that remainder is said to be a frustum; therefore find the areas of the bases respectively; multiply them together; and to the square root of the product, add the two areas; then multiply that sum by $\frac{1}{3}$ of the height for the solidity.

EXAMPLES.

1. In the frustum of a square pyramid let the side of the greater base be 6 feet, and that of the lesser 4, let the height be 66 feet; required the solidity?

Answer, $4 \times 4 = 16$ and $6 \times 6 = 36$. Then $16 \times 36 = 576$. $576 \surd^{2} = 24$. Then $24 + 16 + 36 = 76$, and $76 \times 22 = 1672$.

2. In the frustum of a triangular pyramid each side of the greater base is 25 inches, each side of the lesser base 9 inches, and the length 15 feet; required the solidity?

Answer, 13.99 feet.

3. In the frustum of an octogonal pyramid, each side of the greater base is 9 inches, each side of the lesser 5, and the height 10 feet 6 inches; required the solidity?

Answer, 17.72 feet.

PROBLEM VIII.

To find the solidity of the frustum of a cone.

RULE.

If a part be struck off from the top by a plane parallel to the base, the remainder will the frustum of a cone: to find the solidity of which, to the product of the diameters of the two bases add the squares of the diameters; multiply the sum by .7854, and that product by $\frac{1}{3}$ of the height for the solidity.

EXAMPLES.

1. In the frustum of a cone the diameter of the greater base is 6 feet, and of the lesser 4, and length 12 feet; required the solidity. *Answer*, $6 \times 4 + 60 \times .7854 \times 4 = 263.8944$.

2. In the frustum of a cone, the greater diameter is 18 inches, and the lesser 9, but the length is 14 feet 3 inches; required the solidity?

Answer, 14.6775 feet.

PROBLEM IX.

To measure the frustum of a right angled prismoid, whose bases are parallel, but not in proportion.

RULE.

To the less length add half the greater, and multiply the sum by the breadth of the greater base, reserving the product. Then to the lesser length add $\frac{1}{2}$ the greater, and multiply the sum by the breadth of the lesser base, and add the product to the reserved product, then will their sum multiplied by $\frac{1}{3}$ of the height give the solidity.

EXAMPES.

1. There is a prismoid, in which the greater base is 6 feet in length, and in breadth 2; the length of the lesser base is 4 feet, and breadth $1\frac{1}{2}$; what is the solidity, the length being 12 feet?

Answer, $6 + 2 \times 2 + 4 + 3 \times 1.5 \times 4 = 92$.

2. In a prismoid the length of the greater base is 38 inches, and breadth 16; and the length of the lesser base is 30, and breadth 12; what is the solidity, supposing the height 6 feet?

Answer, 19.94 feet.

PROBLEM X.

To find the solidity of a cylindroid.

RULES.

1. This is a frustum of a cone, whose bases are parallel, but not equal. The solidity is found by adding to the longest diameter of the greater base the longest diameter of the lesser base, and multiplying the sum by the shortest diameter of the greater base, reserving the product.

2. Then to the longest diameter of the lesser base add half the diameter of the greater base, and multiply the sum by the shortest diameter of the lesser base, to which sum add the product, per 1st Rule; multiply this sum by .7854, and that product again by the height for the solidity.

EXAMPLES.

1. In a cylindroid, whose base is an oval, of which the transverse diameter is 20 feet, and the conjugate 12: But the upper base is a circle whose diameter is 15 feet. The cylindroid is 30 feet; required the solidity?

Ans. $\left.\begin{array}{ll} 20 \times 7.5 \times 12 & = 330 \\ 15 + 10 \times 15 & = 375 \end{array}\right\} \times .7854 \times 15 = 8298.105.$

2. In a cylindroid, the base being an oval, the transverse diameter is 44 inches, and conjugate 14; the diameter of the circle at top is 26 inches, and the height of the cylindroid 9 feet; required the solidity?

Answer, 33.47 feet.

PROBLEM XI.

To find the ſolidity of a ſphere or globe.

RULE.

A globe is a ſolid body, each part of the ſurface of which is equidiſtant from the central point within it. To find the ſolidity, multiply the axis, or diameter, into the circumference, and that product into $\frac{1}{6}$ of the axis for the anſwer.

EXAMPLES.

1. What is the ſolidity of the earth, whoſe axis is 7964 miles?

Cir.	$25015\frac{3}{7}$	=	4.398529
Axis	7964	=	3.901131
$\frac{1}{6}$	$1337\frac{1}{3}$	=	3.122889

264434860396.5 11.422549 ſolid miles.

Or the proportion may be as 21 : 11 :: the cube of the axis : the ſolidity.

For 7964 =	3.901131
	3
	11.703393
11 =	1.041393
	12.744786
21	1.322219
Nearly as before,	11.422567

2. There is a globe whoſe diameter meaſures 20 feet; required its ſolidity?

Anſwer, $4190\frac{1}{2}$ feet nearly, by Rule 2d.

PROBLEM XII.

To find the ſolidity of the fruſtum of a globe, the chord and height of a fruſtum given.

PROBLEM XV.

To measure a parabolic conoid.

RULE.

Multiply the square of the base's diameter by $\frac{1}{2}$ the height, and that product by .7854, the last product is the answer.

EXAMPLES.

1. Suppose the diameter of a bowl, in the form of a parabolic conoid be 20 inches, and the height 16 inches; required the solidity?

Answer, $20 \times 20 \times .7854 \times 16 = 5026.56$.

2. Supose the diameter 36 inches, and height 33 inches; required the solidity?

Answer, 16794.9936.

PROBLEM XVI.

To find the solidity of a parabolic spindle.

RULE.

Multiply the square of its greatest circle by $\frac{8}{15}$ of $.7854 = .41888$, and that product by the length for the answer.

EXAMPLES.

1. Suppose the diameter of the greatest circle 20 inches, and length 80 inches; required the solidity?

Answer, $20 \times 20 \times .41888 \times 80 = 13404.168$.

2. Let the greatest diameter be 36 inches, and length 99 inches; required the solidity?

Answer, 53744 inches nearly.

MENSURATION.

III. OF BOARDS AND TIMBER.

PROBLEM I.

I. To measure a board.

RULE I.

If the breadth at both ends be equal, multiply the length by the breadth.

EXAMPLES.

1. There is a board 13 inches long throughout, and 10 feet 9 inches long; how many feet are contained therein?

$$\begin{array}{r} 10.75 \\ 895 \\ \hline 11.645 \end{array}$$ *Answer.*

2. There is a board 13 feet by 16 inches; required the content in feet? *Answer*, $17\frac{1}{3}$ feet.

RULE II.

If the breadth at the one end be greater than the breadth at the other, add the breadths, and multiply $\frac{1}{2}$ the sum by the length for the content.

EXAMPLES.

1. A board at one end is 16 inches broad, and at the other 12, is 14 feet 6 inches in length; required the content?

$$\frac{1 : 4 + 1}{2} = \qquad \begin{array}{r} 14.5 \\ 1\frac{1}{6} \\ \hline 14.5 \\ 2.616 \\ \hline 17.116 \end{array}$$ *Answer.*

2. A board 18 feet 10 inches in length is 18 inches broad at one end, and 14 inches at the other; required the content?

Anſwer, 25.111.

3. If a board be 8 inches in breadth, what length will make a foot?

$$8\,)\,144$$

$$18 \text{ inches.}$$ *Anſwer.*

4. If a board be 14 inches in breadth, how many inches will make a foot? *Anſwer*, $10\frac{2}{7}$.

PROBLEM II.

To meaſure ſquared timber.

RULE I.

If the baſes are equal, and ſides ſtraight and parallel, multiply the area of the baſe by the length.

EXAMPLES.

1. There is a piece of mahogany 2 feet 8 inches broad, 15 inches deep, and 8 feet 6 inches long; how many ſolid feet doth it contain?

$$\begin{array}{r} 8.5 \\ 2\frac{2}{3} \\ \hline 17.0 \\ 2.833 \\ 2.833 \\ \hline 22.666 \\ 1\frac{1}{4} \\ \hline 22.666 \\ 5.666 \\ \hline 28.333 \end{array}$$ Feet.

2. There is a piece of mahogany 18 inches ſquare, and 10 feet 10 inches long; how many ſolid feet doth it contain?

Anſwer, $24.4\frac{1}{2}$.

3. What length of a piece of timber 18 inches ſquare will make a ſolid foot?

$$18 \times 18 = 324\,)\,1728 \text{ solid inches in a foot.}$$

$$5\frac{1}{3} \text{ inches.}$$ *Anſwer.*

$$\text{Or,}\quad 2.25\,)\,1.000$$

$$4 = 5\frac{1}{3} \text{ inches as before.}$$

4. If a piece of timber be 20 inches broad, and 15 deep; how much length will a solid foot require?

Answer, 5.76 inches.

RULE II.

If the bases are unequal, the best way of computing the content, is to consider such timber as the frustums of pyramids, and calculate accordingly.

EXAMPLES.

1. Given a piece of timber 30 inches square at the one end, and 20 inches square at the other, whose length is 18 feet; required the solidity?

$30 \times 20 = 600$ and $30 - 20 = 10$ diff. of the sides.

$10 \times 10 = 100$ and $\frac{100}{3} = 33 \cdot \dot{3}$

To 33·333
Add 600.

$$\frac{633 \cdot 333 \times 18}{144} = 79 \cdot 166 \text{ } \textit{Answer.}$$

2. If a piece of timber be 32 inches by 20 at the greater end, and 10 inches by 6 at the lesser end, and 24 feet long; what is its solidity in feet?

$32 \times 20 \times 6 \times 10 = 38400$ whose square root is 195·959 a mean proportional.

Then $32 \times 20 = 640$
and 6×10 60

895·959 the sum.
8 = $\frac{1}{3}$ of the length.

12) 7167.672

12) 597·306

49·775 *Answer.*

Note, The common or customary method by taking an arithmetical mean, is generally short of the truth, as will appear by resuming the last example.

$$\frac{32 + 10}{2} = 21 \text{ and } \frac{20 + 6}{2} = 13$$

$$\text{Then } \frac{21 \times 13 \times 24}{144} = 45.5 \text{ feet.}$$

Which is short of the true content 4.275 feet.

PROBLEM III.

To measure round timber.

RULE II.

If the bases be equal, it is considered as a cylinder, and the content calculated accordingly.

EXAMPLES.

1. Suppose a piece of timber 36 inches in diameter, and 20 feet long; required the solidity?

Answer, $3 \times 3 \times .7854 \times 20 = 14.1372$.

2. Supose the circumference 96 inches, and length 30 feet; to find the solidity.

Answer, $96 \times 96 \times .07958 = 733.40928$

and $\dfrac{733.40928 \times 30}{144} = 152.8$ nearly.

Note 1. The common or customary way in this case, is also erroneous, by taking $\frac{1}{4}$ of the girt for the side of a square, which will be evident by resuming the last example.

$$\frac{96}{4} = 24 \text{ inches or } 2 \text{ feet,}$$

Then $2 \times 2 = 4$ area of the base.
and $4 \times 30 = 120$ solidity.
Short of the true content 32.8 feet nearly.

Note 2. If the bases are unequal, such timber ought to be consider-ed as frustums of cones, and the content found accordingly.

EXAMPLES.

1. If a piece of timber be 12 inches in diameter at the one end, and 36 at the other, what is the solid content, supposing the length 30 feet?

3 feet	3	Then $\dfrac{2 \times 2}{3} = 1.333$
1	1	
Product 3	2 diff.	Add product 3

$$
\begin{array}{r}
4.333 \\
.7854 \\
\hline
1.444 \\
31.416 \\
\hline
3.2861 \\
30 \\
\hline
98.5830
\end{array}
\text{ *Answer.*}
$$

Note, In this case the common method is also erroneous; for let the last question be resumed.

$$
\begin{array}{r}
1 \times 1 \times .7854 = .7854 \\
3 \times 3 \times .7854 = 7.0686 \\
\hline
2\ )\ 7.854 \\
\hline
3.927 \text{ mean area.} \\
30 \\
\hline
117.810 \text{ feet, } \textit{Answer.}
\end{array}
$$

2. If a piece of timber be 136 inches in circumference at the one end, 32 inches in circumference at the other, and 21 feet long; how many solid feet of timber doth it contain?

Answer, 92 34 feet.

MENSURATION.

IV. OF ARTIFICERS WORK.

MENSURATION applied to the work of artificers, with the customs to be observed relative thereto.

I. *Of Carpenters Work.*

RULE.

The measurable branches of carpenters work consists in flooring, partitioning and roofing, all which are measured by the square of 10 feet long, and 10 feet broad = 100 square feet

PROBLEM I.

Of flooring and partitioning.

EXAMPLES.

1. If a floor be 42 feet 9 inches, by 30 feet 6 inches; how many squares of flooring doth it contain?

$$\begin{array}{r} 42.75 \\ 30\frac{1}{2} \\ \hline 21.375 \\ 128250 \\ \hline \end{array}$$

Answer, 13.03875 squares.

2. A floor 57 feet 3 inches by 28 feet 6 inches, is to be done at L. 3 : 10s. the square; how much will it cost?

Answer, 16.31625 at 70 | = L. 57 : 2 : $1\frac{1}{2}$.

3. There is a partition 91 feet 9 inches by 12 feet 6; how many squares doth it contain?

Answer, 91.75 × 12.5 = 11.46875.

4. A partition 44 feet 8 inches by 30 feet 6 inches, is to be done at L. 4 : 15s. per square; what will be the charge?

Answer, L. 64 : 14 : $2\frac{1}{2}$.

PROBLEM II.

Of roofing.

RULE.

When a roof is truly pitched, the measure is estimated from the square content of the floor, being accounted $\frac{1}{2}$ thereof.

EXAMPLES.

1. The floor of a house is 52 feet 10 inches by 35 feet 6 inches, how many squares of roofing does it contain?

$$\begin{array}{r} 52.833 \\ 35\frac{1}{2} \\ \hline 264.166 \\ 1585.000 \\ 26.416 \\ \hline 2\ )\ 1875.5833 \\ 937.7916 \\ \hline 28.13375 \end{array}$$

Answer.

2. There is a house 44 feet 9 inches, by 20 feet 3 inches, within the walls; how many squares of roofing does it contain, and what will be the charge at L. 4 : 17 : 6 per square?

Answer, 24.84 nearly.

Note, Cornices, doors, and cases, window frames, guttering, lintels, sommers, skirt boards, *&c.* are measured by the lineal foot.

II. *Bricklayers Work.*

PROBLEM I.

Tiling.

RULE.

In general for hips and vallies double measure is allowed. When double measure is allowed for gutters, measure the length along the ridge tile, and that gives double measure. Double measure is also allowed at the eves, for the projection over the plate.

EXAMPLES.

1. There is a roof covered with tiles, whoſe depth on both ſides is 40 feet 6 inches, and length 50 feet; how many ſquares of tiling will it make?

$$\frac{40.5 \times 50}{100} = 20.25 \quad \textit{Anſwer.}$$

2. There is a roof covered with tiles, 62 feet 8 inches by 30 feet in height on either ſide; how many ſquares doth it make?

Anſwer, 18.8 ſquares.

PROBLEM II.

Walling.

RULE.

In moſt caſes of this kind the thickneſs of the wall is reduced to the common ſtandard of ½ brick thick, wherefore multiply the ſuperficial content in feet by the number of bricks in thickneſs, and divide the product by 3 for ſtandard thickneſs; then divide that quotient.

For ſquare rods by 272.25 the ſquare of $16\frac{1}{2}$.
For ditto of 18 feet by 324.
And for 21 by 3 by 63.

EXAMPLES.

1. If a wall be 82 feet 6 inches long, 20 feet 3 inches high, and 5 bricks thick; how many rods of $16\frac{1}{2}$ feet doth it contain?

$$\begin{array}{r} 82.5 \\ 20\frac{1}{4} \\ \hline 20.625 \\ 16500 \\ \hline 272.25\ )\ 1670.625 \\ \hline 61.14 \end{array}$$

61.14 *Anſwer.*

2. If a wall be 245 feet 9 inches long, $16\frac{1}{2}$ feet high, and $2\frac{1}{2}$ bricks thick, how many rods of brick-work are contained therein, when reduced to ſtandard thickneſs, and as it is uſual, omitting the decimal in the diviſor and dividend? *Anſwer*, 24.84 nearly.

Note, Chimneys are reckoned at double work.

III. *Plaiſterers Work.*

RULE.

This work is meaſured by the lineal foot, and the ſquare feet reduced to ſquare yards.

EXAMPLES.

1. If a ceiling be 82 feet 9 inches long, and 32 feet 9 inches broad; how many ſquare yards doth it meaſure?

$$\frac{82.75 \times 32.75}{9} = 381.118 \text{ ſquare yards.}$$

2. The partitions between the rooms of a houſe meaſure $141\frac{1}{2}$ feet by $11\frac{1}{4}$ in height; how many ſquare yards are there?
Anſwer, 176.87.

IV. *Joiners Work.*

RULE.

Joiners, in taking their dimenſions, begin at the top, and with a ſtring, girt over all the mouldings, but in taking the perimeter, they take it as it is upon the floor.

EXAMPLES.

1. A room being meaſured over the mouldings, turned out 16 feet 8 inches in height, and the perimeter meaſured 96 feet 6 inches; how many ſquare yards did it contain?

$$\frac{96.5 \times 16\frac{2}{3}}{9} = 178.9$$

2. By the ſame kind of meaſurement, a room was found to be in compaſs 137 feet 6 inches, and in height 16 feet 3 inches; how many ſquare yards doth it contain? *Anſwer*, 248.2.

Note, In meaſuring doors, window ſhutters, and all ſuch work as is wrought on both ſides, work and half muſt be charged, conſequently, after the content is found as before, multiply by 3, and divide by 2 for the meaſurement to be charged.

IV. *Painters Work.*

RULE.

This work is meaſured every way over the mouldings, wherever the bruſh touches, and is charged by the ſquare yard at a certain rate for one, two, or three coats.

EXAMPLES.

1. A room was painted, whoſe height being taken, was found to be 18 feet 6 inches, and circumference 135 feet 9 inches, two doors 8 feet 6 inches by 5 feet 3 inches each, and 4 windows 7 feet 3 inches by 5 feet 9 inches each. The doors and windows being painted on both ſides are to be charged double; what will the painting amount to at 8d. the ſquare yard?

$$135.75 \times 18.5 = 2511.375$$
$$8.5 \times 5\tfrac{1}{4} \times 2 = 89.25$$
$$7.25 \times 5.75 \times 4 = 166.75$$

$$9\,)\,2767.375$$

$$30\,)\,307.486 \text{ ſquare yards.}$$

$$\text{L. } 10.249 \text{ charge.}$$

2. A room 22 feet 6 inches high, and 155 feet 9 inches in circumference, in which there are 4 windows, each 8 feet 3 inches by 6 feet 4 inches, and 3 doors 10 feet 3 inches by 6 feet 4 inches; how many ſquare yards doth it contain, and what will be the charge of painting it at $9\frac{1}{4}$d. per ſquare yard?

Anſwer, 434.236 = L. 17 : 12 : 10.

VI. *Glaziers Work.*

RULE.

Glaziers measure and charge their work by the foot square.

EXAMPLES.

1. A window consists of 12 panes each, measuring 2 feet $8\frac{3}{4}$ inches long, and 1 foot $4\frac{1}{4}$ inches broad; how many feet of glass doth it contain? *Answer*, $2.729 \times 1.354 \times 12 = 44.34$ square feet.

2. A window consists of 16 panes, each measuring 4 feet $5\frac{1}{2}$ inches in length, and 1 foot $4\frac{3}{4}$ inches in breadth; how many feet of glass doth it contain? *Answer*, 96 feet $6\frac{5}{6}$ inches.

VII. *Masons Work.*

RULE.

This work is measured and charged sometimes by the foot square, sometimes by the foot solid, and also sometimes by the rod 21 feet by 3.

EXAMPLES.

1. If a wall be 220 feet 6 inches long, and 12 feet $6\frac{1}{2}$ inches high; how many superficial feet doth it measure?

F. I.

Answer, $220.5 \times 12\ 6\frac{1}{2} = 2765.4375$.

2. If a wall be 97 feet 5 inches long, 18 feet 3 inches high, and 2 feet 3 inches thick; how many solid feet doth it contain?

Answer, 4000.185, &c.

3. A wall is 112 feet 3 inches by 16 feet 6 inches; how many roods does it contain?

Answer, $29\frac{52}{63}$.

THE NATURE AND USE OF LOGARITHMS.

LOGARITHMS are artificial numbers ſo proportioned to each other, and ſo adapted to the natural numbers, 1, 2, 3, 4, 5, &c. that the ſame anſwer may be obtained by addition and ſubtraction of logarithms, as by the multiplication and diviſion of their correſpondent natural numbers.

For an eaſy method of conſtructing a table of logarithms, ſee the Univerſal Acccountant. It may, however, be proper to obſerve here, that the figure of every logarithm before the ſeparating point, is the index, or characteriſtic of the logarithm, becauſe it points out the higheſt or remoteſt place of that number from unity, in the infinite ſcale of proportionals towards the left. Thus all logarithms of numbers below 10 will have 0 for an index; between 10 and 100, will have 1; between 100 and 1000, 2, &c. And in general, the index will always be one ſhort of the number of places in the natural number, correſponding to the logarithm. Hence, in any table of logarithms, it is quite unneceſſary to prefix the index ſince that is immediately diſcoverable from the number of places, of which the natural number conſiſts; and *vice verſa*, the number of integral places repreſented by the logarithm is alſo known from the index. The following examples will ſerve for an illuſtration.

Nos.			
5555	= 3·74468	·5555	= — 1·74468
555,5	= 2·74468	·05555	= — 2·74468
55,55	= 1·74468	·005555	= — 3·74468
5,555	= 0·74468	·0005555	= — 4·74468

Here it is to be obſerved, that the logarithmic part is always the ſame, and the index decreaſes, according as the number of places decreaſe. When the number falls below the common barrier, *viz.* 1, the index then becomes negative, and ſerves to demonſtrate how far the firſt ſignificant figure of the decimal is below 1. For the value of every fraction is known by dividing its number by its denominator, wherefore $.05555 = \frac{5555}{100000}$.

But if from	5555 =	3.74468
You take	100.000 —	5.00000

The remainder will be 2.74468, and so of any other.

From the nature of these negative quantities, it is evident that the addition of a negative quantity amounts to the same thing as the subtraction of a positive quantity, and *vice versa*, that the subtraction of a negative quantity is the same as the addition of an affirmative.

PROBLEM I.

To find the number corresponding to any logarithm in the table.

RULE.

Seek the three first figures in the column entitled Num., and if it consist but of three figures, its logarithm will be found in a line with it under 0. If it consist of four figures, find the first three figures as before, and the units place at the top, in the column of which, even in a line with the preceding part of the number, will the logarithm be found. If the number exceed 4 figures or places, subtract the logarithm of the first four places, from the logarithm next above it, which is always the next in order in the same line, unless the units place be 9, and then it is the first in the next column; multiply the remainder considered as a whole number, by the residue of the natural number, considered as a decimal, and the product added to the right hand side of the logarithm of the first 4 figures, with the proper index, will be the logarithm complete.

EXAMPLES.

1. Find the logarithm of 59. As the table commences at 100, look for 590, and you will find .77085, to which prefix 1, for an index, and the logarithm will be 1.77085.

2. Find the logarithm of 3784 = .57795, prefix 3 for an index, and it will be 3.57795.

3. Find the logarithm of 5. Seek for 500 = .69897, and prefix — 1, and it will be — 1.69897.

4. Find the logarithm of 34.75 = .54095 to which, as there were only 2 integral places, prefix 1 for an index, and it will be 1.54095.

5. Find the logarithm of 38545.

	3855 =	58602
	and 3854 =	58591
Remainder,		11
Rem. figure,		·5
		5·5

Therefore 3.58602 + 1.00005 = 4.58607 the logarithm required

6. Find the logarithm of 594785.

	5947 =	3·77430
	5948 =	3·77437
Remainder,		7
Rem. figure,		.85
		5·95
		77430
		5·77435 *Answer.*

PROBLEM II.

To find the number corresponding to any logarithm.

RULE.

Seek the logarithm in the table, and the first 3 figures will be in a line with it in the first column on the left, and the units place will be found at the top of the column of the logarithmic place. The numbers will be integral mixt, or a pure decimal as the index points out.

EXAMPLES.

1. What is the natural number answering to logarithm 3·76110?

The figures of the logarithms rise progressively with the numbers, wherefore look for the figure next the point 7, then for 6, and the whole logarithm will be found in a line with 576, and in the column of 9, so that the whole number will be 5769. Had the index been 2, 9 would have been a decimal, and had it been 1, the two last figures, *viz.* 69 would have been a decimal, &c.

2. What is the natural number corresponding to 1.69897?

Answer, .5.

If the index exceed 3, the natural number will exceed the bounds of the table, in which case, take the difference of the two nearest logarithms to it on each side, and also the difference between it and the lesser logarithm of the two former. To this last difference annex cyphers, and divide by the first difference, then will the quotient added to the first four figures found constitute the number required.

EXAMPLE.

Value 5.97061 $\qquad$ 3.97061 = 9345

97063 — 97058 = 5

and 97061 — 97058 = 3 $\quad$ Then $\frac{300}{5}$ = 60 and 5.97061 = 934560.

PROBLEM III.

To multiply whole or mixt numbers logarithmically.

RULE.

Add the logarithms of the numbers given.

EXAMPLES.

1. Multiply 375 into 45.

```
  375
   45
 ----
 1875
1500
-----
16875
```

375 = 2.57403
45 = 1.65321
4.22724

2. Multiply 5785 into .85.

```
  5785
   .85
 -----
 28925
46280
------
491725
```

5785 = 3.76230
.85 = — 1.92942
3.69172

3. Multiply .0375 into .085.

.0375	.0375 = — 2.57403
.085	.085 = — 2.92942
1875	— 3.50345
3000	
.0031875	

PROBLEM IV.

To divide whole or mixt numbers by logarithms.

RULE.

Subtract the logarithm of the diviſor from the logarithm of the dividend, and the remainder will be the logarithm of the quotient.

EXAMPLES.

1. Divide 934560 by 162.

934560 = 5.97061
162 = 2.20951

5769 = 3.76110

2. Divide 4917.25 by .85.

4917.25 = 3.69172
.85 = — 1.92942

5785 = 3.76230

PROBLEM V.

To reſolve queſtions in proportion logarithmically.

RULE.

Having ſtated the queſtion as before, add the terms to be multiplied, and from that ſum take the dividing term, the remainder will be the logarithm of the anſwer.

EXAMPLES.

1. Bought 374 bolls of barley, and paid L. 365, what did it cost me per chalder?

$$\begin{aligned} 374 &= 2.57287 \\ 16 &= 1.20412 \\ 365 &= 2.56229 \\ \hline 15.613 &= 1.19354 \end{aligned} \quad \textit{Answer.}$$

Note, If every figure of the first term but the last be subtracted from 9, and the last from 10, we will then have what is called the arithmetical complement, and the sum of the terms — 10 will then be the answer, as in the last example resumed.

$$\begin{array}{l} 7.42713 \\ 1.20412 \\ 2.56229 \\ \hline 1.19354 \text{ as before.} \end{array}$$

2. When Madeira wine cost L. 80 per pipe, it was retailed in dozens at 36s. what should the dozen be charged at when Madeira cost L. 95 per pipe?

$$\begin{array}{lr} 95 \text{ Ar. Com.} = & 8.12228 \\ 36 \quad\quad\quad = & 1.55630 \\ 80 & 1.90309 \\ \hline 38.16 & 1.58167 \end{array}$$

3. A garrison consisting of 3600 men have bread to serve them 35 days, at 24 ounces each day; but supposing 1200 added to their number; how much ought each man to be allowed, that the same quantity may serve 45 days?

Men. D.

State 3600 : 35 :: 24
480 : 45 —

$$\begin{aligned} 3600 &= 3.55630 \\ 35 &= 154407 \\ 24 &= 1.38021 \\ \hline & \qquad\qquad = 6.48058 \\ \left.\begin{aligned} 4800 &= 3.68124 \\ 45 &= 1.65321 \end{aligned}\right\} & \qquad\qquad = 5.33445 \\ \hline 14 \text{ Ounces} &\qquad\qquad = 1.14631 \; \textit{Ans.} \end{aligned}$$

4. If 248 men in $5\frac{1}{2}$ days dig a trench $23\frac{1}{4}$ yards long, $2\frac{1}{3}$ deep, $3\frac{2}{3}$ wide; in how many days will 24 men dig a trench $33\frac{3}{4}$ yards long, $3\frac{1}{2}$ deep, and $5\frac{1}{3}$ wide?

M.		D.		Leng.		Dep.		Breadth.
248	:	5.5	: :	23.25	: :	2.3	:	3.6
24	:	—	: :	33.75	: :	3.5	:	5.6

248 = 2.39445		24 = — 2.61979	Ar. Com.
5.5 = 0.74036		23.35 = — 2.63358	
23.75 = 1.52827		2.333 = — 1.63202	
3.5 = 0.54407		5.666 = — 1.43573	
5.6 = 0.74819		180 days = 2.27656	

Note, The arithmetical compliments are found, according to Mr. Jones, thus; increase the index by 1; change its sign, subtract each figure from 9, excepting the last, which must be subtracted from 10. But if the logarithmic part consist only of cyphers, nothing more is necessary than to change the sign of the index.

5. A bankrupt is indebted, *viz.*

	L.	s.	d.		
To A.	L. 1628	14	10	=	3.21185
To B.	362	16	8	=	2.55970
To C.	28369	14	9	=	4.45285
To D.	46	13	10	=	1.66924
To E.	9	12	$8\frac{1}{2}$	=	0.98387
To F.	436	16	10 —	=	2.64033
	30854	9	$5\frac{1}{2}$		

× — 1.79913 =

3.01098	=	1025.6087	A's share.
2.35883	=	228.4735	B's
4.25918	=	17864.2468	C's
1.46037	=	29.4015	D's
0.78300	=	6.0674	E's
2.43946	=	275.0762	F's
		19428.8741	Proofs.

Log of 19428.075	=	4.28845
Ar. Com. Log. 30854.473	=	— 5.51068
Due to L. 1		— 1.79913

6. A banker in Paris remits to his factor in Amsterdam 455 crowns Tournois; first to London at 30d. per ecu; thence to Rome at 65d. per stamped crown; thence to Venice at 100 Roman crowns per 140 ducats banco; thence to Leghorn at 100 ducats banco, per 100 piasters of Leghorn; and thence to Amsterdam at 86 groats per piastre: how many guilders banco will be received at Amsterdam without reckoning charges?

Neglecting equalities on either side, and abridging the terms, the antecedents and consequents will stand as under.

65 pence	=	1.81291
100 crowns	=	2.....
		3.81291

30 pence	=	1.47712	
140 ducats	=	2.14613	
86 grotes	=	1.93450	
455 crowns	=	2.65801	
		8.21576	
		3.81291	
			Guil. St.
25284 grotes	=	4.0285	= 632 2

PROBLEM VI.

To perform involution by logarithms.

RULE.

Multiply the logarithm of the No. by the exponent of the power required.

EXAMPLES.

1. Involve 78 to the second power, or the square.

78	=	1.89209
78		2
624	=	3.78418
546		
6084		

2. Involve 78 to the third power or cube 1.89209

$$\frac{\begin{array}{r}1.89209\\ 3\end{array}}{}$$

$$474552 = 5.67627 = 78 \times 78 \times 78$$

3. Involve 356 to the surſolid, or fifth power.

$$356 = 2.55145$$
$$5$$

$$12.75725 = 5718076875767 \quad \textit{Anſwer.}$$

PROBLEM VII.

To perform evolution by logarithms.

RULE.

Divide the logarithm of the given power by its exponent.

EXAMPLES.

1. What is the ſquare-root of 6083?

$$6084 = \frac{3.78418}{2} = 1.89209 = 78$$

2. What is the cube root of 474552?

$$474552 = 5.67627$$

$$\text{and } \frac{5.67627}{3} = 1.89029 = 78$$

3 What is the surſolid root of 5718076875776?

$$5718076875776 = 12.75725$$

$$\text{and } \frac{12.75725}{5} = 2.55145 = 356$$

TABLE I.

The amount of One Pound in any number of Years.

Yrs	3 per cent	3½ per cent	4 per cent	4½ per cent	5 per cent.	Yrs
1	1.0300000	1.0350000	1.0400000	1.0450000	1.0500000	1
2	1.0609000	1.0712250	1.1816000	1.0920250	1.1025000	2
3	1.0927270	1.1087178	1.1248640	1.1411661	1.1576250	3
4	1.1255088	1.1475230	1.1698585	1.1925186	1.2155062	4
5	1.1592740	1.1876863	1.2166529	1.2461819	1.2762815	5
6	1.1940523	1.2292553	1.2653190	1.3022601	1.3400996	6
7	1.2298738	1.2722792	1.3159317	1.3608618	1.4071004	7
8	1.2667700	1.3168090	1.3683690	1.4221006	1.4774554	8
9	1.3047731	1.3628973	1.4283118	1.4860951	1.5513282	9
10	1.3439162	1.4105987	1.4802442	1.5529694	1.6288946	10
11	1.3842338	1.4599607	1.5394540	1.6228530	1.7103393	11
12	1.4257608	1.5110686	1.6010322	1.6958814	1.7958563	12
13	1.4685337	1.5639560	1.6650735	1.7721961	1.8856491	13
14	1.5125897	1.6186945	1.7316764	1.8519449	1.9799316	14
15	1.5579674	1.6753488	1.8009435	1.9352824	2.0789281	15
16	1.6047064	1.7339860	1.8729812	2.0223701	2.1828745	16
17	1.6528476	1.7946755	1 9479005	2.1133768	2.2920183	17
18	1.7024330	1.8574892	2.0258165	2.2084787	2.4066192	18
19	1.7535060	1.9225013	2.1068491	2.3078603	2.5269502	19
20	1.8061112	1.9897888	2.1911231	2.4117140	2.6532977	20
21	1.8602045	2.0594314	2.2787680	2.5202411	2.7859625	21
22	1.9161034	2.1315115	2.3699187	2.6336520	2.9252607	22
23	1.9735865	2.2061124	2.4647155	2.7521663	3.0715237	23
24	2.0327941	2.2833284	2.5633641	2.8760138	3.2250999	24
25	2.0937779	2.3632449	2.6658363	3.0054344	3.3863549	25
26	2.1565912	2.4459585	2.7724697	3.1406790	3.5556026	26
27	2.2212890	2.5315671	2.8833685	3.2820095	3.7334563	27
28	2.2879276	2.6201719	2.9987033	3.4296999	3.9201291	28
29	2.3565655	2.7118779	3.1186514	3.5840364	4.1161356	29
30	2.4272624	2.8067937	3.2433975	3.7453181	4.3219423	30
31	2.5000803	2.9050314	3.3731334	3.9138574	4.5380394	31
32	2.5750827	3.0067075	3.5080587	4.0899810	4.7649414	32
33	2.6523352	3.1119423	3.6483811	4.2740301	5.0031885	33
34	2.7319053	3.2208603	3.7943163	4.4663615	5.2533479	34
35	2.8138624	3.3335904	3.9460889	4.6673478	5.5160153	35
36	2.8982783	3.4502661	4.1039325	4.8773784	5.7918161	36
37	2.9852266	3.5710254	4.2680898	5.0968604	6.0814069	37
38	3.0747834	3.6960113	4.4388134	5.3262192	6.3854772	38
39	3.1670269	3.8553717	4.6163659	5.5658990	6.7047511	39
40	3.2620377	3.9592597	4.8010206	5.8163645	7.0399887	40
41	3.3598989	4.0978338	4.9930614	6.0781009	7.3919881	41
42	3.4606958	4.2412579	5.1927839	6.3516154	7.7615875	42
43	3.5645167	4.3897020	5.4004952	6.6374381	8.1496669	43
44	3.6714522	4.5433416	5.6165150	6.9361229	8.5571502	44
45	3.7815958	4.7023585	5.8411756	7.2482484	8.9850077	45
46	3.8950437	4.8669411	6.0748227	7.5744196	9.4342581	46
47	4.0118950	5.0372840	6.3178156	7.9152684	9.9059710	47
48	4.1322518	5.2135889	6.5705282	8.2714555	10.4012696	48
49	4.2562194	5.3960645	6.8333493	8.6436710	10.9213331	49
50	4.3839060	5.5849268	7.1066833	9.0326362	11.4673997	50

TABLE II.

The present Value of One Pound payable at the end of any number of years.

Yrs	3 p.cent	$3\frac{1}{2}$ p.cent	4 p.cent	$4\frac{1}{2}$ p.cent	5 p.cent	Yrs
1	.9708737	.9661835	.9615384	.9569378	.9523809	1
2	.9425959	.9335107	.9245562	.9157299	.9070294	2
3	.9151416	.9019427	.8889963	.8762966	.8638376	3
4	.8884870	.8714422	.8548041	.8285613	.8227024	4
5	.8626087	.8419731	.8219271	.8024510	.7835261	5
6	.8374842	.8135006	.7903145	.7678957	.7462154	6
7	.8130915	.7859909	.7599178	.7348284	.7106813	7
8	.7894072	.7594115	.7306902	.7031851	.6768393	8
9	.7664167	.7337309	.7025867	.6729044	.6446089	9
10	.7440939	.7089188	.6755641	.6439276	.6139132	10
11	.7224212	.6849457	.6495809	.6161987	.5846792	11
12	.7013798	.6617883	.6245970	.5896638	.5568374	12
13	.6809513	.6394041	.6005740	.5642716	.5303213	13
14	.6611178	.6177817	.5774750	.5399728	.5050679	14
15	.6418619	.5968906	.5552645	.5167204	.4810171	15
16	.6231669	.5767059	.5339081	.4944693	.4581115	16
17	.6050164	.5572037	.5133732	.4731763	.4362966	17
18	.5873946	.5833611	.4936281	.4528003	.4155206	18
19	.5702860	.5201556	.4746424	.4333017	.3957339	19
20	.5536757	.5025658	.4563869	.4146428	.3768894	20
21	.5375492	.4855709	.4388336	.3967874	.3589423	21
22	.5218925	.4691506	.4219553	.3797008	.3418498	22
23	.5066917	.4532856	.4057263	.3633501	.3255713	23
24	.4919337	.4379571	.3901214	.3477034	.3100679	24
25	.4776055	.4231469	.3751168	.3327306	.3953027	25
26	.4636947	.4088376	.3606892	.3184024	.2812407	26
27	.4501890	.3950122	.3468165	.3046913	.2678483	27
28	.4370767	.3816543	.3334774	.2915706	.2550936	28
29	.4243463	.3687481	.3206514	.2790150	.2429463	29
30	.4119867	.3562784	.3083186	.2670000	.2313774	30
31	.3999871	.3442303	.2964602	.2555024	.2203594	31
32	.3883370	.3325897	.2850579	.2444999	.2098661	32
33	.3770262	.3213427	.2740941	.2339712	.1998725	33
34	.3660449	.3104760	.2635520	.2238958	.1903548	34
35	.3553834	.2999768	.2534154	.2142544	.1812902	35
36	.3450324	.2898327	.2436687	.2050281	.1726574	36
37	.3349829	.2800316	.2342968	.1961992	.1644356	37
38	.3252261	.2705619	.2252854	.1877504	.1566053	38
39	.3157535	.2614125	.2166206	.1796654	.1491479	39
40	.3065568	.2525724	.2082890	.1719287	.1420456	40
41	.2976280	.2440313	.2002779	.1645250	.1352816	41
42	.2889592	.2357791	.1925749	.1574402	.1288396	42
43	.2805429	.2278059	.1851682	.1506605	.1227044	43
44	.2723717	.2201023	.1780463	.1441727	.1168613	44
45	.2644386	.2126592	.1711984	.1379643	.1112965	45
46	.2567365	.2054678	.1646138	.1320233	.1059966	46
47	.2492587	.1985196	.1582825	.1263381	.1009492	47
48	.2419988	.1918064	.1521947	.1208977	.0961421	48
49	.2349502	.1853202	.1463411	.1156915	.0915639	49
50	.2281070	.1790533	.1407126	.1107096	.0872037	50

TABLE III.

The amount of One Pound per annum for any number of years.

Yrs	3 per cent	3 ½ per cent	4 per cent	4 ½ per cent	5 per cent	Yrs
1	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1
2	2.0300000	2.0350000	2.0500000	2.0500000	2.0500000	2
3	3.0909000	3.1062250	3.1216000	3.1370250	3.1525000	3
4	4.1836270	4.2149428	4.2464640	4.2781911	4.3101250	4
5	5.3091358	5.3624658	5.4163225	5.4707097	5.5256312	5
6	6.4684098	6.5501521	6.6329754	6.7168916	6.8019128	6
7	7.6624621	7.7794075	7.8982944	8.0191517	8.1420084	7
8	8.8923360	9.0516867	9.2142262	9.3800136	9.5491088	8
9	10.1591061	10.3684958	10.5827953	10.8021142	11.0265643	9
10	11.4638793	11.7313931	12.0061071	12.2882093	12.5778925	10
11	12.8077956	13.1419919	13.4863514	13.8411787	14.2067871	11
12	14.1920295	14.6019616	15.0258054	15.4640318	15.9171265	12
13	15.6177904	16.1130303	16.6268376	17.1599132	17.7129828	13
14	17.0863241	17.6769863	18.2919111	18.9321093	19.5986319	14
15	18.5989138	19.2956808	20.0235876	20.7840542	21.5785635	15
16	20.1568813	20.9710297	21.8245311	22.7193367	23.6574917	16
17	21.7615877	22.7050157	23.6975123	24.7417068	25.8403663	17
18	23.4144353	24.4996913	25.6454128	26.8550837	28.1323846	18
19	25.1168684	26.3571805	27.6712294	29.0635624	30.5390039	19
20	26.8703744	28.2796818	29.7780785	31.3714227	33.0659541	20
21	28.6764857	30.2694706	31.9692017	33.7831368	35.7192518	21
22	30.5367803	32.3289021	34.2479697	36.3033779	38.5052144	22
23	32.4528837	34.4604137	36.6178885	38.9370299	41.4304751	23
24	34.4264702	36.6665282	39.0826041	41.6891963	44.5019988	24
25	36.4592643	38.9498566	41.6459083	44.5652101	47.7270988	25
26	38.5530422	41.3131016	44.3117446	47.5706446	51.1134537	26
27	40.7096325	43.7590602	47.0842144	50.7113236	54.6691264	27
28	42.9309225	46.2906273	49.9675829	53.9933331	58.4025827	28
29	45.2188502	48.9107993	52.9662863	57.4230331	62.3227119	29
30	47.5754157	51.6226272	56.0849377	61.0070696	66.4388475	30
31	50.0026781	54.4294709	59.3283352	64.7523877	70.7607898	31
32	52.5027585	57.3345024	62.7014686	68.6662452	75.2988293	32
33	55.0778412	60.3412100	66.2095274	72.7562262	80.0637708	33
34	57.7301765	63.4531524	69.8579085	77.0302564	85.0669593	34
35	60.4620818	66.9740127	73.6522248	81.4966180	90.3203073	35
36	63.2759442	70.0076031	77.5983138	86.1639658	95.8363227	36
37	66.1742225	73.4578694	81.7022464	91.0413442	101.6281388	37
38	69.1594492	77.0288694	85.9703362	96.1382047	107.7095457	38
39	72.2342327	80.7249060	90.4091497	101.4644239	114 0950230	39
40	75.4012597	84.5502777	95.0255157	107.0303230	120.7997742	40
41	78.6632975	88.5095374	99.8265363	112.8466875	127.8397629	41
42	82.0231964	92.6073712	104.8195978	118.9247885	135.2317510	42
43	85.4838923	96.8486292	110.0123817	125.2764040	142.9933386	43
44	89.0484091	101.2383313	115.4128769	131.9138422	151.1430055	44
45	92.7198613	105.7816729	121.0293930	138.8499651	159.7001558	45
46	96.5014572	110.4840314	126.8705677	146.0982135	168.6851636	46
47	100.3965009	115.3509725	132.0453904	153.6726331	178.1194218	47
48	104.4083959	120.3882565	139.2632060	161.5879016	188.0253929	48
49	108.5406473	125.6018455	145.8337343	169.8593572	198.4266625	49
50	112.7968672	130.9979101	152.6670836	178.5030282	209.3479957	50

TABLE IV.

The present Value of One Pound per annum for any number of years to come.

Yrs	3 per cent	3½ per cent	4 per cent	4½ per cent	5 per cent	Yrs
1	.9708737	.9661835	.9615384	.9569378	.9523809	1
2	1.9134696	1.8996942	1.8860946	1.8726677	1.8594104	2
3	2.8286113	2.8016369	2.7750910	2.7489643	2.7232480	3
4	3.7170984	3.6730792	3.6298952	3.5875257	3.5459505	4
5	4.5797071	4.5150523	4.4518223	4.3899767	4.3294766	5
6	5.4171914	5.3285530	5.2421360	5.1578724	5.075[illegible]	6
7	6.2302829	6.1145439	6.0020546	5.8927009	5.7863734	7
8	7.0196921	6.8739555	6.7327448	6.5958860	6.463[illegible]127	8
9	7.7861089	7.6076865	7.4353316	7.2687904	7.1078216	9
10	8.5302028	8.3166053	8.1108957	7.9127181	7.7217349	10
11	9.2526241	9.0015510	8.7604767	8.5289169	8.3064142	11
12	9.9540039	9.6633343	9.3850737	9.1185807	8.8632516	12
13	10.6349553	10.3027384	9.9856478	9.6828524	9.3935729	13
14	11.2960731	10.9205202	10.5631229	10.2228252	9.8986409	14
15	11.9379350	11.5174108	11.1183874	10.7395457	10.3796580	15
16	12.5611020	12.0941168	11.6522956	11.2340150	10.8377695	16
17	13.1661184	12.6513205	12.1656688	11.7071914	11.2740662	17
18	13.7535130	13.1896817	12.6592969	12.1599918	11.6895869	18
19	14.3237990	13.7098374	13.1339394	12.5932935	12.0853208	19
20	14.8774748	14.2124033	13.5903263	13.0079364	12.4622103	20
21	15.4150241	14.6979742	14.0291599	13.4047238	12.8211527	21
22	15.9369166	15.1671248	14.4511153	13.7844247	13.1630025	22
23	16.4436083	15.6204104	14.8568416	14.1477748	13.4885738	23
24	16.9355421	16.0583676	15.2469631	14.4954783	13.7986417	24
25	17.4131476	16.4815145	15.6220799	14.8282089	14.0939455	25
26	17.8768423	16.8903522	15.9827691	15.1466114	14.3751853	26
27	18.3270314	17.2853645	16.3295875	15.4513028	14.6430336	27
28	18.7641082	17.6670188	16.6630632	15.7428735	14.8981272	28
29	19.1884545	18.0357670	16.9837146	16.0218885	15.1410735	29
30	19.6004413	18.3920454	17.2920333	16.2888885	15.3724510	30
31	20.0004284	18.7362757	17.5884935	16.5443909	15.5928105	31
32	20.3887655	19.0688654	17.8735515	16.7888908	15.8026766	32
33	20.7657917	19.3902081	18.1476456	17.0228620	16.0025492	33
34	21.1318366	19.7006842	18.4111977	17.2467579	16.1929040	34
35	21.4872200	20.0006610	18.6646132	17.4610124	16.3741942	35
36	21.8322524	20.2904938	18.9082819	17.6660405	16.5468517	36
37	22.1672354	20.5705254	19.1425788	17.8622397	16.7112873	37
38	22.4924615	20.8410873	19.3678642	18.0499902	16.8678927	38
39	22.8082151	21.1024998	19.5844848	18.2296557	17.0170406	39
40	23.1147719	21.3550723	19.7927738	18.4015844	17.1590863	40
41	23.4123999	21.5991037	19.9930518	18.5661094	17.2943679	41
42	23.7013591	21.8348828	20.1856267	18.7235497	17.4232075	42
43	23.9819021	22.0626887	20.3707949	18.8742102	17.5459119	43
44	24.2542738	22.2827910	20.5488412	19.0183830	17.6627733	44
45	24.5187125	22.4954502	20.7200397	19.1563474	17.7740698	45
46	24.7754490	22.7009181	20.8846535	19.2883707	17.8800665	46
47	25.0247078	22.8994378	21.0429361	19.4147088	17.9810157	47
48	25.2667066	23.0912442	21.1951308	19.5356065	18.0771578	48
49	25.5016568	23.2765644	21.3414720	19.6512981	18.1687217	49
50	25.7297639	23.4556178	21.4821846	19.7620077	18.2559254	50

TABLE V.

The Annuity which One Pound will purchase for any number of years to come.

Yrs	3 per cent	3½ per cent	4 per cent	4½ per cent	5 per cent	Yrs
1	1.0300000	1.0350000	1.0400000	1.0450000	1.0500000	1
2	1.5226108	1.5264004	1.5301960	1.5339975	1.5378048	2
3	1.3535303	1.3569341	1.3603485	1.3637733	1.3672085	3
4	1.2690270	1.2722511	1.2754900	1.2787436	1.2820118	4
5	1.2183545	1.2214813	1.2246271	1.2277916	1.2309748	5
6	1.1845975	1.1876682	1.1907619	1.1938783	1.1970174	6
7	1.1605063	1.1635444	1.1666096	1.1697014	1.1728198	7
8	1.1424563	1.1454766	1.1485278	1.1516096	1.1547218	8
9	1.1284338	1.1314460	1.1344929	1.1375744	1.1406900	9
10	1.1172305	1.1202413	1.1232909	1.1263788	1.1295045	10
11	1.1080774	1.1110919	1.1141490	1.1172481	1.1203888	11
12	1.1004620	1.1034839	1.1065521	1.1096661	1.1128254	12
13	1.0940295	1.0970615	1.1001437	1.1032753	1.1064557	13
14	1.0885263	1.0915707	1.0946689	1.0978203	1.1010239	14
15	1.0837665	1.0868250	1.0899411	1.0931138	1.0963422	15
16	1.0796108	1.0826848	1.0858200	1.0890153	1.0922699	16
17	1.0759525	1.0790431	1.0821985	1.0854175	1.0886991	17
18	1.0727087	1.0758168	1.0789933	1.0822369	1.0855462	18
19	1.0698138	1.0729403	1.0761386	1.0794073	1.0827450	19
20	1.0672157	1.0703610	1.0735817	1.0768761	1.0802425	20
21	1.0648717	1.0680365	1.0712801	1.0746005	1.0779961	21
22	1.0627473	1.0659320	1.0691988	1.0725456	1.0759705	22
23	1.0608139	1.0640188	1.0673090	1.0706824	1.0741368	23
24	1.0590474	1.0622728	1.0655868	1.0689870	1.0724709	24
25	1.0574278	1.0606740	1.0640119	1.0674390	1.0709524	25
26	1.0559382	1.0592054	1.0625673	1.0660213	1.0695643	26
27	1.0545642	1.0578524	1.0612385	1.0647194	1.0682918	27
28	1.0532923	1.0566026	1.0600129	1.0635208	1.0671225	28
29	1.0521146	1.0554453	1.0588799	1.0624146	1.0660455	29
30	1.0510192	1.0543713	1.0578301	1.0613915	1.0650514	30
31	1.0499989	1.0533724	1.0568553	1.0604434	1.0641321	31
32	1.0490466	1.0524415	1.0559485	1.0595632	1.0632804	32
33	1.0481561	1.0515724	1.0551035	1.0587445	1.0624900	33
34	1.0473219	1.0507596	1.0543147	1.0579819	1.0617554	34
35	1.0465392	1.0499983	1.0535773	1.0572704	1.0610717	35
36	1.0458037	1.0492841	1.0528868	1.0528868	1.0604344	36
37	1.0451116	1.0486132	1.0522395	1.0522395	1.0598397	37
38	1.0444593	1.0479821	1.0516319	1.0554016	1.0592842	38
39	1.0438438	1.0473877	1.0510608	1.0548556	1.0587646	39
40	1.0432623	1.0468272	1.0505234	1.0543431	1.0582781	40
41	1.0427124	1.0462982	1.0500173	1.0538615	1.0578222	41
42	1.0421916	1.0457982	1.0495402	1.0534086	1.0573947	42
43	1.0416981	1.0453253	1.0490898	1.0529823	1.0569933	43
44	1.0412298	1.0448776	1.0486645	1.0525807	1.0566162	44
45	1.0407851	1.0444534	1.0482624	1.0522020	1.0562617	45
46	1.0403625	1.0440510	1.0478820	1.0518447	1.0559282	46
47	1.0399605	1.0436691	1.0475218	1.0515073	1.0556142	47
48	1.0395777	1.0433064	1.0471806	1.0511885	1.0553184	48
49	1.0392131	1.0429616	1.0468571	1.0508872	1.0550396	49
50	1.0388654	1.0426337	1.0465502	1.0506021	1.0547767	50

THE END.

ERRATA.

Page 48 Queſt. 8 read to A 25 butts, 1 barrel, 11 gallons, to B 13 butts, 1 barrel, 10 gallons.

52 8 Anſ. 552960.

54 7 Anſ. L. 2 : 7 : $11\frac{1}{2}$.

55 10 Anſ. 378 acres, 2 roods, 22 poles, $12\frac{1}{2}$ ſq. yards, $6\frac{3}{4}$ ſq. feet.

57 8 Anſ. $94782\frac{6}{37}$.

ib. 13 for L. 676 : 1 : 3, read L. 676 : 19 : 3.

ib. 14 Anſ. 573 acres, 0 ſq. roods, 9 ſq. poles, $29\frac{3051}{3148}$ ſq. yards.

62 1 r. $5 \times 11 + 1 = 56$

64 4 Case VI. Anſ. $\frac{61326}{8}$

92 3 Var. 3. Quot .3852500.

104 2 for 2 r. 1 = .05.

121 2 r. $18 \times 24 = 432$.

162 2 Case II. for $7\frac{5}{9}$, r. $7\frac{7}{9}$.

170 4 Anſ. L. 730 : 9 : $7\frac{1}{4}$.

175 3 for ſch, r. ſh.

176 5 Anſ. L. 907 : 8 : 10 Ster.

177 2 Anſ. L. 42 : 6 : $6\frac{3}{4}$ coſt.

ib. 4 Anſ. 1449.275 guilders.

179 2 Anſ. 1754 millr. 473 reas.

ib. 3 Anſ. 1246 millr. 298 reas.

180 4 Anſ. 3743 piaſt. 6 rials, prime coſt in Spain.

189 2 for political, r. arbitrated par.

190 2 Anſ. $568\frac{328}{779}$.

193 1 Anſ. $754\frac{38}{53}$.

ib. 2 Anſ. $444\frac{4}{9}$.

212 2 for 2297, r. 2997.

217 4 Anſ. 7073276.

220 Prob. II. for year, r. years.

ib. 2 Anſ. L. 319.972.

224 5 for 11.38, r. 11.83.

ib. 5 for in reverſion, r. and reverſion.

230 2 Anſ. L. 2845.52.